Channel Estimation Based on Approximated Power Iteration Subspace Tracking for Massive MIMO Systems

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Abstract. Traditional semi-blind channel estimator is based on eigen value decomposition (EVD) or singular value decomposition (SVD), which effectively reduces the interference through dividing the observed signal into signal subspace and noise subspace. Due to the large computation, Massive MIMO systems could not afford the cost of traditional algorithms in spite of the high performance. In this paper, we propose a channel estimation algorithm based on subspace tracking, in which the signal subspace is obtained by approximating power iteration algorithm. Without sacrificing the estimation performance, the complexity is greatly reduced compared with the traditional semi-blind channel estimation algorithm, which improves the applicability of the estimator.

Keywords: Massive MIMO · Channel estimation · Semi-blind
Subspace tracking

1 Introduction

Massive MIMO technology greatly improves the system capacity and spectrum efficiency [1–4] through installing hundreds or thousands of antennas at BSs. It has become one of the key technologies of 5G now. The dimension of the channel state matrix increases with the number of antennas, which results in higher requirements for the channel estimation algorithm. Pilot contamination is particularly prominent in Massive MIMO system, so it's a serious problem to seek low complexity and anti-pilot contamination channel estimation algorithm.

The pilot-based channel estimation algorithms can't completely eliminate the effects of pilot contamination [5–7], while full-blind or semi-blind channel estimation algorithms don't require pilots or transmit fewer short pilots, thus avoiding pilot contamination. The subspace based channel estimation algorithm divides observation signal into signal subspace and noise subspace, which effectively reduces the interference and obtains the excellent estimation performance. Ngo. B. Q proposed a EVD based channel estimation algorithm to transform the channel estimation problem into the problem of ambiguous matrix. Through the eigenvalue decomposition of the received vector covariance matrix, the channel vector can be expressed as a corresponding eigenvector multiplying a scalar ambiguous factor, and the ambiguous factors constitute an ambiguous diagonal matrix [8]. The estimation performance and error

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term of EVD based algorithm are theoretically deduced and analyzed in [9], then the generalized linear (WL) algorithm is proposed. Dr. Hu proposed a semi-blind channel estimation algorithm based on SVD for Massive MIMO systems, like the method in [8], singular value decomposition of the received vector covariance matrix is needed. The ambiguity matrix of SVD based channel estimator is not a diagonal matrix, but a square matrix, which reduces the error caused by the non-orthogonal channel [10].

EVD and SVD algorithm have a large computational complexity O(M^3), where M is the dimension of the received vector. When the number of antennas in the BS reaches hundreds, the huge complexity of EVD or SVD based algorithm is unacceptable in Massive MIMO systems. In this paper, a subspace tracking based channel estimation algorithm is proposed, which uses the approximation power iteration algorithm to obtain the signal subspace with fast convergence and low complexity, the computational complexity to solve signal subspace of each iteration is O(MK^2) using API algorithm, FAPI algorithm needs only O(MK) operations for each update [11], K is the number of users in each cell.

2 System Model

Consider a multiuser Massive MIMO system with L cells that share the same band of frequencies, each cell contains K single-antenna users and one central BS equipped with M antennas. The system works in time-division duplex, so the uplink channel matrix is just the transpose of the downlink matrix because of the channel reciprocity. We consider the uplink where the users in the system synchronously send signals to BSs, the received signal vector at the BS of the jth cell can be expressed as

$$\mathbf{y}_j = \sqrt{p_{\mathrm{u}}}\sum_{i=1}^{L}\mathbf{G}_{ji}\mathbf{x}_i + \mathbf{w}_j \tag{1}$$

$$\mathbf{G}_{ji} = \mathbf{H}_{ji}\mathbf{D}_{ji}^{1/2} \tag{2}$$

where $\mathbf{x}_i$ is the transmitted symbols by the K users from the ith cell. p_{u} is the average power used by each user. $\mathbf{H}_{ji}$ is the $M \times K$ matrix of fast fading coefficients between K users in the ith cell and the jth BS. $\mathbf{D}_{ji}^{1/2}$ is a $K \times K$ diagonal matrix representing the geometric attenuation and shadow fading, diagonal elements are $\left[\mathbf{D}_{ji}\right]_{kk} = \beta_{jik}$.$\mathbf{w}_j$ is additive Gaussian white noise with zero mean and unit variance.

3 Traditional Semi-blind Channel Estimation

In this section, EVD and SVD based semi-blind channel estimator will be introduced.

3.1 EVD Based Estimator

The covariance matrix of the received vector $\mathbf{y}_j$ can be expressed as

$$\mathbf{R}_{\mathrm{y}} \triangleq \mathrm{E}\{\mathbf{y}_j\mathbf{y}_j^{\mathrm{H}}\} = p_{\mathrm{u}} \sum_{i=1}^{L} \mathbf{H}_{ji}\mathbf{D}_{ji}\mathbf{H}_{ji}^{\mathrm{H}} + \mathbf{I}_M \tag{3}$$

The channel vectors are approximately orthogonal in the Massive MIMO systems. multiplying (3) from the right by $\mathbf{H}_{jj}$, then we can obtain

$$\mathbf{R}_{\mathrm{y}}\mathbf{H}_{jj} \approx \mathbf{H}_{jj}(Mp_{\mathrm{u}}\mathbf{D}_{jj} + \mathbf{I}_K) \tag{4}$$

When M trends to infinity, the columns of $\mathbf{H}_{jj}$ are approximately orthogonal, and $Mp_{\mathrm{u}}\mathbf{D}_{jj} + \mathbf{I}_K$ is a diagonal matrix. So Eq. (4) can be considered as a characteristic equation for the covariance matrix $\mathbf{R}_{\mathrm{y}}$, the kth column of $\mathbf{H}_{jj}$ is the eigenvector corresponding to the eigenvalue $Mp\beta_{jjk} + \sigma_w^2$ of $\mathbf{R}_{\mathrm{y}}$. Each column of $\mathbf{H}_{jj}$ can be expressed as a corresponding eigenvector multiplying a scalar ambiguous factor, which is

$$\hat{\mathbf{H}}_{jj}^{\mathrm{EVD}} = \mathbf{U}_j\mathbf{C} \tag{5}$$

where $\mathbf{U}_j$ is the $M \times K$ eigenvector matrix. Ambiguity matrix $\mathbf{C}$ is K-order diagonal matrix, the ambiguity can be solved by using a short pilot sequence.

In practice, this covariance matrix is unavailable. Instead, we use the sample data covariance matrix $\hat{\mathbf{R}}_{\mathrm{y}}$ as the estimate of $\mathbf{R}_{\mathrm{y}}$,

$$\hat{\mathbf{R}}_{\mathrm{y}} \triangleq \frac{1}{N_d}\sum_{n=1}^{N_d} \mathbf{y}_j(n)\mathbf{y}_j(n)^{\mathrm{H}} \tag{6}$$

The EVD-based channel estimation algorithm is as follows,

(1) Given the number of samples N_d, compute $\hat{\mathbf{R}}_{\mathrm{y}}$.
(2) Perform EVD of $\hat{\mathbf{R}}_{\mathrm{y}}$, then obtain $\mathbf{U}_j$.
(3) Obtain the estimate $\hat{\mathbf{C}}$ of ambiguity matrix using a short pilot sequence.
(4) Obtain the channel estimate as $\hat{\mathbf{H}}_{jj}^{\mathrm{EVD}} = \mathbf{U}_j\hat{\mathbf{C}}$.

3.2 SVD Based Estimators

The channel estimation based on EVD algorithm utilizes orthogonality of the channel vectors, However, the antenna number M in the actual system is not infinite, The channel vectors are not perfectly orthogonal. The ambiguity matrix of SVD based channel estimator is not a diagonal matrix, but a square matrix, which reduces the error caused by the non-orthogonal channel.

The channel matrix can be expressed as

$$\mathbf{H}_{ji} = \tilde{\mathbf{H}}_{ji}\Gamma_i \tag{7}$$

where $\Gamma_i \in \mathbb{C}^{K\times K}$ represents the error between the real channel matrix $\mathbf{H}_{ji}$ and the orthogonal channel matrix $\tilde{\mathbf{H}}_{ji}$. Substituting into (3), then $\mathbf{R}_{\mathrm{y}} = p_{\mathrm{u}}\sum_{i=1}^{L}\tilde{\mathbf{H}}_{ji}\mathbf{A}_{ji}\tilde{\mathbf{H}}_{ji}^{\mathrm{H}} + \mathbf{I}_M$, where the k-order normal matrix $\mathbf{A}_{ji} = \Gamma_i\mathbf{D}_{ji}\Gamma_i^{\mathrm{H}}$ and its SVD form $\mathbf{A}_{ji} = \mathbf{V}_i\tilde{\Sigma}_i\mathbf{V}_i^{\mathrm{H}}$, $\mathbf{V}_i \in \mathbb{C}^{K\times K}$ is the left-singular matrix. Therefore, $\mathbf{R}_{\mathrm{y}}$ can be expressed as

$$\mathbf{R}_{\mathrm{y}} = p_{\mathrm{u}}\sum_{i=1}^{L}\tilde{\mathbf{H}}_{ji}\mathbf{V}_i\tilde{\Sigma}_i\mathbf{V}_i^{\mathrm{H}}\tilde{\mathbf{H}}_{ji}^{\mathrm{H}} + \mathbf{I}_M \tag{8}$$

$\mathbf{R}_{\mathrm{y}}$ is also a normal matrix and its SVD can be expressed as

$$\mathbf{R}_{\mathrm{y}} = \mathbf{Q}_j\Sigma_j\mathbf{Q}_j^{\mathrm{H}} \tag{9}$$

where $\mathbf{Q}_j$ contains M singular vectors; Σ_j is a real diagonal matrix which contains M singular values with descending order. $\mathbf{Q}_j = \left[\mathbf{Q}_j^{\mathrm{s}}, \mathbf{Q}_j^{\mathrm{n}}\right]$, where $\mathbf{Q}_j^{\mathrm{s}} \in \mathbb{C}^{M\times K}$, $\mathbf{Q}_j^{\mathrm{n}} \in \mathbb{C}^{M\times(M-K)}$. The columns of $\frac{1}{\sqrt{M}}\tilde{\mathbf{H}}_{ji}\mathbf{V}_i$ are the left-singular vectors that correspond to the largest KL singular values of $\mathbf{R}_{\mathrm{y}}$. Assuming $\beta_{jjk} \gg \beta_{jik}$, $\mathbf{Q}_j^{\mathrm{s}}$ can be denoted as $\mathbf{Q}_j^{\mathrm{s}} = \frac{1}{\sqrt{M}}(\tilde{\mathbf{H}}_{jj} + \mathbf{F}_j)\mathbf{V}_j\mathbf{B}_j.\mathbf{F}_j = \sqrt{M}\mathbf{O}_j\mathbf{V}_j^{\mathrm{H}}$ corresponds to the ICI in the received data symbols, $\mathbf{B}_j$ is a permutation matrix. Despite $\mathbf{F}_j$, we obtain

$$\mathbf{Q}_j^{\mathrm{s}} = \frac{1}{\sqrt{M}}\mathbf{H}_{jj}\mathbf{E}_j \tag{10}$$

where $\mathbf{E}_j = \Gamma_j^{-1}\mathbf{V}_j\mathbf{B}_j$, ambiguity matrix $\mathbf{E}_j$ is approximately a unitary matrix, the estimate of $\mathbf{E}_j$ can be resolved by pilot,

$$\hat{\mathbf{E}}_j = \frac{1}{\sqrt{M}}(\hat{\mathbf{H}}_{jj}^{\mathrm{LS}})^{\mathrm{H}}\mathbf{Q}_j^{\mathrm{s}} \tag{11}$$

From (10) and (11), we obtain the channel estimate,

$$\hat{\mathbf{H}}_{jj}^{\mathrm{SVD}} = \mathbf{Q}_j^{\mathrm{s}}(\mathbf{Q}_j^{\mathrm{s}})^{\mathrm{H}}\hat{\mathbf{H}}_{jj}^{\mathrm{LS}} \tag{12}$$

The SVD-based channel estimation algorithm is as follows,

(1) Given the number of samples N_d, compute $\hat{\mathbf{R}}_{\mathrm{y}}$.
(2) Perform SVD of $\hat{\mathbf{R}}_{\mathrm{y}}$, then obtain $\mathbf{Q}_j^{\mathrm{s}}$.
(3) Compute the pilot-based channel estimate $\hat{\mathbf{H}}_{jj}^{\mathrm{LS}}$.
(4) Obtain the channel estimate as $\hat{\mathbf{H}}_{jj}^{\mathrm{SVD}} = \mathbf{Q}_j^{\mathrm{s}}(\mathbf{Q}_j^{\mathrm{s}})^{\mathrm{H}}\hat{\mathbf{H}}_{jj}^{\mathrm{LS}}$.

4 Subspace Tracking Based Channel Estimation

Although EVD and SVD-based algorithms effectively reduce the interference, but the complexity is too large to apply to Massive MIMO system.

The subspace tracking based channel estimation algorithm is similar to the principle based on SVD, except that the method of obtaining the signal subspace is different. The algorithm steps are as follows,

(1) Given the number of samples N_d.
(2) Obtain the signal subspace estimate $\tilde{\mathbf{Q}}_j^{\mathrm{s}}$ using subspace tracking algorithm.
(3) Compute the pilot-based channel estimate $\hat{\mathbf{H}}_{jj}^{\mathrm{LS}}$.
(4) Obtain the channel estimate as $\hat{\mathbf{H}}_{jj}^{\mathrm{ST-CE}} = \tilde{\mathbf{Q}}_j^{\mathrm{s}}(\tilde{\mathbf{Q}}_j^{\mathrm{s}})^{\mathrm{H}}\hat{\mathbf{H}}_{jj}^{\mathrm{LS}}$.

4.1 API Subspace Tracking Algorithm

The approximated power iteration subspace algorithm is an improvement to the power iteration algorithm. Firstly, we introduce the idea of the power iteration subspace tracking algorithm.

The covariance matrix of the received vector $\mathbf{y}(n)$ can be expressed as

$$\mathbf{R}_{\mathrm{yy}}(n) = \sum_{m=-\infty}^{n} \tau^{n-m}\mathbf{y}(n)\mathbf{y}(n)^{\mathrm{H}} \tag{13}$$

where τ is the forgetting factor. The covariance matrix can be recursively updated according to the following scheme,

$$\mathbf{R}_{\mathrm{yy}}(n) = \tau\mathbf{R}_{\mathrm{yy}}(n-1) + \mathbf{y}(n)\mathbf{y}(n)^{\mathrm{H}} \tag{14}$$

Let the $M \times K$ orthogonal matrix $\mathbf{Q}(n)$ be transformed into the dominant subspace of $\mathbf{R}_{\mathrm{yy}}(n)$, then the compressed received vector $\mathbf{r}(n) = \mathbf{Q}(n-1)^{\mathrm{H}}\mathbf{y}(n)$. The power iteration method tracks the dominant subspace by the following compression step and orthonormalization step,

$$\mathbf{R}_{\mathrm{yr}}(n) = \mathbf{R}_{\mathrm{yy}}(n)\mathbf{Q}(n-1) \tag{15}$$

$$\mathbf{Q}(n)\Psi(n) = \mathbf{R}_{\mathrm{yr}}(n) \tag{16}$$

where $\Psi(n)$ a non-negative Hermitian matrix, and satisfying $\Psi(n)^{\mathrm{H}}\Psi(n) = \mathbf{R}_{\mathrm{yr}}(n)^{\mathrm{H}}\mathbf{R}_{\mathrm{yr}}(n)$. If $\mathbf{R}_{\mathrm{yy}}(n)$ remains constant and its first K eigenvalues are strictly larger than the M-K others, the power iteration method converges globally and exponentially to the dominant subspace.

By introducing the compensation matrix and the auxiliary matrix, the API algorithm makes $\mathbf{Q}(n)$ and $\mathbf{R}_{\mathrm{yr}}(n)$ independent recursive operations, and avoids the complicated process of solving $\Psi(n)$. The steps of the API algorithm are shown in Table 1.

Table 1. API algorithm

Step	Complexity
Initialization: $\mathbf{Q}(0) = [\mathbf{I}_k; \mathbf{0}_{(M-k)\times k}]$, $\mathbf{Z}(0) = \mathbf{I}_k$	
FOR $n = 1, 2, \cdots, N_d$	
$\mathbf{r}(n) = \mathbf{Q}(n-1)^{\mathrm{H}}\mathbf{y}(n)$	MK
$\mathbf{h}(n) = \mathbf{Z}(n-1)\mathbf{r}(n)$	K^2
$\mathbf{g}(n) = \frac{\mathbf{h}(n)}{\tau + \mathbf{r}(n)^{\mathrm{H}}\mathbf{h}(n)}$	$2K$
$\mathbf{e}(n) = \mathbf{y}(n) - \mathbf{Q}(n-1)\mathbf{r}(n)$	MK
$\mathbf{\Theta}(n) = (\mathbf{I}_k + \Vert\mathbf{e}(n)\Vert^2\mathbf{g}(n)\mathbf{g}(n)^{\mathrm{H}})^{-\frac{1}{2}}$	$M + \mathrm{O}(K^3)$
$\mathbf{Z}(n) = \frac{1}{\tau}\mathbf{\Theta}(n)^{\mathrm{H}}(\mathbf{I}_k - \mathbf{g}(n)\mathbf{y}(n)^{\mathrm{H}})\mathbf{Z}(n-1)\mathbf{\Theta}(n)^{-\mathrm{H}}$	$\mathrm{O}(K^3)$
$\mathbf{Q}(n) = (\mathbf{Q}(n-1) + \mathbf{e}(n)\mathbf{g}(n)^{\mathrm{H}})\mathbf{\Theta}(n)$	$MK^2 + MK$
End	

4.2 FAPI Subspace Tracking Algorithm

The fast approximated power iteration algorithm optimizes the solution process of the compensation matrix, thus speeding up the convergence. The steps of the FAPI algorithm are shown in Table 2.

Table 2. FAPI algorithm

Step	Complexity
Initialization: $\mathbf{Q}(0) = [\mathbf{I}_k; \mathbf{0}_{(M-k)\times k}]$, $\mathbf{Z}(0) = \mathbf{I}_k$	
FOR $n = 1, 2, \cdots, N_d$	
$\mathbf{r}(n) = \mathbf{Q}(n-1)^{\mathrm{H}}\mathbf{y}(n)$	MK
$\mathbf{h}(n) = \mathbf{Z}(n-1)\mathbf{r}(n)$	K^2
$\varepsilon^2(n) = \Vert\mathbf{y}(n)\Vert^2 - \Vert\mathbf{r}(n)\Vert^2$	$M+K$
$\vartheta(n) = \frac{\varepsilon^2(n)}{1+\varepsilon^2(n)\Vert\mathbf{g}(n)\Vert^2 + \sqrt{1+\varepsilon^2(n)\Vert\mathbf{g}(n)\Vert^2}}$	K
$\eta(n) = 1 - \vartheta(n)\Vert\mathbf{g}(n)\Vert^2$	1
$\mathbf{r}'(n) = \mathbf{r}(n)\eta(n) + \mathbf{g}(n)\vartheta(n)$	$2K$
$\mathbf{h}'(n) = \mathbf{Z}(n-1)^{\mathrm{H}}\mathbf{r}'(n)$	K^2
$\varepsilon(n) = \frac{\vartheta(n)}{\eta(n)}(\mathbf{Z}(n-1)\mathbf{g}(n) - \mathbf{h}'(n)^{\mathrm{H}}\mathbf{g}(n)\mathbf{g}(n))$	$K^2 + 3K$
$\mathbf{Z}(n) = \frac{1}{\tau}(\mathbf{Z}(n-1) - \mathbf{g}(n)\mathbf{h}'(n)^{\mathrm{H}} + \varepsilon(n)\mathbf{g}(n)^{\mathrm{H}})$	$2K^2$
$\mathbf{e}'(n) = \mathbf{y}(n)\eta(n) - \mathbf{Q}(n-1)\mathbf{r}'(n)$	$MK + M$
$\mathbf{Q}(n) = \mathbf{Q}(n-1) + \mathbf{e}'(n)\mathbf{g}(n)^{\mathrm{H}}$	MK
End	

4.3 Complexity Analysis

API algorithm has a computational complexity $MK^2 + O(MK)$ for each update, FAPI algorithm needs only $3MK + O(M)$ operations for each update. The number of samples is N_d, the complexity to obtain $K \times K$-dimensional signal subspace based on different algorithms is shown in Table 3.

Table 3. Algorithm complexity

Algorithm	Complexity
SVD	$O(M^3) + N_d M^2$
API-CE	$N_d MK^2 + N_d O(MK)$
FAPI-CE	$3N_d MK + N_d O(M)$

Obviously, the FAPI based channel estimation algorithm greatly reduces the complexity of the SVD based algorithm, and the simulation analysis based on the subspace tracking channel estimation algorithm will be introduced in the next section.

5 Simulation Results

Let $M = 128$, $K = 4$, $L = 3$. The large scale fading of the main cell take the random value of 0.6–1, and the large scale fading of the adjacent cells take the random value of 0.1–0.4. The modulation mode is BPSK. The estimation accuracy of the various algorithms is measured by the normalized mean square error (NMSE), which is defined as follows,

$$\mathrm{NMSE} = \frac{\|\hat{\mathbf{H}} - \mathbf{H}\|_F^2}{\|\mathbf{H}\|_F^2} \tag{10}$$

where $\hat{\mathbf{H}}$ is the channel estimate of $\mathbf{H}$.

The simulation results of the channel estimation algorithm based on subspace tracking are shown in Fig. 1, and the performance curves of EVD and SVD are also given for comparative analysis.

As the simulation shown, the API-CE and FAPI-CE channel estimation algorithms approach to the estimation performance of the SVD-based algorithm, and outperform the EVD-based algorithm. When SNR is 10 dB, the performance of the proposed algorithm is improved by nearly 10 dB compared with EVD algorithm. With the increase of SNR, the estimation accuracy of the proposed algorithm is higher, but the performance of EVD algorithm is not improved significantly because of the nonorthogonality part of the channel vectors.

As shown in Fig. 2, with the increase of antenna number, the estimation accuracy of API-CE and FAPI-CE algorithm both improve. When M grows from 100 to 300, the NMSE curve decreased significantly, the main source of error at this time is the channel

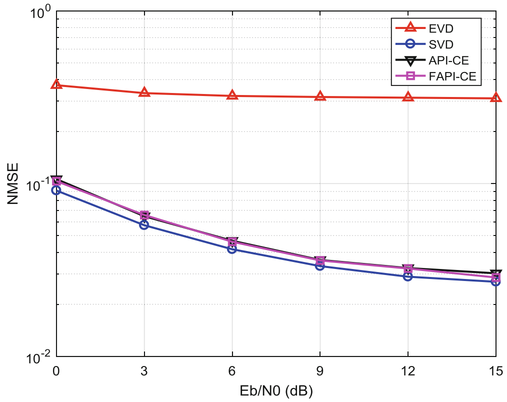

Fig. 1. NMSE versus Eb/N0 for M = 128, k = 4, L = 3, N_d = 100.

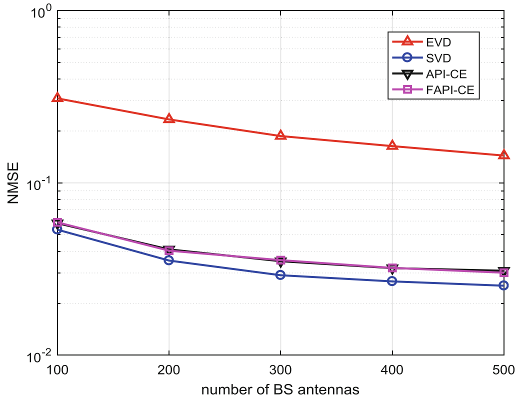

Fig. 2. NMSE versus M for Eb/N0 = 5 dB, k = 4, L = 3, Nd = 100.

nonorthogonality. When M is 300 to 500, the decrease of NMSE is gentle, the main factor that limits the performance is the error between the sample data covariance matrix and the real covariance matrix.

6 Conclusion

In this paper, we propose a channel estimation algorithm based on approximation power iteration subspace tracking. The computational complexity to solve signal subspace of each iteration is MK^2 + O(MK) using API algorithm, 3MK + O(M) using FAPI algorithm. The proposed channel estimation algorithms approach to the estimation performance of the SVD based algorithm, and outperform the EVD based algorithm in terms of the normalized mean square error, while greatly reduce the computational complexity. As the number of antennas increases, the estimation accuracy of API-CE and FAPI-CE algorithm improves. Therefore, the low complexity subspace tracking based channel estimation algorithm is very suitable for Massive MMO systems.

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