

# Research on the Pre-coding Technology of Broadcast Stage in Multi-user MIMO System

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**Abstract.** For the poor transmission reliability of data, this paper focuses on the precoding techniques of the multi-user MIMO system in the broadcast phase in, and the single-user MIMO and multi-user MIMO precoding techniques have been studied. In the multi-user MIMO system, BD-MMSE-VP coding algorithm is proposed to improve the BER performance of the system by using the MMSE criterion to optimize the perturbation vector, and the system uses QR technique to decompose BD channel matrix. Simulation results verify the effective of the proposed precoding algorithm.

**Keywords:** Multi-user · MIMO system · Precoding · Block diagonalization

## 1 Introduction

With advent of the era of “Internet +”, the mobile Internet has been seen everywhere and played an important role in the development of human society. People enjoy the convenience of mobile communications at the same time, but also put forward higher requirements on the mobile communication technology, which promoted the rapid development of mobile communications [1]. It has developed from the first generation mobile communication (1st-generation, 1G) based on analog cellular technology to the fourth generation mobile communication (4th-generation, 4G) mainly based on MIMO systems through 30 years' development. 1G only provided high quality speech services, but now mobile communication has to meet all aspects of the demand, including work, leisure and entertainment. Mobile communication brings people endless convenience and well-being [2]. However, 4G cannot meet the current requirements of achieving interconnectedness of all things. Therefore, the next generation of communication technology - the fifth generation mobile communication (5th-generation, 5G) has also started to be studied.

In this paper, we focus on the preprogramming in the multi-user MIMO system broadcast phase. The inter-user interference can be eliminated by precoding the signals sent to the multiuser at the base station.

## 2 The Precoding Technique of Multi-user MIMO System

### 2.1 Traditional Precoding Technology

Multi-user MIMO system can achieve the communication between base station and multiple users in the same frequency domain or time domain, increasing the spectrum efficiency and the reliability of communication system [3]. Multi-user MIMO as the 5G core technology has attracted many researchers' attention. In the broadcast link, the base station needs to send multiple data streams to multiple users. The signals obtained by each user contain not only the signals sent by the base station to them, but also the interference signals of other users. In addition, the users cannot cooperate among each other. The multi-user interference (MUI) occurs. Therefore, it is necessary to encode the transmitted signal before the signal is transmitted or to performing the detection operation at the receiving end, in order to eliminate the interference between the users and separate the information required by each user.

The precoding in Multi-user MIMO system is classified into linear and non-linear precoding. The linear precoding mainly includes channel inversion (CI) and block diagonalization (BD). The nonlinear precoding includes Dirty Paper Coding (DPC) and Tomlinson-Harashima Precoding (THP) [4].

CI precoding can be divided into the following two types: channel inversion precoding (ZF-CI) based on zero forcing and channel inversion precoding (MMSE-CI) based on minimum mean square error. Similar to the linear precoding of single-user MIMO, CI preprogramming is simple, with low computational complexity and thus easy to be implemented in communication systems, but weak at the anti-jamming. Block diagonalization precoding uses singular value decomposition of the channel matrix and is more suitable for the case of multi-antenna users. The computational complexity of Block diagonalization precoding is much smaller than other precoding algorithms, but it increases significantly as the number of users and receiving antennas increases. Therefore, it is important to find a new precoding algorithm that can reduce the block diagonalization precoding complexity and improve the system performance.

For non-linear dirty paper coding, the key idea is to treat other user signals as interference in addition to the effective user. Dirty paper coding can achieve close to the total channel capacity of multiple users, but is too complicated in computation to be applied to real communication system. Nonlinear THP precoding is formed on the basis of dirty paper coding. Although it does a compromise in terms of computational complexity and performance, it is still necessary to perform multiple iterations to eliminate user interference. THP is not suitable to the real communication system due its computational complexity. Therefore, in this section, we go into more detail on BD precoding and propose an optimization scheme.

### 2.2 An Optimal Coding Scheme Based on Block Diagonalization

The traditional block diagonalization coding (BD) uses the singular value decomposition (SVD) to decompose the channel matrix and obtains the zero matrix of the unitary matrix as the coding matrix. However, computational complexity of the singular value decomposition increases sharply as the number of transmitting and

receiving antennas increases [5]. The study on precoding with low complexity and ideal performance is very important. In this paper, a block diagonalization scheme based on Perturbation Vector (VP) is proposed. The optimization of the perturbation vector is based on the MMSE criterion. Therefore, we name the proposed code scheme as BD-MMSE-VP.

In this paper, we use the multiuser MIMO system as a model to study the BD pre-coding optimization scheme. The base station transmits the perturbation signal vector, and for the optimization of the perturbation vector, the MMSE is used as the criterion. The client needs to carry out simple modulo operation on the received signal to restore the signal. Unlike traditional BD precoding, BD-MMSE-VP encoding uses QR decomposition channel matrix, while traditional BD precoding uses SVD.

$\mathbf{H}$  is the channel matrix between the base station and all users,  $\mathbf{H} = [\mathbf{H}_1^T \mathbf{H}_2^T \cdots \mathbf{H}_K^T]^T$ ,  $\hat{\mathbf{H}}$  is  $\mathbf{H}$ 's pseudo inverse matrix,  $\hat{\mathbf{H}} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H)^{-1}$ , and  $\hat{\mathbf{H}} = [\hat{\mathbf{H}}_1 \hat{\mathbf{H}}_2 \cdots \hat{\mathbf{H}}_K]$ , so

$$\begin{aligned} \mathbf{H}\hat{\mathbf{H}}_1 &= \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} [\hat{\mathbf{H}}_1 \cdots \hat{\mathbf{H}}_K] = \begin{bmatrix} \mathbf{H}_1 \hat{\mathbf{H}}_1 & \cdots & \mathbf{H}_1 \hat{\mathbf{H}}_K \\ \vdots & \ddots & \vdots \\ \mathbf{H}_K \hat{\mathbf{H}}_1 & \cdots & \mathbf{H}_K \hat{\mathbf{H}}_K \end{bmatrix} \\ &= \begin{bmatrix} I_{N_{R,1}} & & \\ & \ddots & \\ & & I_{N_{R,K}} \end{bmatrix} = I_{N_R} \end{aligned} \quad (1)$$

From the above can be seen when  $j \neq k$ ,  $\mathbf{H}_j \hat{\mathbf{H}}_k = 0$ . The channel matrix of all other users except the user  $j$  is constructed as follows:

$$\tilde{\mathbf{H}}_j = [\mathbf{H}_1^T \cdots \mathbf{H}_{j-1}^T \mathbf{H}_{j+1}^T \cdots \mathbf{H}_K^T]^T \quad (2)$$

If you want to avoid other users on the user  $j$  caused by user interference, then the user  $j$  pseudo inverse matrix must fall in the matrix of zero space, so

$$\tilde{\mathbf{H}}_j \hat{\mathbf{H}}_j = 0, \quad j = 1, 2, \dots, K \quad (3)$$

Now, QR decomposition of the pseudo inverse matrix of user  $j$  is performed

$$\hat{\mathbf{H}}_j = \hat{\mathbf{Q}}_j \hat{\mathbf{R}}_j \quad j = 1, \dots, K \quad (4)$$

According to QR decomposition properties,  $\hat{\mathbf{R}}_j$  is the upper triangular matrix, and  $\hat{\mathbf{Q}}_j$  column vector can form the standard orthogonal basis of the matrix  $\hat{\mathbf{H}}_j$ . In order to eliminate inter-user interference,  $\tilde{\mathbf{H}}_j \hat{\mathbf{H}}_j = \tilde{\mathbf{H}}_j \hat{\mathbf{Q}}_j \hat{\mathbf{R}}_j = 0$ , because  $\hat{\mathbf{R}}_j$  is reversible matrix, so  $\tilde{\mathbf{H}}_j \hat{\mathbf{Q}}_j = 0$ .  $\hat{\mathbf{Q}}_j$  is removing the orthogonal base forming the zero space.

Let  $W_j = \hat{\mathbf{Q}}_j$ , construct a valid matrix  $H_{eff,j} = W_j H$ , BD-MMSE-VP coding principle is shown in Fig. 1:

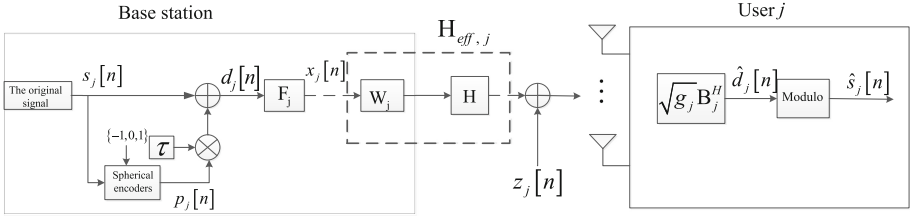


Fig. 1. BD-MMSE-VP precoding diagram

The optimal perturbation vector and power constraint factor are found by using MMSE criterion,  $d_j[n]$  is the user  $j$  wants to get the signal,  $\hat{d}_j[n]$  is the user  $j$  actually receives the signal, The minimum homogeneous error of the two signals is expressed by the formula (4), while  $\mathbf{B}_j^H$  is the user receives the detection matrix, if it is unit array, then the system optimization problem is transformed into the base station transmitter optimization problem, and we know  $H_{eff,j}$ ,  $d_j[n]$ ,  $tr(R_n)$ . So the problem to be solved is when  $p_j[n]$ ,  $x_j[n]$ ,  $g_j$  are what values, the  $d_j[n]$  and  $\hat{d}_j[n]$  mean square error is minimal, and power constraint is  $\sum_{n=1}^{N_B} x_j^H[n]x_j[n] = P$ ,  $P$  is the base station total transmit power.

$$\begin{aligned}
 \varepsilon(p_j[n], x_j[n], g_j) &= \sum_{n=1}^{N_B} E\left(\|\hat{d}_j[n] - d_j[n]\|^2\right) \\
 &= \sum_{n=1}^{N_B} E\left(\left\|g_j \mathbf{B}_j^H (H_{eff,j} x_j[n] + z_j[n]) - d_j[n]\right\|^2\right) \\
 &= \sum_{n=1}^{N_B} E\left(\left(g_j \mathbf{B}_j^H (H_{eff,j} x_j[n] + z_j[n]) - d_j[n]\right)^H \right. \\
 &\quad \times \left. \left(g_j \mathbf{B}_j^H (H_{eff,j} x_j[n] + z_j[n]) - d_j[n]\right)\right) \\
 &= \sum_{n=1}^{N_B} E\left(\begin{aligned} &g_j^2 x_j^H[n] H_{eff,j}^H H_{eff,j} x_j[n] - g_j d_j^H[n] \mathbf{B}_j^H H_{eff,j} x_j[n] \\ &+ g_j^2 tr(R_n) - g_j x_j^H[n] H_{eff,j}^H \mathbf{B}_j d_j[n] + d_j^H[n] d_j[n] \end{aligned}\right)
 \end{aligned} \tag{5}$$

Firstly, the Lagrangian function is constructed by using the Lagrangian algorithm:

$$f(p_j[n], x_j[n], g_j, \lambda) = \varepsilon(p_j[n], x_j[n], g_j) + \lambda \left( \sum_{n=1}^{N_B} x_j^H[n]x_j[n] - P \right) \tag{6}$$

And then  $p_j[n]$ ,  $x_j[n]$ ,  $\lambda$  were partial guide:

$$\frac{\partial f(\cdot)}{\partial x_j[n]} = g_j^2 x_j^H[n] H_{eff,j}^H H_{eff,j} - g_j d_j^H[n] \mathbf{B}_j^H H_{eff,j} + \lambda x_j^H[n] = 0 \tag{7}$$

$$\begin{aligned} \frac{\partial f(\cdot)}{\partial g_j} &= \sum_{n=1}^{N_B} 2g_j x_j^H[n] H_{eff,j}^H H_{eff,j} x_j[n] - d_j^H[n] B_j^H H_{eff,j} x_j[n] \\ &+ 2g_j \text{tr}(R_n) - x_j^H[n] H_{eff,j}^H B_j d_j[n] = 0 \end{aligned} \quad (8)$$

$$\frac{\partial f(\cdot)}{\partial \lambda} = \sum_{n=1}^{N_B} x_j^H[n] x_j[n] - P = 0 \quad (9)$$

Let  $\xi = \frac{N_B \text{tr}(R_n)}{P}$ , finally we can obtain

$$x_j[n] = F_j d_j[n] = \frac{1}{g_j} H_{eff,j}^H \left( H_{eff,j} H_{eff,j}^H + \xi I \right)^{-1} B_j d_j[n] \quad (10)$$

$$g_j = \sqrt{\frac{1}{P} \sum_{n=1}^{N_B} d_j^H[n] B_j^H H_{eff,j} \left( H_{eff,j} H_{eff,j}^H + \xi I \right)^{-2} H_{eff,j}^H B_j d_j[n]} \quad (11)$$

Relative to the perturbation vector signal  $d_j[n]$  coding matrix is  $F_j = \frac{1}{g_j} H_{eff,j}^H \left( H_{eff,j} H_{eff,j}^H + \xi I \right)^{-1} B_j$ , and the perturbation vector  $p_j[n]$  can be formed by a spherical encoder.

$$p_j[n] = \arg \min \left\| B_j \left( s_j[n] + \tau p_j'[n] \right) \right\| \quad (12)$$

The user  $j$  receive signal can be expressed as

$$\begin{aligned} y_j[n] &= H_{eff,j} x_j[n] + z_j[n] \\ &= \frac{1}{g_j} H_{eff,j} H_{eff,j}^H \left( H_{eff,j} H_{eff,j}^H + \xi I \right)^{-1} B_j (s_j[n] + \tau p_j[n]) + z_j[n] \\ &\approx \frac{1}{g_j} B_j (s_j[n] + \tau p_j[n]) + z_j'[n] \end{aligned} \quad (13)$$

The equation  $z_j'[n]$  contains the Gaussian redundant interference noise, which is multiplied by the power of the user  $j$  to remove the power scaling, and the user  $j$  knows the value of the spherical encoder  $\tau$ , so the receiver can eliminate  $\tau p_j[n]$  influence by modulo operation.

### 3 Implementation and Performance Analysis

In this section, the performance of the proposed algorithm and system will be evaluated and analyzed in terms of the optimized coding sum rate and complexity of optimized coding.

### 3.1 Optimized Coding Sum Rate Analysis

The sum of the rates of the multiuser MIMO systems is the sum of all single user rates. Through the use of BD-MMSE-VP precoding in multi-user MIMO systems,  $H_{eff,j}$  is the equivalent matrix of base station and user  $j$ , transmit a signal is  $x_j[n] = \frac{1}{g_j} H_{eff,j}^H (H_{eff,j} H_{eff,j}^H + \xi I)^{-1} B_j d_j[n]$ , the receiving signal where in the user is  $y_j[n] = H_{eff,j} x_j[n] + z_j[n]$ . According to feature decomposition, we can easily obtain  $H_{eff,j} H_{eff,j}^H = Q \Lambda Q^H$ ,  $H_{eff,j} x_j[n]$  is decomposed:

$$H_{eff,j} x_j[n] = \frac{1}{\sqrt{g_j}} Q \Phi Q^H B_j d_j[n] \tag{14}$$

While  $Q$  is the unit matrix,  $\Phi$  is the diagonal matrix, and the value on the diagonal is  $\frac{\lambda_j}{\lambda_j + \xi}$ ,  $\lambda_j$  is  $\Lambda$  matrix's diagonal elements. According to (14), and it can be seen that the user  $j$  input signal can be expressed as:

$$\begin{aligned} [H_{eff,j} x_j[n]]_k &= \frac{1}{\sqrt{g_j}} \begin{bmatrix} q_{k,1} \frac{\lambda_1}{\lambda_1 + \xi} \cdots q_{k,N_{R,j}} \frac{\lambda_{N_{R,j}}}{\lambda_{N_{R,j}} + \xi} \\ q_{1,1}^H \cdots q_{N_{R,j},1}^H \\ \vdots \vdots \vdots \\ q_{1,N_{R,j}}^H \cdots q_{N_{R,j},N_{R,j}}^H \end{bmatrix} \begin{bmatrix} c_{j,1}[n] \\ \vdots \\ c_{j,N_{R,j}}[n] \end{bmatrix} \end{aligned} \tag{15}$$

So the user  $j$ 's  $k$ -th received signal is represented by the following equation:

$$y_{j,k}[n] = \frac{1}{\sqrt{g_j}} \left( \sum_{l=1}^{N_{R,j}} \frac{\lambda_l}{\lambda_l + \xi} |q_{k,l}|^2 \right) c_{j,k}[n] + z''_{j,k}[n] \tag{16}$$

While  $z''_{j,k}[n] = \frac{1}{\sqrt{g_j}} \sum_{m=1, m \neq k}^{N_{R,j}} \left( \sum_{l=1}^{N_{R,j}} \frac{\lambda_l}{\lambda_l + \xi} q_{k,l} q_{m,l}^H \right) c_{j,k}[n] + z_{j,k}[n]$ , including internal data stream interference and signal noise  $z_{j,k}[n]$ , so the user  $j$  SINR is:

$$SINR_{j,k} = \sum_{n=1}^{N_B} \frac{\|s_{c_{j,k}}[n]\|^2}{\|t_{c_{j,m}}[n]\|^2 + \|z_{j,k}[n]\|^2} \tag{17}$$

While  $s = \frac{1}{\sqrt{g_j}} \left( \sum_{l=1}^{N_{R,j}} \frac{\lambda_l}{\lambda_l + \xi} |q_{k,l}|^2 \right)$ ,  $t = \frac{1}{\sqrt{g_j}} \sum_{m=1, m \neq k}^{N_{R,j}} \left( \sum_{l=1}^{N_{R,j}} \frac{\lambda_l}{\lambda_l + \xi} q_{k,l} q_{m,l}^H \right)$ , so the BD-MMSE-VP coding sum rate is:

$$R_{BD-MMSE-VP} = \sum_{j=1}^K \sum_{k=1}^{N_{R,j}} \log_2(1 + SINR_{j,k}) \tag{18}$$

### 3.2 Complexity Analysis of Optimized Coding

For multiuser MIMO precoding complexity analysis, this section considers only the complexity of the base station precoding algorithm and measures the computational complexity using floating-point arithmetic. Traditional BD precoding uses the SVD decomposition, its complexity:

$$C_{SVD} = K \times \left( 4N_B^2 N_R + 13(N_R - N_{R,j})^3 \right) \tag{19}$$

The BD-MMSE-VP coding uses QR decomposition and perturbation vector optimization, QR decomposition complexity is

$$C_{QR} = \frac{11}{3}N_B^3 + \frac{5}{3}N_B^2 + KN_{R,j}^2 \times \left( N_B - \frac{1}{3}N_{R,j} \right) \tag{20}$$

It can be seen that the QR operation is much smaller than the SVD by comparing the SVD and QR operations, because the QR is decomposed  $N_B \times N_{R,j}$ , and the matrix of the traditional BD precoding SVD is  $(N_R - N_{R,j}) \times N_B$ .

### 3.3 Simulation Results

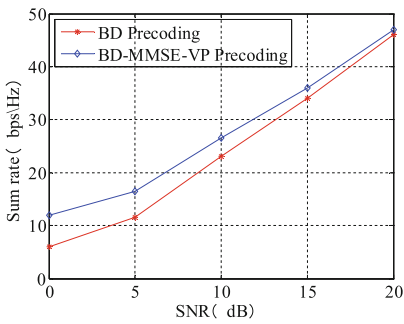
In this paper, BD-MMSE-VP precoding and traditional BD precoding algorithm is simulated by Matlab. In the multi-user MIMO system broadcast stage, QPSK is used for modulation and demodulation. The number of base stations is 8, and each user adopts two receiving antennas, the number of users is 4. The simulation compares the system and the rate and BER performance, the specific simulation parameters are shown in Table 1 below:

**Table 1.** Multi-user MIMO precoding simulation parameter

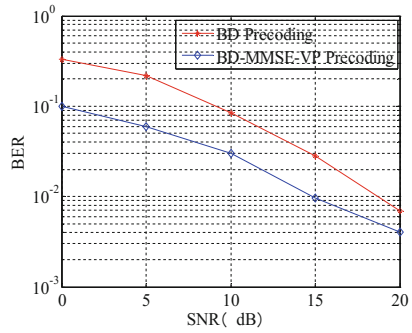
Parameter	Set values
System	Multiple-MIMO
Antenna configuration/User number	8 * 2/K = 4
Channel condition	Zero mean complex Gaussian random channel
Noise	White Gaussian Noise
Modulation mode	QPSK
Precoding algorithm	Traditional BD/BD-MMSE-VP algorithm

It can be seen from Figs. 2 and 3 that the BD-MMSE-VP precoding proposed in this paper has a great advantage over traditional BD precoding, both BER performance and system sum rate performance, and the signal-to-noise ratio In the case of no more than 10 dB, BD-MMSE-VP pre-coding performance advantage is more obvious. When the signal-to-noise ratio is 5 dB, the BD-MMSE-VP precoding is improved by 5 bps/Hz compared with the traditional BD precoding algorithm. When the SNR is

15 dB, the BD-MMSE-VP precoding is better than the traditional BD precoding. The algorithm improves the rate of 2 bps/Hz. This is because the BD-MMSE-VP uses the Lagrangian algorithm to allocate the transmit power, allocates more transmit power for subchannels with good channel states, and the channel subcarriers are allocated less or do not allocate transmit power. Noise ratio is maximized, but with the increase of the signal-to-noise ratio, the BD-MMSE-VP precoding is almost evenly distributed to all subchannels, so its rate performance is high. Noise ratio is similar to the traditional BD precoding. BD-MMSE-VP precoding BER performance is better than traditional BD precoding. At BER =  $10^{-2}$ , BD-MMSE-VP precoding obtains a gain of 2.7 dB relative to traditional BD algorithm.



**Fig. 2.** The contrast of Multi-user MIMO precoding sum rate



**Fig. 3.** The contrast of Multi-user MIMO precoding BER performance

### 4 Conclusions

In the multiuser MIMO system, two kinds of linear precoding of CI and BD are preliminarily studied, and two kinds of nonlinear codes such as DPC and THP coding are difficult to be applied because of the high complexity of non-linear coding DPC and THP coding. The performance of BD pre-coding is better than that of the CI pre-coding, and the BD-MMSE-VP precoding is proposed at the same time, which is combined with the BD algorithm of QR decomposition, and the MMSE criterion is used to optimize the perturbation vector. Simulation results show that the performance of sum rate and BER has been greatly improved by comparison.

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