Mutual Coupling Estimation in DOA Estimation for Mixed Wideband Signals

Jiaqi Zhen(✉), Yong Liu, and Yanchao Li

College of Electronic Engineering,
Heilongjiang University, Harbin 150080, China
zhenjiaqi2011@163.com

Abstract. With the electromagnetic frequency getting higher and higher, the distance between the sensors is becoming smaller and smaller, so the mutual coupling is increasingly obvious, it will lead to the mismatch between actual and ideal array manifold. Therefore, a novel mutual coupling error calculation approach in direction of arrival (DOA) estimation for mixed wideband signals is provided in this paper. First, the signals are transformed on the focusing frequency. Then root finding is employed for determining the far-field signals. Finally, mutual coupling error can be calculated according to the orthogonality of far-field signal subspace and noise subspace.

Keywords: DOA estimation · Mutual coupling · Far-field signals
Near-field signals · Wideband signals

1 Introduction

Direction of arrival (DOA) estimation has developed very rapidly in recent years [1–5], and many excellent algorithms has been proposed, such as multiple signal classification (MUSIC) [6], ESPRIT [7], maximum likelihood [8] and so on, all of them can achieve a high precision and resolution capability under ideal condition, but they are always not in common use due to the complex circumstance. One of the reasons is the mutual coupling effect in the array, it is the interference among sensors. Due to the unambiguous demand for the array in direction of arrival (DOA) estimation, the interval of the adjacent sensor is often not allowed larger than half of the wavelength, which leads to the serious mutual coupling, then estimation result will deviate the actual direction, so we need to fundamentally resolve this kind of problem.

In recent years, mutual coupling error calculation has attracted many scholars. Generally speaking, they can be classified into active calibration and passive

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calibration, the former needs a known source in advance: Zhang and Zhu [9] proposed two kinds of compensation algorithms, but they both need to set a standard source; Ng and See [10] presented a maximum algorithm to compensate the no calibration damage, and it has very obvious improvement. The latter omits the hardware, but increase the computation: Friedlander and Weiss [11] opened the door to the passive calibration; then Sellone and Serra [12] provided a method through minimum mean square iteration; In [13], researchers studied the calibration methods for the special array structure, comparing with the active calibration, this kind of method has more computation. All the researchers above have shown the effect of the mutual coupling to the DOA estimation, they also greatly promoted the practical application of corresponding techniques, but there are rare published literatures in DOA estimation for mixed signals.

A novel mutual coupling calculation method in DOA estimation for mixed far-field and near-field wideband signals (abbreviate as FS and NS) is provided in this paper. First, the signals are transformed on the focusing frequency. Then root finding is employed for determining the far-field sources. Finally, mutual coupling error can be calculated according to the orthogonality of far-field signal subspace and noise subspace.

2 Array Signal Model

Define the wavelength of the signal is λ, D is the array aperture, l is the distance between the signal and the reference. Generally speaking, if $l > > 2D^2/\lambda$, it will be in the far-field; if $l \in (\lambda/2\pi,\ 2D^2/\lambda)$, it will be in the near-field. As is shown in Fig. 1, assume that N_1 far-field and N_2 near-field wideband signals impinge onto a uniform linear array with $2M+1$ sensors from directions of $\theta = [\theta_1, \cdots, \theta_{N_1}, \theta_{N_1+1}, \cdots, \theta_N]$, the middle sensor is deemed to be the origin, and $N = N_1 + N_2$, $0 < \theta < \pi$, the space of sensors d is half of the wavelength of the center frequency,

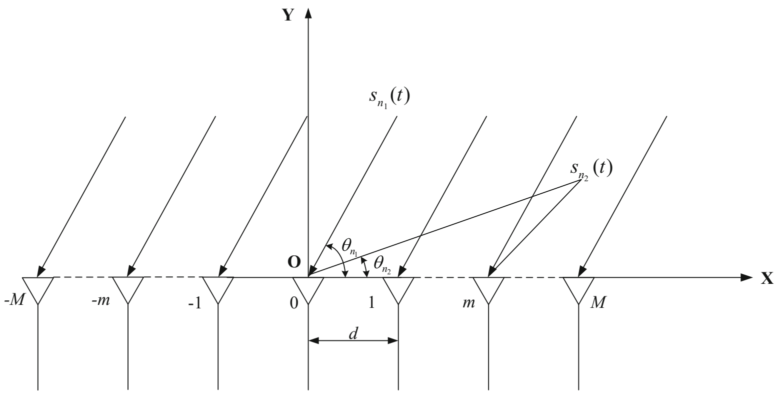

Fig. 1. Signal model

and N_1, N_2 is assumed to be known beforehand, we divide output of array into J parts, that is

$$\boldsymbol{X}(f_i) = \boldsymbol{A}(f_i, \theta)\boldsymbol{S}(f_i) + \boldsymbol{E}(f_i) \quad (i = 1, 2, \cdots, J) \tag{1}$$

where $\boldsymbol{A}(f_i, \theta)$ is the array manifold

$$\begin{aligned}\boldsymbol{A}(f_i, \theta) &= [\boldsymbol{a}_{FS}(f_i, \theta_1), \cdots, \boldsymbol{a}_{FS}(f_i, \theta_{N_1}), \boldsymbol{a}_{NS}(f_i, \theta_{N_1+1}), \cdots, \boldsymbol{a}_{NS}(f_i, \theta_N)] \\ &= [\boldsymbol{A}_{FS}(f_i), \boldsymbol{A}_{NS}(f_i)]\end{aligned} \tag{2}$$

where $\boldsymbol{A}_{FS}(f_i) = [\boldsymbol{a}_{FS}(f_i, \theta_1), \cdots, \boldsymbol{a}_{FS}(f_i, \theta_{N_1})]$ is the array manifold of FS; $\boldsymbol{A}_{NS}(f_i) = [\boldsymbol{a}_{NS}(f_i, \theta_{N_1+1}), \cdots, \boldsymbol{a}_{NS}(f_i, \theta_N)]$ is the array manifold of NS, here

$$\boldsymbol{a}_{FS}(f_i, \theta_{n_1}) = [\exp(-\mathrm{j}2\pi f_i \tau_{-M}(\theta_{n_1})), \cdots, \exp(-\mathrm{j}2\pi f_i \tau_M(\theta_{n_1}))]^{\mathrm{T}} \tag{3}$$

and

$$\begin{aligned}\tau_m(\theta_{n_1}) = m\frac{d}{c}\cos\theta_{n_1} (m &= -M, \cdots, -m, \cdots, 0, \cdots, m, \cdots, M; \\ n_1 &= 1, 2, \cdots, N_1)\end{aligned} \tag{4}$$

is the delay for $s_{n_1}(t)$ arriving at the mth sensor with respect to the origin, meanwhile

$$\boldsymbol{a}_{NS}(f_i, \theta_{n_2}) = [\exp(-\mathrm{j}2\pi f_i \tau_{-M}(\theta_{n_2})) \cdots, \exp(-\mathrm{j}2\pi f_i \tau_M(\theta_{n_2}))]^{\mathrm{T}} \tag{5}$$

according to the geometrical relationship, we can deduce the $\tau_m(\theta_{n_2})$ from Fig. 1.

$$\tau_m(\theta_{n_2}) = \frac{l_{n_2} - \sqrt{l_{n_2}^2 + (md)^2 - 2l_{n_2} md \cos\theta_{n_2}}}{c} \tag{6}$$

reference Taylor series [14], Eq. (6) can be transformed into

$$\tau_m(\theta_{n_2}) = \frac{m^2 d^2}{4l_{n_2} c}\cos 2\theta_{n_2} + \frac{1}{c} md \cos\theta_{n_2} - \frac{m^2 d^2}{4l_{n_2} c} \tag{7}$$

and

$$\boldsymbol{S}(f_i) = [\boldsymbol{S}_{FS}(f_i), \boldsymbol{S}_{NS}(f_i)]^{\mathrm{T}} = [\boldsymbol{S}_1(f_i), \cdots, \boldsymbol{S}_{N_1}(f_i), \boldsymbol{S}_{N_1+1}(f_i), \cdots, \boldsymbol{S}_N(f_i)]^{\mathrm{T}} \tag{8}$$

here $\boldsymbol{S}_{FS}(f_i) = [\boldsymbol{S}_1(f_i), \cdots, \boldsymbol{S}_{N_1}(f_i)]^{\mathrm{T}}$ is signal vector of FS, $\boldsymbol{S}_{NS}(f_i) = [\boldsymbol{S}_{N_1+1}(f_i), \cdots, \boldsymbol{S}_N(f_i)]^{\mathrm{T}}$ is that of NS. $\boldsymbol{E}(f_i)$ is the Gaussian white noise matrix with mean 0 and variance σ^2, then corresponding covariance matrix is

$$\begin{aligned} \boldsymbol{R}(f_i) &= \frac{1}{Z}\boldsymbol{X}(f_i)\boldsymbol{X}^{\mathrm{H}}(f_i) \\ &= \frac{1}{Z}\boldsymbol{A}(f_i,\,\theta)\boldsymbol{S}(f_i)\boldsymbol{S}^{\mathrm{H}}(f_i)\boldsymbol{A}^{\mathrm{H}}(f_i,\,\theta) + \sigma^2(f_i)\boldsymbol{I} \\ &= \boldsymbol{R}_{FS}(f_i) + \boldsymbol{R}_{NS}(f_i) + \sigma^2(f_i)\boldsymbol{I} \end{aligned} \tag{9}$$

the covariance matrix of FS is $\boldsymbol{R}_{FS}(f_i) = \frac{1}{Z}\boldsymbol{A}_{FS}(f_i)\boldsymbol{S}_{FS}(f_i)\boldsymbol{S}_{FS}^{\mathrm{H}}(f_i)\boldsymbol{A}_{FS}^{\mathrm{H}}(f_i)$, that of NS is $\boldsymbol{R}_{NS}(f_i) = \frac{1}{Z}\boldsymbol{A}_{NS}(f_i)\boldsymbol{S}_{NS}(f_i)\boldsymbol{S}_{NS}^{\mathrm{H}}(f_i)\boldsymbol{A}_{NS}^{\mathrm{H}}(f_i)$, Z is the sampling times at f_i.

The degree of mutual coupling is closely related to signal frequency, when there is mutual coupling among sensors, perturbation matrix can be expressed by $\boldsymbol{W}(f_i)$, we itemize Q corresponding the freedom degree of the array, according to the property of uniform linear array, the mutual coupling among sensors is independent with one another, then we know $\boldsymbol{W}(f_i)$ can be expressed as

$$\boldsymbol{W}(f_i) = \begin{bmatrix} 1 & & c_1(f_i) & & \cdots & c_Q(f_i) & \\ c_1(f_i) & 1 & & c_1(f_i) & & & \ddots \\ & c_1(f_i) & & & & & c_Q(f_i) \\ \vdots & & \ddots & & \ddots & \ddots & \\ c_Q(f_i) & & & & & & \\ & \ddots & & & & 1 & c_1(f_i) \\ & & c_Q(f_i) & & & c_1(f_i) & 1 \end{bmatrix} \tag{10}$$

where $c_q(f_i)$ $(q = 1, 2, \cdots, Q)$ is the mutual coupling coefficient, when the distance between two sensor is q, signal frequency is f_i, the steering vector of the array can be revised to

$$\boldsymbol{a}'(f_i,\theta_n) = \boldsymbol{W}(f_i)\boldsymbol{a}(f_i,\theta_n) \quad (n = 1, 2, \cdots, N) \tag{11}$$

corresponding array manifold is

$$\boldsymbol{A}'(f_i,\,\theta) = [\boldsymbol{a}'(f_i,\,\theta_1), \cdots, \boldsymbol{a}'(f_i,\,\theta_N)] = \boldsymbol{W}(f_i)\boldsymbol{A}(f_i,\theta) \tag{12}$$

so the array output is

$$\boldsymbol{X}'(f_i) = \boldsymbol{A}'(f_i,\,\theta)\boldsymbol{S}(f_i) + \boldsymbol{E}(f_i) = \boldsymbol{W}(f_i)\boldsymbol{A}(f_i,\,\theta)\boldsymbol{S}(f_i) + \boldsymbol{E}(f_i) \tag{13}$$

3 Proposed Method

The covariance matrix at the present time is

$$\begin{aligned} \boldsymbol{R}'(f_i) &= \frac{1}{Z}\boldsymbol{X}'(f_i)(\boldsymbol{X}'(f_i))^{\mathrm{H}} \\ &= \frac{1}{Z}\boldsymbol{A}'(f_i, \theta)\boldsymbol{S}(f_i)\boldsymbol{S}^{\mathrm{H}}(f_i)(\boldsymbol{A}'(f_i, \theta))^{\mathrm{H}} + \sigma^2(f_i)\boldsymbol{I} \\ &= \frac{1}{Z}\boldsymbol{W}(f_i)\boldsymbol{A}(f_i, \theta)\boldsymbol{S}(f_i)\boldsymbol{S}^{\mathrm{H}}(f_i)\boldsymbol{A}^{\mathrm{H}}(f_i, \theta)\boldsymbol{W}^{\mathrm{H}}(f_i) + \sigma^2(f_i)\boldsymbol{I} \\ &= \boldsymbol{R}'_{FS}(f_i) + \boldsymbol{R}'_{NS}(f_i) + \sigma^2(f_i)\boldsymbol{I} \end{aligned} \tag{14}$$

then we can employ coherent signal method (CSM) [15] to transform the information on the focusing frequency

$$\boldsymbol{R}''(f_0) = \frac{1}{J}\sum_{i=1}^{J}\boldsymbol{T}(f_i)\boldsymbol{R}'(f_i)\boldsymbol{T}^{\mathrm{H}}(f_i) \tag{15}$$

here $\boldsymbol{T}(f_i) = \boldsymbol{U}'_S(f_0)\left(\boldsymbol{U}'_S(f_i)\right)^{\mathrm{H}}$ is the transforming matrix, $\boldsymbol{U}'_S(f_i)$ is the signal eigenvector of $\boldsymbol{R}'(f_i)$, f_0 is the center frequency, then we can deduce the MUSIC spatial spectrum of FS

$$\begin{aligned} P_{MU-F}(\theta) &= \frac{1}{\left(\boldsymbol{a}'_{FS}(f_0, \theta)\right)^{\mathrm{H}}\boldsymbol{U}_E(f_0)\boldsymbol{U}_E^{\mathrm{H}}(f_0)\boldsymbol{a}'_{FS}(f_0, \theta)} \\ &= \frac{1}{\boldsymbol{a}_{FS}^{\mathrm{H}}(f_0, \theta)\boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{U}_E(f_0)\boldsymbol{U}_E^{\mathrm{H}}(f_0)\boldsymbol{W}(f_0)\boldsymbol{a}_{FS}(f_0, \theta)} \\ &= \frac{1}{Y} \end{aligned} \tag{16}$$

where $\boldsymbol{U}_E(f_0)$ is the noise subspace of $\boldsymbol{R}''(f_0)$, for the sake of analyzing the equation easily, we express the mutual coupling error with another shape

$$\boldsymbol{w}(f_i) = [1, c_1(f_i), \cdots, c_Q(f_i)]^{\mathrm{T}} = [1, \boldsymbol{w}_1^{\mathrm{T}}(f_i)]^{\mathrm{T}} \tag{17}$$

combining [11], $\boldsymbol{W}(f_0)\boldsymbol{a}_{FS}(f_0, \theta)$ can be written as another form

$$\boldsymbol{W}(f_0)\boldsymbol{a}_{FS}(f_0, \theta) = \boldsymbol{G}(f_0, \theta)\boldsymbol{w}(f_0) \tag{18}$$

where $\boldsymbol{G}(f_0, \theta) = \boldsymbol{G}_1(f_0) + \boldsymbol{G}_2(f_0)$, and

$$[\boldsymbol{G}_1(f_0)]_{\alpha,\beta} = \begin{cases} [\boldsymbol{a}(f_0, \theta)]_{\alpha+\beta-1}, & \alpha + \beta \le 2M \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

$$[\boldsymbol{G}_2(f_0)]_{\alpha,\beta} = \begin{cases} [\boldsymbol{a}(f_0,\ \theta)]_{\alpha-\beta+1}, & \alpha \geq \beta \geq 2 \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

then we can abbreviate Y as

$$\begin{aligned} Y &= \boldsymbol{a}_{FS}^{\mathrm{H}}(f_0,\ \theta)\boldsymbol{W}^{\mathrm{H}}(f_0)\boldsymbol{U}_E(f_0)\boldsymbol{U}_E^{\mathrm{H}}(f_0)\boldsymbol{W}(f_0)\boldsymbol{a}_{FS}(f_0,\ \theta) \\ &= \boldsymbol{w}^{\mathrm{H}}(f_0)\boldsymbol{G}^{\mathrm{H}}(f_0,\ \theta)\boldsymbol{U}_E(f_0)\boldsymbol{U}_E^{\mathrm{H}}(f_0)\boldsymbol{G}(f_0,\ \theta)\boldsymbol{w}(f_0) \\ &= \boldsymbol{w}^{\mathrm{H}}(f_0)\boldsymbol{D}(f_0,\ \theta)\boldsymbol{w}(f_0) \end{aligned} \tag{21}$$

where $\boldsymbol{D}(f_0,\ \theta) = \boldsymbol{G}^{\mathrm{H}}(f_0,\ \theta)\boldsymbol{U}_E(f_0)\boldsymbol{U}_E^{\mathrm{H}}(f_0)\boldsymbol{G}(f_0,\ \theta)$, the DOA of FS can be solved by minimizing (21). $\boldsymbol{w}(f_0)$ is not null matrix, so $\boldsymbol{w}^{\mathrm{H}}(f_0)\boldsymbol{D}(f_i,\ \theta)\boldsymbol{w}(f_0) = 0$ sets up only if $\boldsymbol{D}(f_0, \theta)$ is singular, then $\theta_1, \cdots \theta_{N_1}$ can be estimated by solving N_1 roots of the determinant of $\boldsymbol{D}(f_0, \theta)$ below

$$\det[\boldsymbol{D}(f_0, \theta)] = 0 \tag{22}$$

Then we can use the orthogonality between noise and signal space of FS, that is

$$\left(\boldsymbol{U}_E'(f_i)\right)^{\mathrm{H}}\boldsymbol{a}_{FS}'(f_i,\ \theta_{n_1}) = \left(\boldsymbol{U}_E'(f_i)\right)^{\mathrm{H}}\boldsymbol{W}(f_i)\boldsymbol{a}_{FS}(f_i,\ \theta_{n_1}) = \boldsymbol{0}_{(2M+1-N)\times 1} \tag{23}$$

combining the information of $\theta_1, \cdots, \theta_{N_1}$, we have

$$\begin{aligned} &\begin{bmatrix} \left(\boldsymbol{U}_E'(f_i)\right)^{\mathrm{H}}\boldsymbol{a}_{FS}'(f_i,\ \theta_1) \\ \vdots \\ \left(\boldsymbol{U}_E'(f_i)\right)^{\mathrm{H}}\boldsymbol{a}_{FS}'(f_i, \theta_{N_1}) \end{bmatrix} = \begin{bmatrix} \left(\boldsymbol{U}_E'(f_i)\right)^{\mathrm{H}}\boldsymbol{G}(f_i,\ \theta_1) \\ \vdots \\ \left(\boldsymbol{U}_E'(f_i)\right)^{\mathrm{H}}\boldsymbol{G}(f_i,\ \theta_{N_1}) \end{bmatrix}\boldsymbol{w}(f_i) \\ &= \boldsymbol{\Theta}(f_i)\boldsymbol{w}(f_i) = \left[\boldsymbol{B}_1(f_i) \vdots \boldsymbol{B}_2(f_i)\right]\begin{bmatrix} 1 \\ \boldsymbol{w}_1(f_i) \end{bmatrix} = \boldsymbol{0}_{N_1(2M+1-N),1} \end{aligned} \tag{24}$$

where $\boldsymbol{B}_1(f_i)$ is the first column of $\boldsymbol{\Theta}(f_i)$, and $\boldsymbol{B}_2(f_i)$ is the other parts of $\boldsymbol{\Theta}(f_i)$, then we can estimate the mutual coupling error as

$$\hat{\boldsymbol{w}}_1(f_i) = -\mathrm{pin}(\boldsymbol{B}_2(f_i))\boldsymbol{B}_1(f_i) \tag{25}$$

here pin() means solving the pseudo-inverse, then $\boldsymbol{W}(f_i)$ can also be obtained according to (10), and the number of sensors and the signals must satisfy $2M + 1 > N_1 + N_2$, the method we proposed is suitable for mutual coupling error calculation in DOA estimation for mixed wideband signals, so we call it MCW for short.

4 Simulations

The structure of the array is illustrated as Fig. 1, consider two FS and two NS impinge on a uniform linear array with 8 sensors from $(50°,\ 60°)$ and $(70°,\ 80°)$, the frequency of the signals limited in 0.8 GHz–1.0 GHz. The band is divided into 9 frequency bins, suppose that the freedom degree among sensors $Q = 2$, mutual coupling perturbation vector $\boldsymbol{w}_{(1)}(f_i) = [c_1(f_i), c_2(f_i)]^{\mathrm{T}} = [a_1(f_i) + b_1(f_i)\mathrm{j}, a_2(f_i) + b_2(f_i)\mathrm{j}]^{\mathrm{T}}$, $a_1(f_i)$ and $b_1(f_i)$ is selected between (−0.5–0.5) randomly, $a_2(f_i)$ and $b_2(f_i)$ is selected between (−0.25–0.25) randomly; SNR is 12 dB, the sampling times at every frequency is 30, 300 Monte-Carlo simulations are repeated, their average values are deemed as the final results, Figs. 2 and 3 have shown the mutual couple estimation of $c_1(f_i)$ and $c_2(f_i)$ at every frequency, where fi − A means actual value at the ith ($i = 1, \cdots, 9$) frequency bin, and fi − E is the corresponding estimation.

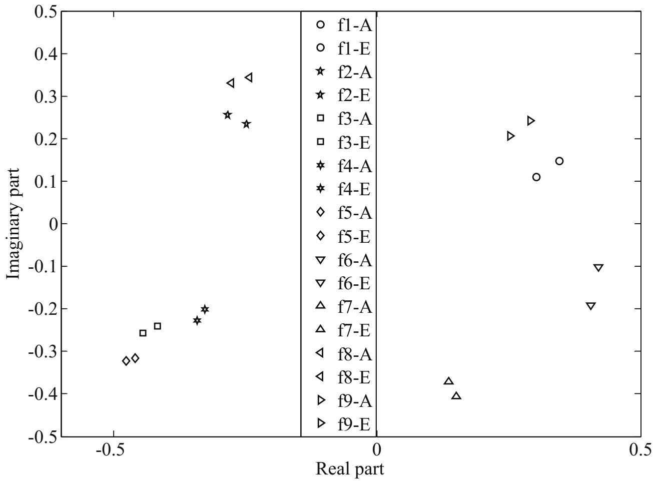

Fig. 2. Mutual couple error estimation of $c1$

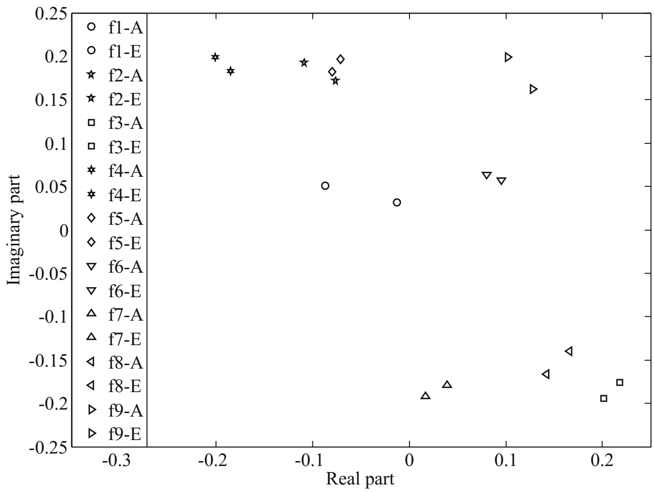

Fig. 3. Mutual couple error estimation of $c2$

It can be seen from Figs. 2 and 3, the MCW method can effectively estimate the mutual couple perturbations, especially when the frequency is near to the center frequency. As the center frequency corresponds to half of the wavelength, so it is more precise than the other parts. Then we can use these results to calibrate the array and acquire the actual DOA of the wideband signals.

5 Conclusion

A novel mutual couple error calculation approach in DOA estimation for mixed wideband signals is provided in this paper. First, the signals are transformed on the focusing frequency. Then root finding is employed for determining the far-field sources. Finally, mutual couple error can be calculated according to the orthogonality of far-field signal subspace and noise subspace. However, it just applies to uniform linear array, we will be committed to study the technique for planar array in future.

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