A Review on Poly-Phase Coded Waveforms for MIMO Radar with Increased Orthogonality

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Abstract. Multiple input multiple output (MIMO) radar efficiently works with orthogonal signals. Designing of these orthogonal signals affect radar parameters such as range resolution, angle resolution, doppler resolution etc. The paper represents survey of various optimization algorithms used for designing polyphase waveforms. Autocorrelation main lobe width and side lobe peak influence the pulse compression goodness of a code. Autocorrelation side lobe peak(ASP) and cross correlation peak(CP) parameters of code sets are compared.

Keywords: MIMO radar · Optimization · Polyphase waveform

1 Introduction

Compared to standard phased-array radar systems, MIMO radar systems being capable of sending an independent waveforms from various transmit antennas, offer more degrees of freedom which leads to improved angular resolution and parameter identifiability [2], and provides more flexibility for transmit beam pattern design. Parameter identifiability increases M_t times as compared to phased array radar where M_t is number of transmit antennas. The estimation of several target parameters such as range, Doppler, and Direction-of-Arrival (DOA) etc. are the main issue of interest. Since the information of the targets is obtained from the echoes of the transmitted signals, it is straightforward that the design of the waveforms plays an important role in the system accuracy.

The resolution,distance,characteristics of the received signal etc. strongly depends on the shape of the pulse. To achieve better performance in the context of resolution, pulse compression techniques can be employed. Digital pulse compression techniques such as binary phase codes, polyphase codes and frequency codes are judged by their autocorrelation properties [8]. Mutually orthogonal waveforms should be transmitted through various transmit antennas in order to avoid interference as well as acquiring independent information from various target returns.

Our aim is to achieve high range resolution and multiple target resolution. Hence we must concentrate on designing sequences with good autocorrelation and cross-correlation properties. Autocorrelation side lobe peak (ASP) and cross-correlation (CP) must be as small as possible. Smaller ASP contribute towards

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reduction in the probability of false alarm, while a narrower main lobe enhances range resolution. Cross correlation property has a significant role in lowering the probability of intercept [3].

The rest of the paper is organized as follows. In Sect. 2 polyphase waveform design problem is formulated. Literature survey of Optimization Algorithms are introduced in Sect. 3. Simulation results are presented in Sect. 4 and finally Sect. 5 concludes the paper.

2 Polyphase Coded Waveform

The designed complex phase waveforms should have a property of constant modulus over all time duration. Each waveform with code length N consists of N samples. The l^{th} waveform of the set of L orthogonal polyphase waveforms is represented by

$$s_l(t) = e^{2\pi j \Phi_l(n)/M} \tag{1}$$

where $\Phi_l(n) \in (0 \leq \Phi_l(n) \leq (M-1))$ with M distinct phases, $l = 1, 2, 3, ..., L$ and $n = 1, 2, 3, ..., N$. L represents the maximum number of radar stations can be accommodated in the radar system. The Autocorrelation function of polyphase sequence s_l with discrete time index k can be represented as

$$\begin{aligned} A(\Phi_l, k) &= \frac{1}{N} \sum_{n=1}^{N-k} e^{j[\Phi_l(n) - \Phi_l(n+k)]} = 0, \quad for \qquad 0 < k < N \\ &= \frac{1}{N} \sum_{n=-k+1}^{N} e^{j[\Phi_l(n) - \Phi_l(n+k)]} = 0, \quad for \qquad -N < k < 0 \end{aligned} \tag{2}$$

Similarly cross correlation function of sequences s_p and s_q is given as

$$\begin{aligned} C(\Phi_p, \Phi_q, k) &= \frac{1}{N} \sum_{n=1}^{N-k} e^{j[\Phi_q(n) - \Phi_p(n+k)]} = 0, \quad for \qquad 0 \leq k < N \\ &= \frac{1}{N} \sum_{n=-k+1}^{N} e^{j[\Phi_q(n) - \Phi_p(n+k)]} = 0, \quad for \qquad -N < k < 0 \end{aligned} \tag{3}$$

The orthogonal waveforms should be chosen which have low autocorrelation side lobe peak and low cross correlation peak. It is very difficult to design three or more polyphase code sets which are having low cross correlation. Different optimization algorithms have been applied previously to not only minimize ASP and CP but also minimize total autocorrelation side lobe energy and cross correlation energy.

The cost function for optimization problem of minimizing ASP and CP given by Deng [4] is as follows:

$$CF_1 = \sum_{l=1}^{L} \max_{k \neq 0} |A(\Phi_l, k)| + \lambda \sum_{p=1}^{L-1} \sum_{q=p+1}^{L} \max_{k} |C(\Phi_p, \Phi_q, k)| \tag{4}$$

Where λ represents weighing factor between autocorrelation and cross correlation function. He carried simulation results by giving equal weight to both the functions. Simulation results resembles that the location of ASP and CP varies with sequences in the optimization process producing abnormal results. For maintaining stability in optimization process, Deng [4] uses cost function considering total energy of autocorrelation side lobes and cross correlation function given by

$$CF_2 = \sum_{l=1}^{L}\sum_{k=1}^{N-1} |A(\Phi_l, k)|^2 + \lambda \sum_{p=1}^{L-1}\sum_{q=p+1}^{L}\sum_{k=-(N-1)}^{N-1} |C(\Phi_p, \Phi_q, k)|^2 \tag{5}$$

He carried out a statistical simulated annealing (SA) algorithm for minimizing CF_2. Results showed that ASP reduces at a rate of $1/\sqrt{N}$ for larger value of $N(> 400)$ allowing more degrees of freedom to minimize cost function. Liu et al. [6] applied Genetic Algorithm (GA)to minimize the following cost function which is side lobe peak and energy based function.

$$\begin{aligned} CF = &\sum_{l=1}^{L}\max_{k\neq 0} |A(\Phi_l, k)| + \sum_{p=1}^{L-1}\sum_{q=p+1}^{L}\max_{k} |C(\Phi_p, \Phi_q, k)| \\ &+ \sum_{l=1}^{L}\sum_{k=1}^{N-1} |A(\Phi_l, k)|^2 + \sum_{p=1}^{L-1}\sum_{q=p+1}^{L}\sum_{k=-(N-1)}^{N-1} |C(\Phi_p, \Phi_q, k)|^2 \end{aligned} \tag{6}$$

For the given values of N, L, and M equally weighted objective functions in Eq. (6) can be minimized. The generated polyphase sequences are automatically constrained by Eqs. (1) and (3).

3 Optimization of MIMO RADAR Waveforms

The population-based optimization algorithms to find near-optimal solutions to the difficult multi-objective optimization problems are used in the literature. Population-based algorithms has advantage of employing fewer control parameters. Aubry et al. [1] reported case studies to design waveforms which maximizes the detection probability considering scenarios like signal-dependent or signal-independent interference. Integrated side lobe and peak side lobe of the cross correlation function for different iterations were presented. Mathematical operations required for the calculation of cost function and constraint function are formulated in [9]. The cost function containing integrated side lobe level ratio and peak side lobe level ratio was optimized using genetic algorithm in [7].

As observed from the Table 1, if more weight ($\lambda = 2$) is given to cross correlation energy in the cost function and all other parameters remained same, then reduction in CP value by 15% is observed as expected. But at the same time ASP value has increased by 13.33% [4]. ASP and CP are also inversely proportional to code length N. Slight improved results can be observed using Genetic Algorithm followed by iterative search method [6]. Iterative search continues till

Table 1. Comparison of optimization algorithms

Author	Year	Algorithm	Parameters	Avg. ASP		Avg. CP	
				Normalized value	in dB	Normalized value	in dB
Deng [4]	2004	Integration of SA with traditional iterative code selection	$L = 4, M = 4, N = 40, \lambda = 1$	0.15	−16.5	0.2	−14
			$L = 4, M = 4, N = 40, \lambda = 2$	0.17	−15.39	0.17	−15.39
			$L = 3, M = 4, N = 128, \lambda = 1$	0.0895	−20.96	0.1113	−19.07
Liu et al. [6]	2006	Integration of GA with traditional iterative code selection	$L = 4, M = 4, N = 40, \lambda = 1$	0.147	−16.7	0.2078	−13.64
Reddy and Uttarakumari [10,11]	2013	PSO	$L = 4, M = 4, N = 40, \lambda = 1$	0.1384	−17.1	0.2018	−13.9
	2014	Modified Ant colony algorithm	$L = 4, M = 4, N = 40, \lambda = 1$	0.129	−17.76	0.2068	−13.68
			$L = 4, M = 4, N = 256, \lambda = 1$	0.0039	−48.16	0.00139	−57.14

no phase change is observed. Reddy and Uttarakumari [10] worked out Particle Swarm Optimization (PSO) and got the improved ASP as well as CP, which reduced by 6% and 3% respectively as compared to [6].

Modified ant colony algorithm in which optimized sequence is followed by hamming scan algorithm. It looks for all hamming neighbours of the sequence which further reduces the objective function. By increasing the code length from 40 to 256, reduction in ASP from −17.76 dB to −48.16 dB and CP from −13.68 dB to −57.14 dB is observed. Increasing the code length significantly improves the result but also increases time complexity.

4 Simulation Results

Simulations are carried out in MATLAB by considering the optimized sequences with 4 code sets [11] having orthogonality property. Polyphase waveform with 4 phases $\{0, \pi/2, \pi, 3\pi/2\}$ and code length of 40 is considered. Figure 1 shows Autocorrelation function of every code set, while Fig. 2 describes Autocorrelation function in decibels(dB). ASP and CP are normalized with respect to sequence length.

The pulse compression goodness of a code is decided by its autocorrelation function since in the absence of noise, the output of the matched filter is proportional to the code autocorrelation. The main lobe width (compressed pulse width) and the side lobe levels for the given autocorrelation function of a certain code are the two factors that need to be considered in order to evaluate the codes pulse compression characteristics.

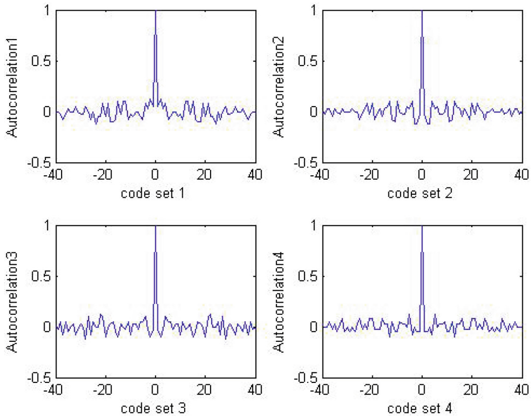

Fig. 1. Autocorrelation function for $L = 4, M = 4, N = 40$.

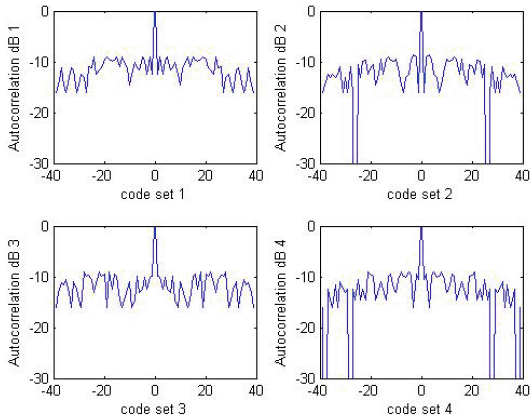

Fig. 2. Autocorrelation function in dB.

The signal design problem for MIMO radar is to generate a pulse with a sharp autocorrelation and low cross correlation. The average ASP from Fig. 1 is 0.129 indicating high probability of detecting weak targets and clutter with the main target. This is because main lobe of the autocorrelation function of weak target or clutter may hide behind side lobe peak of main target which infers miss detection.

The cross correlation function and it's dB equivalents are plotted in Figs. 3 and 4 respectively. The designed waveforms for MIMO radar should have even lower cross correlation in order to detect multiple targets [5]. Cross-correlation peak (CP) gives measure of orthogonality between signals from different antennas. Lesser the CP, lesser the interference between waveforms of different antennas.

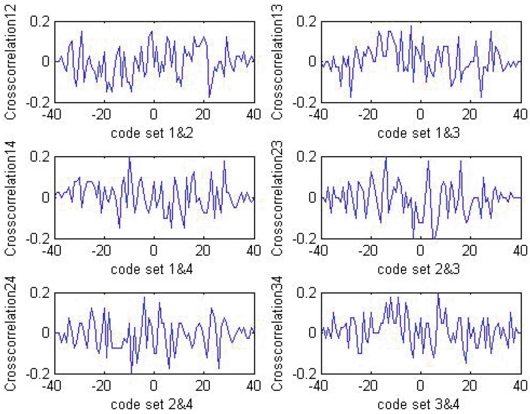

Fig. 3. Crosscorrelation function between code sets.

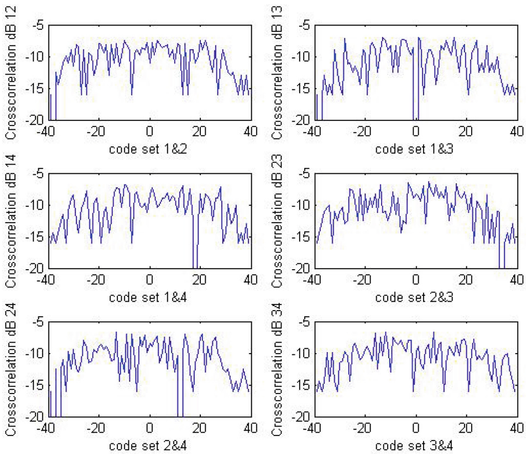

Fig. 4. Crosscorrelation function in dB.

5 Conclusion

The paper compares optimization algorithms (PSO, GA, SA, Modified ant colony) for designing of polyphase waveforms used in MIMO radar. Modified ant colony algorithm [11] appears to be the best among all as far as ASP is concerned. The gradual decrement was observed in ASP and CP values as code length increases. Providing more weights to either ASP or CP in the cost function also improves their values but at the cost of other.

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