

# Application of Time-Varying Filter in Time-Frequency Resource Allocation

Zhongchao Ma<sup>1,2</sup>, Liang Ye<sup>1</sup>, and Xuejun Sha<sup>1</sup>(✉)

<sup>1</sup> School of Electronics and Information Engineering,  
Harbin Institute of Technology, Harbin 150001, China  
2279779526@qq.com, coldwound@163.com,  
shaxuejun@hit.edu.cn

<sup>2</sup> Science and Technology on Information Transmission and Dissemination  
in Communication Networks Laboratory, Shijiazhuang, China

**Abstract.** With the rapid development of wireless communication industry, the problem of spectrum resource scarcity is becoming more and more serious. To improve the spectral efficiency by compressing the adjacent carrier frequency intervals will increase the inter carrier interference. At this point, the performance of time-invariant filter is poor. If the communication system can make full use of the low energy slots in the time-frequency domain, the spectral efficiency could be improved. This paper proposes a time-frequency resource allocation method in which there is a delay of half a symbol between adjacent carriers. Accordingly, a time-varying filter is proposed. The simulation results show that the proposed time-varying filter performs better than the time-invariant filter.

**Keywords:** Time-frequency transform · Time-varying filter  
Root raised cosine signal

## 1 Introduction

In recent years, the mobile communication industry has developed rapidly, and various communication services have greatly facilitated production and life. However, as the number of terminals increases, the problem of spectrum resource scarcity is increasingly serious. It is an effective approach to alleviate this problem by improving spectral efficiency. Conventional resource allocation method is allocating the resources only in frequency domain and applying the time-invariant filter to suppress the adjacent channel interference. But this solution performs poor if the adjacent carrier frequency interval is compressed. An efficient allocation scheme is to sense the low energy slots in the time-frequency domain, and allocate the time-frequency resources according to the time-frequency distributions of adjacent signals. In this case, the time-varying filter is used to mitigate the interference. In addition, time-varying filter can also be applied to cellular systems, such as D2D communication by using guard bands, non-orthogonal multiple access in 5G, and frequency reuse in adjacent cellular cells.

Fourier transform can only be used to analyze stationary signals because it can only tell what frequency components the signal contains, but can not tell the time when the components appear. Short-time Fourier transform [1] is based on Fourier transform,

and the signal is processed by adding analysis window. When the window is short, short-time Fourier transform can show the time-varying spectrum of the signal. In 1996, Stockwell proposed the famous S transform [2]. S transform can be seen as an evolution of short-time Fourier transform. Unlike STFT whose length of time window is fixed, S transform uses the Gaussian window, and the length of the time window is inversely proportional to the frequency. In addition to the linear time-frequency distribution, there are also nonlinear time-frequency distributions [3]. WVD is a typical nonlinear distribution. It has good characteristics in time-frequency concentration, but WVD has serious cross-term problems. In recent years, there have been many studies on weighted-type fractional Fourier transform [4, 5].

In order to improve the efficiency of time-frequency resource utilization, some new technologies have emerged in recent years, such as MIMO and context awareness [6, 7]. The allocation of time-frequency resources is determined by perceiving the user's behavior, such as the type of application, whether the user is interacting with the program, and so on. This method not only improves the spectral efficiency, but also improves the quality of service. In paper [8], a context aware time-frequency resource scheduling algorithm is proposed, which has lower computational complexity. However, this algorithm uses the channel resources fully, but does not change the energy distribution of the signal when transmitting data, and does not use the low energy characteristics in time-frequency domain. When all channels are transmitting data, this method has no ability to further improve resource utilization.

In recent years, researches on time-varying filter have been in full swing. Although there have been some researches on the time-invariant filter [9, 10], this paper forgoes an introduction. In paper [11], a fast algorithm of inverse S transform is proposed. Based on this, a time-varying filter for synthetic aperture radar is designed. Since the root raised cosine signal does not have a relatively fixed time-frequency spectrum under S transform, this filtering method is not applicable to a communication system which employs root raised cosine signal. The time-varying filter based on S transform proposed in [12] has similar problems. Paper [13] studies the interferences of adjacent cell in SC-FDMA system. A time-varying filter is designed based on Choi-Williams distribution. The signal which includes a desired signal and two interfering signals is used in simulation, and the channel is an additive white Gauss noise channel. The time-frequency filtering pass region (Mask) used by the receiver is generated by the received signal in real time. The method of generating Mask is as follows: (1) transform the received signal to time-frequency domain; (2) set the region to Mask whose energy exceeds the threshold value. Since the Mask is determined by the received signal, the characteristics of the received signal can significantly affect the performance of filter. The simulation results show that the time-varying filter has advantages when the signal to noise ratio is in a suitable range. However, when the power of noise and interference is large, this method can't provide the effective pass region, and can't be used to suppress the noise and interference.

In view of these shortcomings and based on the low energy slots of root raised cosine signal in time-frequency domain, a new method of time-frequency resource allocation is proposed in this paper. On this foundation, a time-varying filter is used to eliminate interference and noise. Simulation results show that this method performs better than the time-invariant filter.

## 2 Time-Frequency Resource Allocation and Filter Design

### 2.1 Fundamentals of STFT Analysis

It is very important to learn the signal's energy distribution characteristics at different time for a non-stationary signal. Fourier transform maps the signal to a one-dimensional spectral distribution, so it only shows what frequencies it contains. When the time-frequency analysis method is used, the signal can be mapped to a time-frequency plane. This plane shows the spectral characteristics of non-stationary signals at different times. Short time Fourier transform (STFT) is a common time-frequency analysis method. It is widely used in various fields. The main principle of STFT is to add a window which slides with time. The window intercepts the original signal and considers the signal to be stationary within the window. Then Fourier transform is applied to the signal within the window, and the time-frequency distribution of the signal is formed. For discrete sequence  $x(n)$ , its instantaneous data at the moment  $n$  is defined as

$$x_n(m) = x(m)w(n - m) \quad (1)$$

Where  $w(n)$  is a window function whose length is  $N$ . DFT of (1) can be expressed as

$$STFT(n, \omega) = \sum_{m=-\infty}^{+\infty} x(m)w(n - m)e^{-jm\omega} = \sum_{m=-\infty}^{+\infty} x_n(m)e^{-jm\omega} \quad (2)$$

This is the discrete STFT formula of the signal  $x(n)$ .

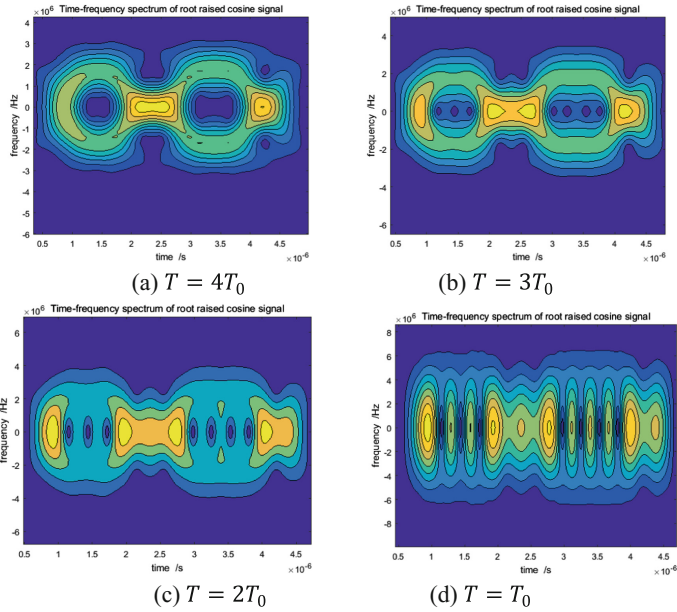
### 2.2 Time-Frequency Distribution of Root Raised Cosine Signal and Resource Allocation

The root raised cosine signal is expressed as

$$h(t) = \frac{(4\alpha t/T) \cos[\pi(1 + \alpha)t/T] + \sin[\pi(1 - \alpha)t/T]}{(\pi t/T)[1 - (4\alpha t/T)^2]} \quad (3)$$

A system which uses root raised cosine signal as the transmit pulse and the receive filter is an optimum receiver without inter-symbol interference. Assume that the symbol transmit rate  $R_b = 3.84$  Mbps, sampling rate  $f_s = 1.61 \times 10^8$  Hz, and the raised cosine coefficient  $\alpha = 0.22$ . The time-frequency spectrum of the signal with different window lengths is shown in Fig. 1 where  $T$  represents the length of the time window and  $T_0 = 1/R_b$ .

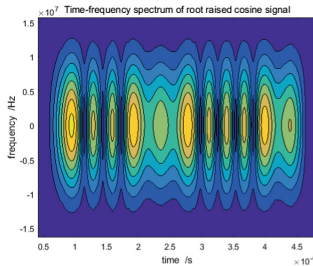
Figure 1 shows that when the window length is longer than the length of symbol, the energy concentration regions and the low energy slots emerge randomly which is determined by the transmit symbol. In other words, the energy concentration region will become low energy slot at the next simulation because of different transmitting symbols. This adds obstacles to time-frequency resource allocation and filtering. With the shortening of time window, the randomness of energy concentration region and low energy slots decrease gradually. Low energy slots are usually present at the junction of



**Fig. 1.** The time-frequency spectrum at different lengths of window

two symbols. It is possible to determine the time-frequency domain occupied by the signal when  $T = T_0$ . The analysis window length used in this paper is  $T = T_0/2$ . The STFT spectrum of root raised cosine signal is shown in Fig. 2.

Figure 2 shows that there are low energy slots in the time-frequency spectrum of



**Fig. 2.** Time-frequency distribution when  $T = T_0/2$

the signals, and the positions of these low energy slots are relatively fixed. When the transmit sequence produces a positive and negative conversion, a low energy slots is generated at the junction of the two symbols. When two consecutive 1 (or -1) occurs, an energy concentration region is generated at the junction of the two symbols. If the communication system allocate the frequency of each channel and adjust the time delay

of the transmitting symbols properly, these low energy slots can be fully utilized, and then the utilization ratio of the resource will be improved. The time-frequency resource allocation is shown in the Fig. 3.

Figure 3 shows that there is a delay of half a symbol between two adjacent signals,

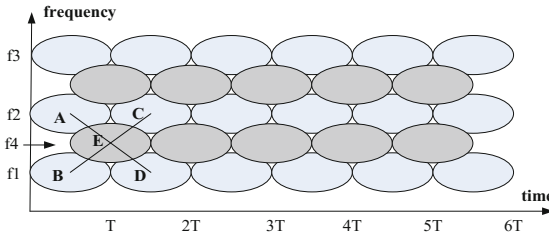


Fig. 3. Time-frequency resource allocation

which can make full use of the low energy slots in the spectrum and enhance the spectral efficiency. However, it will add interference to the original signal. Time-invariant filter can not distinguish two signals overlapped in frequency spectrum very well. So time-varying filter based on time-frequency transform has some advantages.

### 2.3 The Proposed Time-Varying Filter

There are two ways to design time-varying filters: explicit design method and implicit design method. Explicit design method is based on time-frequency weighting function  $M(t, f)$  to calculate the impulse response  $h(t, t')$ . Finally we will get filtering result  $y(t)$  by calculating the convolution of  $x(t')$  and  $h(t, t')$ .

$$y(t) = \int_{-\infty}^{+\infty} h(t, t')x(t')dt' \tag{4}$$

Implicit design method is implemented in three steps: analysis, emphasis and synthesis. First, get the time-frequency distribution  $X(t, f)$  of signal  $x(t)$  by the method of WVD or STFT, and then use the time frequency weighting function  $M(t, f)$  to handle with  $X(t, f)$  where we will get  $X'(t, f)$ . Finally, do inverse transform to convert  $X'(t, f)$  into time domain.

The performance of filter designed by explicit design method and implicit design method is similarly. Therefore, the filter involved in this paper is designed by implicit design method.

In this paper, STFT transform is used to analyze time-frequency spectrum. Four types of time-frequency weighting functions are designed.

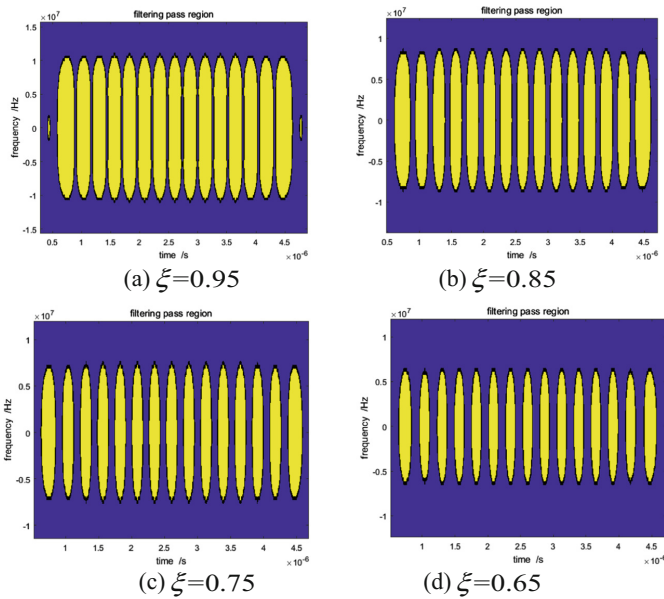
When the spectrum of a signal is relatively fixed, the filtering pass region of the signal can be selected according to the time frequency distribution of the signal with a selection threshold constraint. Moreover, the filtering pass region can seriously affect the performance of filter. In time-frequency spectrum, the energy region is selected

from high to low. When the total energy reaches the threshold, a filter pass region ( $mask_1$ ) can be obtained. The threshold  $\xi$  can be expressed as

$$\xi = \frac{P_{sum}}{P_{total}} \tag{5}$$

where  $P_{sum}$  is the energy of the region selected, and  $P_{total}$  is the energy of the signal. The filter domain is shown in Fig. 4.

Figure 4 shows that the filtering pass region shrinks as the energy threshold



**Fig. 4.** Filtering pass region ( $mask_1$ ) under different energy thresholds

decreases. In general the larger the power of noise and interference, the smaller the optimal threshold. Small energy threshold may filter out some of the useful signal, but it also means filtering out more noise and interference. Using this filtering pass region, the energy between the two energy concentration regions is discarded. But the desired signal has some energy here, so the filtering pass region can be improved. When the energy concentration region is connected to each other, another filtering pass region ( $mask_2$ ) is generated which is show in Fig. 5.

Figure 5 shows that the width of the energy concentration region changes with different energy threshold, and the width of the added area change in proportion. Two filtering pass regions above are generated by alternately sending positive and negative signals. The signal transmitted by actual communication system are random, so the filtering pass region generated by the statistical probability may have some advantages.

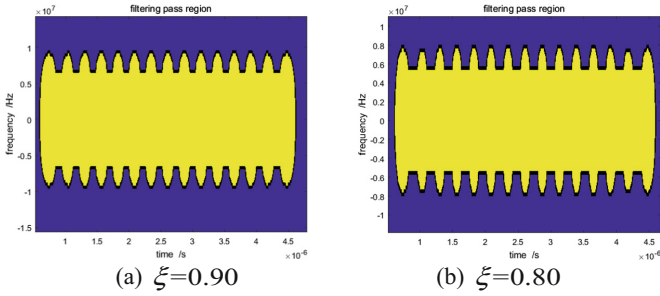


Fig. 5. The filtering pass region ( $mask_2$ ) under different energy thresholds

The process is to send random symbols and generate some filtering pass regions according to the energy threshold  $\xi$ . The filter pass region ( $mask_3$ ) which is shown in Fig. 6 can be generated by superimposing all the filtering pass regions.

Figure 6 shows that the width of filtering pass region shrinks as the energy

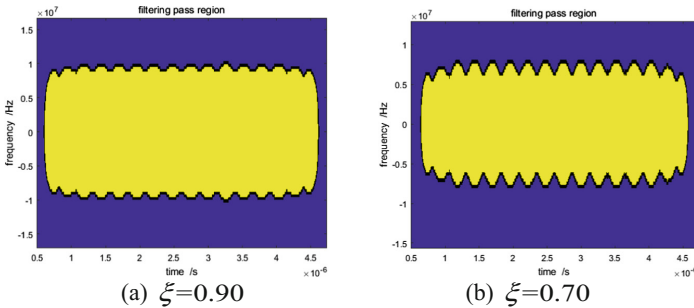


Fig. 6. The filtering pass region ( $mask_3$ ) under different energy thresholds

threshold decreases, and the energy concentration region is connected. It is worth noting that the widest position of  $mask_3$  exactly corresponds to the narrowest position of  $mask_1$  and  $mask_2$ . The filtering pass region is generated by the method of superposition, thus it shows the energy distribution under various transmit sequences. The problem is that there is energy of desired signal at some time-frequency area in  $mask_3$ , but the probability is very small. This region will introduce interference at most time, without much benefit to the desired signal. Then, another filtering pass region is proposed to remove the time-frequency regions with small occurrence probability of the desired signal from  $mask_3$ . The regions are preserved where the occurrence probability is above the threshold  $p$ . Figure 7 shows the filtering pass region ( $mask_4$ ).

Figure 7 shows that the widest part of the region narrows gradually as the probability threshold  $p$  increases. That leads to the filtering pass region becoming isolated.

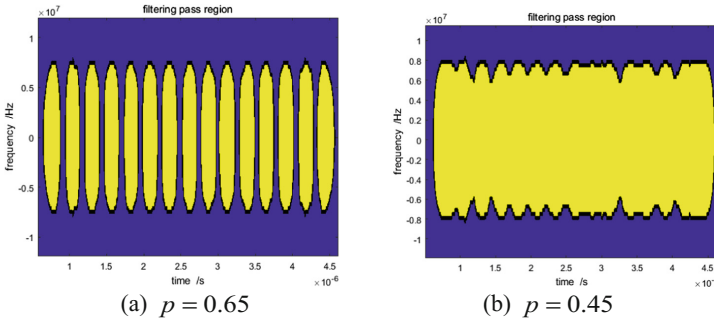


Fig. 7. The filtering pass region ( $mask_4$ ) under different probability thresholds

### 3 Simulation Results

The signal used in this paper includes a desired signal and two interference signals. There is a delay of half a symbol between desired signal and interference signals. The signal transmitted can be expressed as

$$\begin{aligned}
 f(t) &= f_1(t) + f_2(t) + f_3(t) \\
 &= \sum_{n=-\infty}^{+\infty} a_n g(t - \frac{T_0}{2}) \cos(2\pi f t) * \delta(t - nT_0) \\
 &+ \sum_{n=-\infty}^{+\infty} b_n g(t) \cos[2\pi(f + \Delta f)t] * \delta(t - nT_0) \\
 &+ \sum_{n=-\infty}^{+\infty} c_n g(t) \cos[2\pi(f - \Delta f)t] * \delta(t - nT_0)
 \end{aligned} \tag{6}$$

where  $a_n, b_n, c_n$  is the sequences transmitted.  $1/T_0 = R_b = 3.84$  Mbps is the symbol transmission rate.  $g(t)$  is the root raised cosine signal,  $f = 10$  MHz is the carrier frequency of the desired signal,  $f + \Delta f, f - \Delta f$  are the carrier frequencies of the interference signals, and  $\alpha = 0.22$  is raised cosine roll off coefficient.

In this paper, a time-invariant filter is used before time-varying filter. Figure 8 shows the simulation results.

Figure 8(a) shows that several filtering methods perform similarly when  $\Delta f = 5$  MHz. The reason is that the bandwidth of the desired signal is 5 MHz in simulation. The interference is basically mitigated after a low pass filter, so the time-varying filter doesn't provide any advantages. In the case of  $\Delta f = 3.5$  MHz and  $\Delta f = 2.5$  MHz, the time-varying filter with  $mask_2$  performs better. In the case of  $\Delta f = 1.5$  MHz, the time-varying filter with  $mask_1$  performs better. The smaller the adjacent carrier frequency interval, the worse the interference. At this point, although desired signal is attenuated by using smaller filtering pass region, we can also benefits by suppressing more interferences. So it's better to use the time-varying filter with  $mask_1$  when  $\Delta f = 1.5$  MHz.



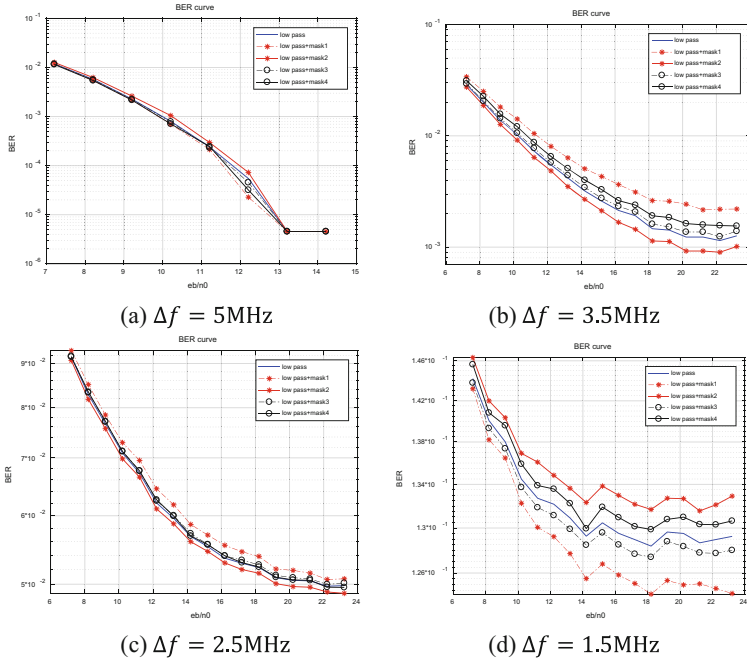


Fig. 8. Simulation under different  $\Delta f$

### 4 Conclusion

In this paper, a method of resource allocation in the time-frequency domain is proposed. On the basis of that, the time-varying filter based on STFT transform is used to suppress the interference. The simulation results show that the method of time-varying filter performs better than the time-invariant filter. Moreover, when the carrier frequency intervals between interference signals and desired signal are small, it is better to use  $mask_1$  in the time-varying filter. When the intervals are big, it is better to use  $mask_2$ .

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