

# Compressive Sensing Based Image Reconstruction

Sherin C. Abraham<sup>1</sup>, Ketki Pathak<sup>2(✉)</sup>, and Jigna J. Patel<sup>1</sup>

<sup>1</sup> Electronics & Communication Department, Dr. S. & S. S. Ghandhy Government College,  
Surat, India

sherincheeranabraham@gmail.com, jigna2012me@gmail.com

<sup>2</sup> Sarvajani College of Engineering and Technology, Surat, India  
ketki.joshi@scet.ac.in

**Abstract.** Compressive Sensing is novel technique where reconstruction of an image can be done with less number of samples than conventional Nyquist theorem suggests. The signal will pass through sensing matrix wavelet transformation to make the signal sparser enough which is a criterion for compressive sensing. Different levels of wavelet decomposition are also analyzed in this paper. The performance further can be improved by using DARC prediction method. The prediction error signal transmitted through OFDM channel. The reconstructed image should be better in both PSNR and bandwidth. Medical field especially in MRI scanning, compressive sensing can be utilized for less scanning time.

**Keywords:** Compressive sensing · Wavelet transform · Sparsity  
DARC prediction · Predictive coding · LZW encoder

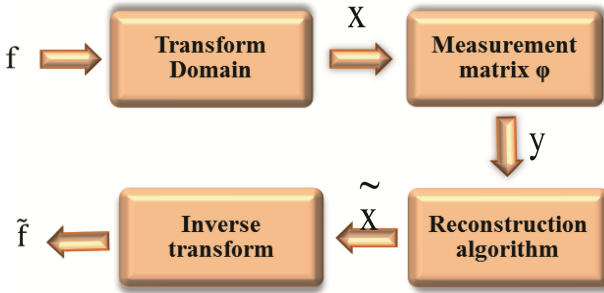
## 1 Introduction

Compressive sensing (CS) is new compression technique where with fewer samples of measurements is enough to reconstruct the image with good visual quality. The samples required are much lesser than Nyquist criterion suggests, but reconstruction is more complex in CS whereas linear in conventional compression. Now CS is actively researched in applications like MRI, RADAR, single pixel camera, etc. [1].

We consider the application in medical field. MRI is slow process due to large number of data need to be collected while scanning a patient. With the help of CS we can reduce the number of samples or skip certain acquisitions, which will benefit patient with less radiation exposure since scan time reduction is exactly proportional to the degree of under-sampling [2].

In CS, there are three main principles – Sparsity, measurements taking and nonlinear reconstruction. The signal should be sparse – Information rate contained in the image should be much less than bandwidth - to undergo CS. If it's not sparse enough; we need to undergo the transformation of the image to make it sparse. We took wavelet transform as sparsity inducing matrix in this paper. The reconstruction of signals from lesser samples can only possible if the chosen sparsity matrix and measurement matrix follows Restricted Isometric Property. The incoherence between these matrices is necessary for this. There are two approaches for reconstructing image at receiver side – basis pursuit

and greedy algorithm. These nonlinear techniques will result in good quality reconstructed image [3]. In short, CS can able to reduce sampling and computation costs for sensing signals that have a sparse or compressible representation. The sampling or acquisition method is by multiplying the sparse signal with the measurement matrix as in Fig. 1.



**Fig. 1.** Basic block diagram of compressive sensing

The channel considered as OFDM, because of its wide use in communication field. The OFDM technique is used for high data rate wireless communication due to high multipath interference rejection [4]. Through this paper, we aim to reconstruct the image with good visual quality which is required in medial field applications but with less bandwidth required for channel transmission. The importance of visual quality in medical field is clear in Fig. 3 and it can be measured through PSNR parameter.

In paper [5], the authors introduced a novel compressive sensing based prediction measurement (CSPM) encoder. The sparse image undergoes CS and these measured values pass to CSPM. In CSPM, the measured matrix undergoes linear prediction and LZW encoding. This CSPM encoder can achieve significant reduction in data storage and save transmission energy.

The wavelet transformed image has high frequency coefficients that are sparse and the low frequency coefficients that are not sparse. The low-frequency coefficients contain most energy of the image and have coherence nature. In paper [6], they measured (CS) the high-frequency sub band coefficients, and keep the low-frequency sub-band coefficients unchanged.

In this paper we have chosen deterministic matrix such as Hadamard matrix, Toeplitz matrix, random matrix such as Gaussian matrix, Bernoulli matrix as measurement matrices and to attain more sparsity, different levels of wavelets are used. The nonlinear reconstruction methods used are Orthogonal Matching Pursuit (OMP) and L1 minimisation technique.

## 2 Basic Block Diagram

The signal which is  $K$ -sparse in one domain can be reconstructed from another domain which has  $cK$  non adaptive linear projections where  $c$  is small constant. The sparsity matrix and measurement matrix should be incoherent for good reconstruction.

Let  $x$  is a real valued, finite length, one dimensional, discrete time signal which is an  $N \times 1$  vector in  $\mathbb{R}^N$ . When  $x$  is  $K$ -sparse, i.e.  $K \ll N$ , the signal  $x$  can undergoes CS.

Thus signal  $x$  can be written as

$$X = \Psi f \quad (1)$$

where  $f$  is the representation of signal  $x$  in another domain  $\Psi$ .

Let  $y$  be the measured vector of size  $M \times 1$  and  $\Phi \in \mathbb{R}^{M \times N}$  be the measurement matrix.

$$y = \Phi \Psi f = Acsf = \Phi x \quad (2)$$

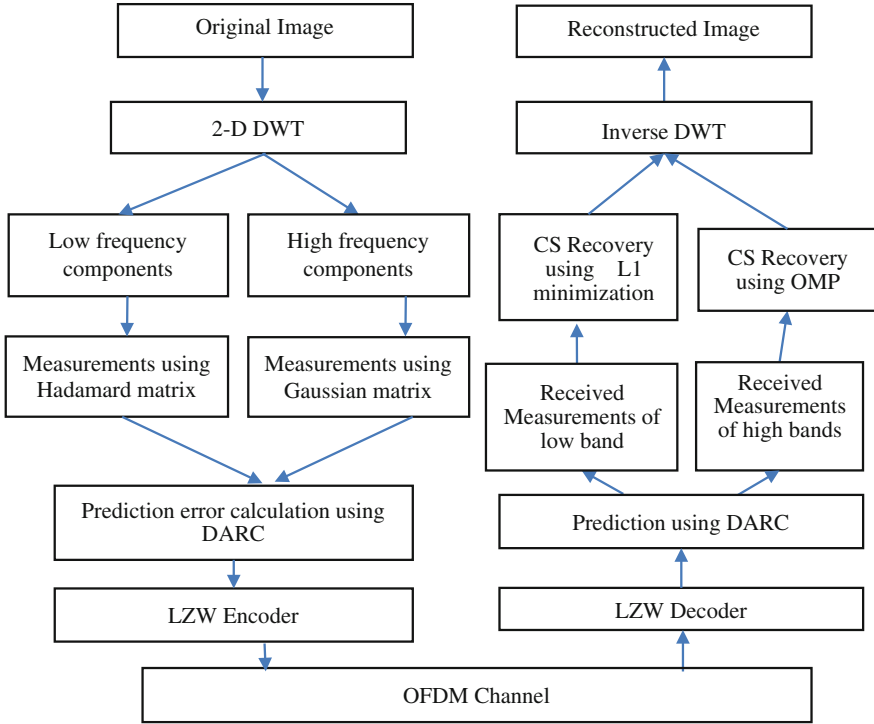
where  $Acs = \Phi \Psi$  is the sensing matrix of  $M \times N$  [7].

## 3 Proposed Block Diagram

In Fig. 2, we propose a new method based on CSPM encoder. First the image will undergo 2-D wavelet transform. Different levels of wavelet decomposition are used to make image sparser. In paper [5], the whole wavelet coefficients are undergoing CS together and measured using a single measurement matrix. Low frequency components contain coarse information and high frequency components contain detail information. The processing of high frequency components together with low frequency components will not exploit the coherent nature of LL band and this lead to performance degradation of the reconstructed image. So in this paper, CS will be applied separately for high frequency bands and low frequency bands. When applying CS to the sparse signal, we need to take a measurement which is inner product of the signal with fixed matrix. This matrix should follow the Restricted Isometry Property and incoherence. Gaussian matrix, Bernoulli matrix, Hadamard matrix and Toeplitz matrix are used as measurement matrices in this paper.

The prediction error calculated from the measured values which add more sparsity, which enhance the performance. Differential adaptive run coding (DARC) prediction method is used in this paper and it gives better results than linear prediction which is used in paper [5]. Before transmission we need to encode the values. This will helps in reducing the number of measurements, storage space and bandwidth. LZW encoding and decoding which has better speed in running dictionary operations is implemented in this paper. Then the encoded signal passes through OFDM channel. The encoded data are interleaved to avoid burst errors without change the throughput or data rate. In order to avoid the inter-symbol interference (ISI) due to multipath delay spread, a cyclic prefix (CP) is inserted in each OFDM symbol prior to transmission.

At receiver side, we need to decode the signal received and predict the values. This is passed for reconstruction of wavelet coefficients using any basic pursuit or greedy



**Fig. 2.** Proposed block diagram

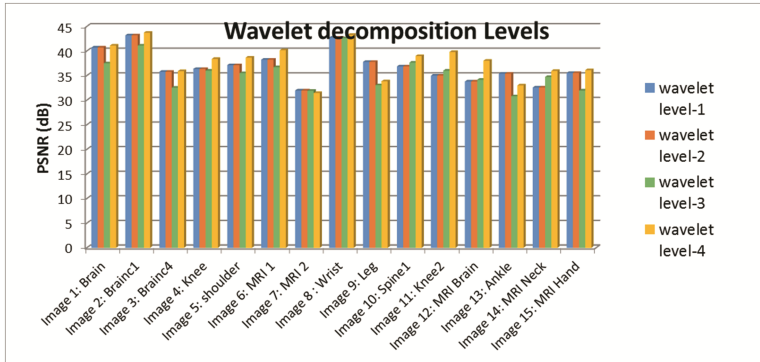
algorithms. The OMP and L1 minimization method is used for reconstruction in this paper. More iteration in OMP will give better results. After recovery, the image undergoes Inverse wavelet transform. The proposed diagram with techniques used in this paper is given in the Fig. 2.

### 4 Results

The image should be sparse to apply CS on it. After applying wavelet transform the signal will get sparser as shown in [7]. As we already discussed the LL band and high bands are processed separately. The high bands are applied with Gaussian measurement matrix since its random nature and LL band applied with different measurement matrix. The prediction error is calculated and the values are encoded by LZW algorithm. This encoded data passed through OFDM channel. For CS recovery we used L1 minimisation technique for LL band and OMP for high bands (Table 1 and Fig. 3).

**Table 1.** Comparison between different levels of wavelet decomposition

|                   | Level 1 | Level 2 | Level 3 | Level 4 |
|-------------------|---------|---------|---------|---------|
| Image 1: Brain    | 40.7    | 40.7    | 37.5    | 41.1    |
| Image 2: Brainc1  | 43.18   | 43.18   | 41.1    | 43.71   |
| Image 3: Brainc4  | 35.77   | 35.77   | 32.54   | 35.89   |
| Image 4: Knee     | 36.34   | 36.34   | 36      | 38.38   |
| Image 5: Shoulder | 37.11   | 37.11   | 35.5    | 38.65   |

**Fig. 3.** Comparison of different levels of wavelet decomposition

The result shows that level 4 wavelet decomposition is giving better PSNR or visual quality for image due to exploiting more frequency divisions of an image (Table 2 and Fig. 4).

**Table 2.** Comparison between prediction techniques

|                   | W/o prediction | Linear prediction | DARC prediction |
|-------------------|----------------|-------------------|-----------------|
| Image 1: Brain    | 37.588         | 37.58             | 37.6            |
| Image 2: Brainc1  | 40.79          | 40.78             | 41.1            |
| Image 3: Brainc4  | 40.14          | 40.75             | 41.3            |
| Image 4: Knee     | 36.63          | 37.34             | 41.5            |
| Image 5: Shoulder | 40.7           | 41.77             | 40.4            |

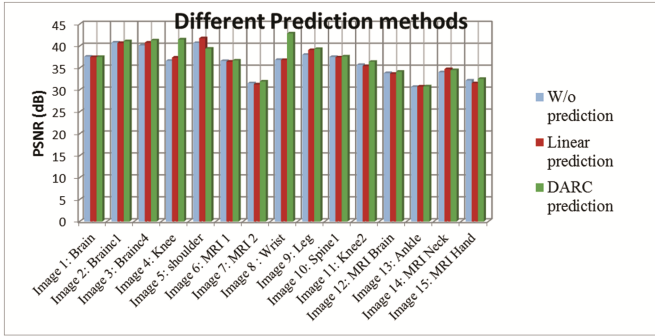


Fig. 4. Comparison between prediction techniques

The results show that DARC prediction technique is better. More clarity of reconstructed image can be achieved because DARC prediction considers more neighbouring pixels than the linear prediction.

The detail nature of High frequency band can be best exploited using Gaussian matrix which are pseudorandom values drawn from the standard normal distribution. The various Bernoulli matrix, Hadamard matrix, Toeplitz matrix and Gaussian matrix applied for LL band and results are given in Fig. 5 (Table 3).

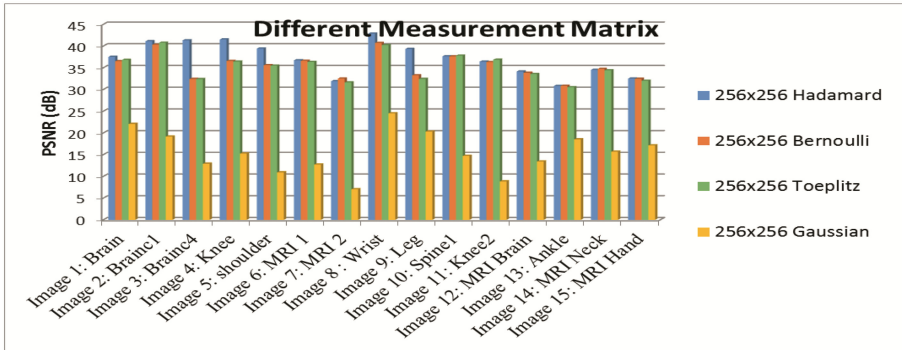


Fig. 5. Comparison of different measurement matrix

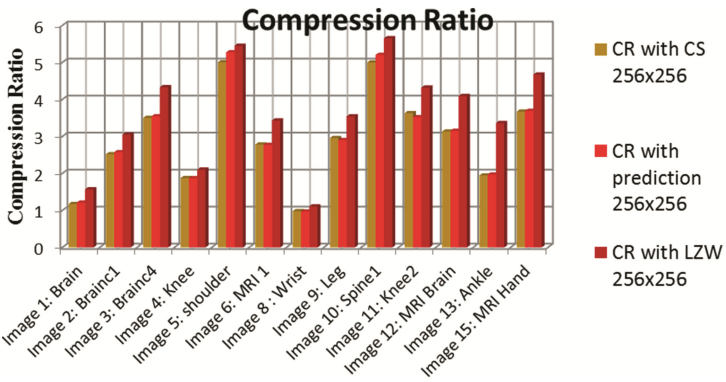
Table 3. Comparison between Bernoulli matrix, Hadamard matrix, Toeplitz matrix and Gaussian matrix applied for LL band

|                   | Hadamard | Bernoulli | Toeplitz | Gaussian |
|-------------------|----------|-----------|----------|----------|
| Image 1: Brain    | 37.5     | 36.5      | 36.8     | 22.06    |
| Image 2: Brainc1  | 41.1     | 40.29     | 40.73    | 19.14    |
| Image 3: Brainc4  | 41.3     | 32.41     | 32.37    | 12.9     |
| Image 4: Knee     | 41.5     | 36.55     | 36.4     | 15.24    |
| Image 5: Shoulder | 39.4     | 35.57     | 35.46    | 10.9     |

Hadamard matrix is giving better results when used for measuring LL band. L1 minimisation works well with these measured values for recovering pixels (Table 4 and Fig. 6).

**Table 4.** Comparison of compression ratio without encoder and with encoder

|                   | CR with LZW | CR with prediction | CR with CS |
|-------------------|-------------|--------------------|------------|
| Image 1: Brain    | 1.57        | 1.21               | 1.17       |
| Image 2: Brainc1  | 3.05        | 2.57               | 2.52       |
| Image 3: Brainc4  | 4.33        | 3.54               | 3.5        |
| Image 4: Knee     | 2.1         | 1.87               | 1.87       |
| Image 5: Shoulder | 5.45        | 5.27               | 5.00       |

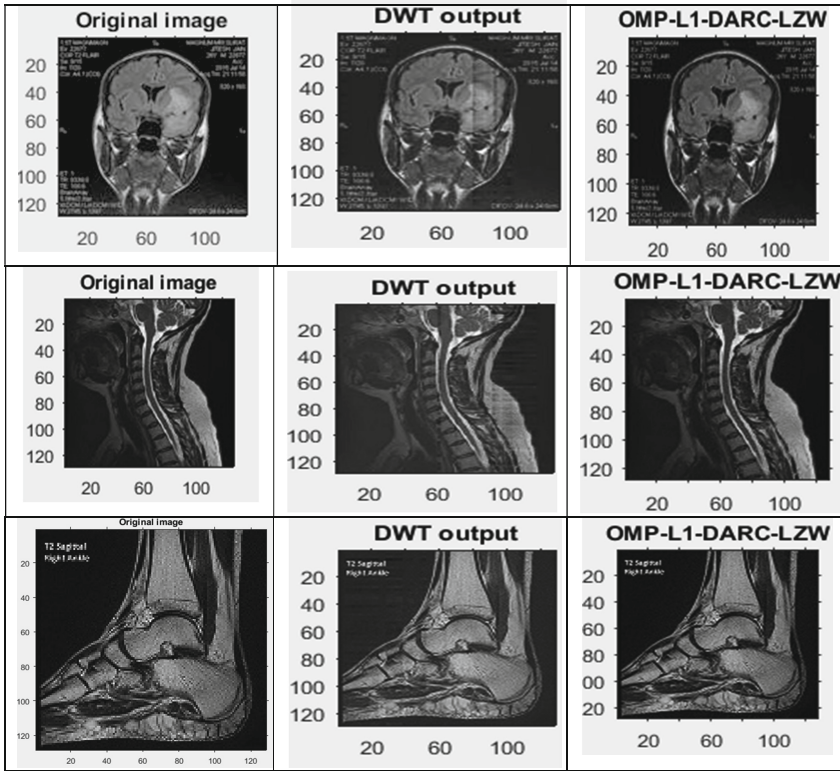


**Fig. 6.** Comparison of compression ratio on JPEG images

By this proposed method compression ratio is increased and thus less bandwidth only required for transmission through channel.

The visual quality is better when using the proposed method as shown in Table 5 when compared to CSPM technique used in [5] and all algorithms are tested on medical database collected from Hospital [8] and shown in result in above Table 5.

**Table 5.** Visual quality of images with DARC prediction and LZW encoder by using Hadamard matrix for measuring LL band and Gaussian matrix for measuring high bands.



## 5 Conclusion

The Compressive sensing measures small number of samples – in medical field this will help to reduce radiation time for MRI. Sparsity, incoherence and nonlinear reconstruction are three main components of CS.

Wavelet transform is proven sparsity domain for many signals. The high-frequency coefficients are sparse while the low frequency coefficients are not sparse. So, both bands processed separately and used different measurement matrix and recovery algorithms. The prediction technique helps in improving the PSNR.

For LL band, Hadamard matrix as measurement matrix, DARC as prediction method and L1 minimisation as recovery algorithm will result in higher PSNR. For high frequency bands, Gaussian matrix as measurement matrix, DARC as prediction method and OMP algorithm as recovery algorithm will result in higher PSNR and better reconstructed image.

The high PSNR, compression ratio and visual quality shows the proposed novel technique will be helpful in medical field especially for patients who undergoes MRI



scan. This proposed method will reduce the scan time but with better visual quality for easy diagnosing. Also the bandwidth reduction will be helpful in channel transmission.

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