

# Gain-Phase Error Calculation in DOA Estimation for Mixed Wideband Signals

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**Abstract.** Gain-phase error is inevitable in direction of arrival (DOA) estimation, it will lead to the mismatch between actual and ideal array manifold. Therefore, a novel gain-phase error calculation approach in DOA estimation for mixed wideband signals is provided in this paper. First, the signals are transformed on the focusing frequency. Then peak searching is employed for determining the far-field sources. Finally, gain-phase error can be calculated according to the orthogonality of far-field signal subspace and noise subspace, simulation results manifest the effectiveness of the proposed approach.

**Keywords:** DOA estimation · Gain-phase error · Far-field signals  
Near-field signals · Wideband signals

## 1 Introduction

With the development of array signal processing, more and more DOA estimation methods are springing up [1–8]. Such as multiple signal classification (MUSIC) [9], ESPRIT [10], maximum likelihood [11] and so on, all of them can achieve a high precision and resolution capability under ideal condition. But as a matter of fact, due to the processing technology and some disturbance, gain-phase error often exists in hardware, which leads to the deviation between actual and ideal array manifold, then most DOA estimation methods have deteriorated, so how to calculate this kind of error is very important.

In recent years, gain-phase error calculation has attracted many scholars: Friedlander [12] analyzed its effect to MUSIC algorithm, then approximate expression of the estimation variance is given; Weiss and Friedlander [13] discussed the first and second order statistical properties of the spatial spectrum, then deduced the resolution threshold; Su et al. [14] inferred the expression of spatial spectrum, the relation

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between gain-phase and resolution capacity; Wang et al. [15] concluded the quadric equation in one unknown of average signal to noise ratio (SNR) resolution threshold for MUSIC algorithm. All the research have shown the effect of the gain-phase error to the DOA estimation, they also greatly promoted the practical application of corresponding techniques, but there are rare published literatures in DOA estimation for mixed signals.

A novel gain-phase error calculation approach in DOA estimation for mixed far-field and near-field wideband signals (abbreviate as FS and NS) is provided in this paper. First, the signals are transformed on the focusing frequency. Then peak searching is employed for determining the far-field sources. Finally, Gain-phase error can be calculated according to the orthogonality of far-field signal subspace and noise subspace.

## 2 Array Signal Model

Define the wavelength of the signal is  $\lambda$ ,  $D$  is the array aperture,  $l$  is the distance between the signal and the reference. Generally speaking, if  $l \gg 2D^2/\lambda$ , it will be in the far-field; if  $l \in (\lambda/2\pi, 2D^2/\lambda)$ , it will be in the near-field. As is shown in Fig. 1, assume that  $N_1$  wideband far-field and  $N_2$  near-field sources impinge onto a  $2M + 1$ -element uniform linear array from directions of  $\theta = [\theta_1, \dots, \theta_{N_1}, \theta_{N_1+1}, \dots, \theta_N]$ , the middle sensor is treated as the reference, where  $N = N_1 + N_2$ ,  $0 < \theta < \pi$ , the space of sensors  $d$  equals half of the wavelength of the center frequency, and  $N_1, N_2$  is assumed to be known in advance, then array output is

$$X(f_i) = A(f_i, \theta)S(f_i) + E(f_i) \quad (i = 1, 2, \dots, J) \tag{1}$$

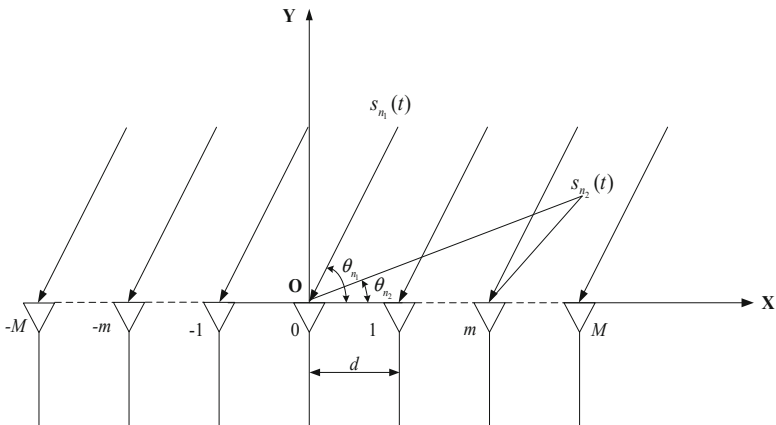


Fig. 1. Signal model

where  $J$  is number of divided narrowband frequency bins,  $\mathbf{A}(f_i, \theta)$  is the array manifold

$$\begin{aligned} \mathbf{A}(f_i, \theta) &= [\mathbf{a}_{FS}(f_i, \theta_1), \dots, \mathbf{a}_{FS}(f_i, \theta_{n_1}), \dots, \mathbf{a}_{FS}(f_i, \theta_{N_1}), \mathbf{a}_{NS}(f_i, \theta_{N_1+1}), \dots, \mathbf{a}_{NS}(f_i, \theta_{n_2}), \dots, \mathbf{a}_{NS}(f_i, \theta_N)] \\ &= [\mathbf{A}_{FS}(f_i), \mathbf{A}_{NS}(f_i)] \end{aligned} \tag{2}$$

where  $\mathbf{A}_{FS}(f_i) = [\mathbf{a}_{FS}(f_i, \theta_1), \dots, \mathbf{a}_{FS}(f_i, \theta_{n_1}), \dots, \mathbf{a}_{FS}(f_i, \theta_{N_1})]$  is the array manifold of FS, and  $\mathbf{a}_{FS}(f_i, \theta_{n_1})$  is the steering vector of  $s_{n_1}(t)$ ;  $\mathbf{A}_{NS}(f_i) = [\mathbf{a}_{NS}(f_i, \theta_{N_1+1}), \dots, \mathbf{a}_{NS}(f_i, \theta_{n_2}), \dots, \mathbf{a}_{NS}(f_i, \theta_N)]$  is the array manifold of NS, and  $\mathbf{a}_{NS}(f_i, \theta_{n_2})$  is the steering vector of  $s_{n_2}(t)$ , here

$$\begin{aligned} \mathbf{a}_{FS}(f_i, \theta_{n_1}) &= [\exp(-j2\pi f_i \tau_{-M}(\theta_{n_1})), \dots, \exp(-j2\pi f_i \tau_{-m}(\theta_{n_1})), \dots, 1, \\ &\quad \dots, \exp(-j2\pi f_i \tau_m(\theta_{n_1})), \dots, \exp(-j2\pi f_i \tau_M(\theta_{n_1}))]^T \end{aligned} \tag{3}$$

and

$$\begin{aligned} \tau_m(\theta_{n_1}) &= m \frac{d}{c} \cos \theta_{n_1} \quad (m = -M, \dots, -m, \dots, 0, \dots, m, \dots, M; \\ n_1 &= 1, 2, \dots, N_1) \end{aligned} \tag{4}$$

is the delay for  $s_{n_1}(t)$  arriving at the  $m$ th sensor with respect to the origin, on the other hand

$$\begin{aligned} \mathbf{a}_{NS}(f_i, \theta_{n_2}) &= [\exp(-j2\pi f_i \tau_{-M}(\theta_{n_2})), \dots, \exp(-j2\pi f_i \tau_{-m}(\theta_{n_2})), \dots, 1, \\ &\quad \dots, \exp(-j2\pi f_i \tau_m(\theta_{n_2})), \dots, \exp(-j2\pi f_i \tau_M(\theta_{n_2}))]^T \end{aligned} \tag{5}$$

according to the geometrical relationship, we can deduce the  $\tau_m(\theta_{n_2})$  from Fig. 1

$$\tau_m(\theta_{n_2}) = \frac{l_{n_2} - \sqrt{l_{n_2}^2 + (md)^2 - 2l_{n_2}md\cos\theta_{n_2}}}{c} \tag{6}$$

reference Taylor series, Eq. (6) can be transformed into [16]

$$\tau_m(\theta_{n_2}) = \frac{m^2 d^2}{4l_{n_2} c} \cos 2\theta_{n_2} + \frac{1}{c} md \cos \theta_{n_2} - \frac{m^2 d^2}{4l_{n_2} c} \tag{7}$$

and

$$\begin{aligned} \mathbf{S}(f_i) &= [\mathbf{S}_{FS}(f_i), \mathbf{S}_{NS}(f_i)]^T \\ &= [\mathbf{S}_1(f_i), \dots, \mathbf{S}_{n_1}(f_i), \dots, \mathbf{S}_{N_1}(f_i), \mathbf{S}_{N_1+1}(f_i), \dots, \mathbf{S}_{n_2}(f_i), \dots, \mathbf{S}_N(f_i)]^T \end{aligned} \tag{8}$$

here  $\mathbf{S}_{FS}(f_i) = [\mathbf{S}_1(f_i), \dots, \mathbf{S}_{n_1}(f_i), \dots, \mathbf{S}_{N_1}(f_i)]^T$  is signal vector of FS,  $\mathbf{S}_{NS}(f_i) = [\mathbf{S}_{N_1+1}(f_i), \dots, \mathbf{S}_{n_2}(f_i), \dots, \mathbf{S}_N(f_i)]^T$  is that of NS.  $\mathbf{E}(f_i)$  is the Gaussian white noise matrix with mean 0 and variance  $\sigma^2$ , then corresponding covariance matrix is

$$\begin{aligned}
\mathbf{R}(f_i) &= \frac{1}{Z} \mathbf{X}(f_i) \mathbf{X}^H(f_i) \\
&= \frac{1}{Z} \mathbf{A}(f_i, \theta) \mathbf{S}(f_i) \mathbf{S}^H(f_i) \mathbf{A}^H(f_i, \theta) + \sigma^2(f_i) \mathbf{I} \\
&= \mathbf{R}_{FS}(f_i) + \mathbf{R}_{NS}(f_i) + \sigma^2(f_i) \mathbf{I}
\end{aligned} \tag{9}$$

the covariance matrix of FS is  $\mathbf{R}_{FS}(f_i) = \frac{1}{Z} \mathbf{A}_{FS}(f_i) \mathbf{S}_{FS}(f_i) \mathbf{S}_{FS}^H(f_i) \mathbf{A}_{FS}^H(f_i)$ , that of NS is  $\mathbf{R}_{NS}(f_i) = \frac{1}{Z} \mathbf{A}_{NS}(f_i) \mathbf{S}_{NS}(f_i) \mathbf{S}_{NS}^H(f_i) \mathbf{A}_{NS}^H(f_i)$ .

We can also model the gain-phase error as

$$\mathbf{W}(f_i) = \text{diag} \left( [W_{-M}(f_i), \dots, W_{-m}(f_i), \dots, 1, \dots, W_m(f_i), \dots, W_M(f_i)]^T \right) \tag{10}$$

where

$$W_m(f_i) = \rho_m(f_i) e^{j\varphi_m(f_i)}, \quad m = -M, \dots, -m, \dots, 0, \dots, m, \dots, M, \tag{11}$$

is the gain-phase error of the sensor  $m$ ,  $\rho_m(f_i)$ ,  $\varphi_m(f_i)$  are the corresponding gain and phase errors, and they are independent with each other, so the array output with gain-phase error is

$$\mathbf{X}'(f_i) = \mathbf{A}'(f_i, \theta) \mathbf{S}(f_i) + \mathbf{E}(f_i) = \mathbf{W}(f_i) \mathbf{A}(f_i, \theta) \mathbf{S}(f_i) + \mathbf{E}(f_i) \tag{12}$$

### 3 Estimation Theory

First, we need to estimate the covariance matrix with gain-phase error

$$\begin{aligned}
\mathbf{R}'(f_i) &= \frac{1}{Z} \mathbf{X}'(f_i) (\mathbf{X}'(f_i))^H \\
&= \frac{1}{Z} \mathbf{A}'(f_i, \theta) \mathbf{S}(f_i) \mathbf{S}^H(f_i) (\mathbf{A}'(f_i, \theta))^H + \sigma^2(f_i) \mathbf{I} \\
&= \frac{1}{Z} \mathbf{W}(f_i) \mathbf{A}(f_i, \theta) \mathbf{S}(f_i) \mathbf{S}^H(f_i) \mathbf{A}^H(f_i, \theta) \mathbf{W}^H(f_i) + \sigma^2(f_i) \mathbf{I} \\
&= \mathbf{R}'_{FS}(f_i) + \mathbf{R}'_{NS}(f_i) + \sigma^2(f_i) \mathbf{I}
\end{aligned} \tag{13}$$

where the covariance matrix of the FS is  $\mathbf{R}'_{FS}(f_i) = \frac{1}{Z} \mathbf{W}(f_i) \mathbf{A}_{FS}(f_i) \mathbf{S}_{FS}(f_i) \mathbf{S}_{FS}^H(f_i) \times \mathbf{A}_{FS}^H(f_i) \mathbf{W}^H(f_i)$ , that of the NS is  $\mathbf{R}'_{NS}(f_i) = \frac{1}{Z} \mathbf{W}(f_i) \mathbf{A}_{NS}(f_i) \mathbf{S}_{NS}(f_i) \mathbf{S}_{NS}^H(f_i) \times \mathbf{A}_{NS}^H(f_i) \mathbf{W}^H(f_i)$ . We can employ some coherent signal subspace methods to transform the received data on the focusing frequency

$$\mathbf{R}''(f_0) = \frac{1}{J} \sum_{i=1}^J \mathbf{T}(f_i) \mathbf{R}'(f_i) \mathbf{T}^H(f_i) \tag{14}$$

here  $\mathbf{T}(f_i) = \mathbf{U}'_S(f_0) (\mathbf{U}'_S(f_i))^H$  is the focusing matrix,  $\mathbf{U}'_S(f_0)$  is the signal subspace of  $\mathbf{R}'(f_i)$ ,  $f_0$  is the focusing frequency, then we can found the MUSIC spatial spectrum of FS

$$\begin{aligned}
 P_{MU-F}(\theta) &= \frac{1}{(\mathbf{a}'_{FS}(f_0, \theta))^H \mathbf{U}_E(f_0) \mathbf{U}_E^H(f_0) \mathbf{a}'_{FS}(f_0, \theta)} \\
 &= \frac{1}{\mathbf{a}_{FS}^H(f_0, \theta) \mathbf{W}^H(f_0) \mathbf{U}_E(f_0) \mathbf{U}_E^H(f_0) \mathbf{W}(f_0) \mathbf{a}_{FS}(f_0, \theta)} \\
 &= \frac{1}{Y}
 \end{aligned} \tag{15}$$

where  $\mathbf{U}_E(f_0)$  is the noise subspace of  $\mathbf{R}''(f_0)$ , in order to be convenient to the derivation, we express the gain-phase error with another form

$$\mathbf{w}(f_i) = [\rho_{-M}(f_i)e^{j\varphi_{-M}(f_i)}, \dots, \rho_{-m}(f_i)e^{j\varphi_{-m}(f_i)}, \dots, 1, \dots, \rho_m(f_i)e^{j\varphi_m(f_i)}, \dots, \rho_M(f_i)e^{j\varphi_M(f_i)}]^T \tag{16}$$

then we can simplify the denominator of the function above

$$\begin{aligned}
 Y &= \mathbf{a}_{FS}^H(f_0, \theta) \mathbf{W}^H(f_0) \mathbf{U}_E(f_0) \mathbf{U}_E^H(f_0) \mathbf{W}(f_0) \mathbf{a}_{FS}(f_0, \theta) \\
 &= \sum_{n_1=1}^{N_1} \mathbf{a}_{FS}^H(f_0, \theta_{n_1}) \mathbf{W}^H(f_0) \mathbf{U}_E(f_0) \mathbf{U}_E^H(f_0) \mathbf{W}(f_0) \mathbf{a}_{FS}(f_0, \theta_{n_1}) \\
 &= \sum_{n_1=1}^{N_1} \mathbf{w}^H(f_0) \left\{ (\text{diag}(\mathbf{a}_{FS}(f_0, \theta_{n_1})))^H \mathbf{U}_E(f_0) \mathbf{U}_E^H(f_0) \text{diag}(\mathbf{a}_{FS}(f_0, \theta_{n_1})) \right\} \mathbf{w}(f_0) \\
 &= \mathbf{w}^H(f_0) \mathbf{D}(f_0, \theta) \mathbf{w}(f_0)
 \end{aligned} \tag{17}$$

where  $\mathbf{D}(f_0, \theta) = \sum_{n_1=1}^{N_1} \left\{ (\text{diag}(\mathbf{a}_{FS}(f_0, \theta_{n_1})))^H \mathbf{U}_E(f_0) \mathbf{U}_E^H(f_0) \text{diag}(\mathbf{a}_{FS}(f_0, \theta_{n_1})) \right\}$ , the DOA of FS can be solved by minimizing (17).  $\mathbf{w}(f_0)$  is not null matrix, so  $\mathbf{w}^H(f_0) \mathbf{D}(f_0, \theta) \mathbf{w}(f_0) = 0$  holds only if  $\mathbf{D}(f_0, \theta)$  is singular, then  $\theta_1, \dots, \theta_{N_1}$  can be estimated by searching  $N_1$  peaks of  $\mathbf{D}(f_0, \theta)$ .

Next, the orthogonality of signal subspace of FS and noise subspace can be utilized

$$(\mathbf{a}'_{FS}(\theta_{n_1}))^H \mathbf{U}'_E = \mathbf{a}_{FS}^H(\theta_{n_1}) \mathbf{W}^H \mathbf{U}'_E = \mathbf{0}_{1 \times (2M+1-N)} \tag{18}$$

it can be transformed into

$$\mathbf{a}_{FS}^H(\theta_{n_1}) \mathbf{W}^H \mathbf{U}'_E = \mathbf{w}^H \{ \text{diag}(\mathbf{a}_{FS}(\theta_{n_1})) \}^H \mathbf{U}'_E = \mathbf{w}^H \mathbf{Q}(\theta_{n_1}) \tag{19}$$

here  $\mathbf{Q}(\theta_{n_1}) = \{ \text{diag}(\mathbf{a}_{FS}(\theta_{n_1})) \}^H \mathbf{U}'_E$ , define  $\mathbf{D}$  as the middle row of  $\mathbf{U}'_E$ , as the middle row of  $\mathbf{a}_{FS}(\theta_{n_1})$  equals 1, the middle element of  $\mathbf{Q}(\theta_{n_1})$  is  $\mathbf{D}$  too. Combining all FS, and let  $\mathbf{Q}(\theta) = [\mathbf{Q}(\theta_1), \dots, \mathbf{Q}(\theta_{n_1}), \dots, \mathbf{Q}(\theta_{N_1})]$ , therefore

$$\mathbf{w}^H \mathbf{Q}(\theta) = \mathbf{w}^H \begin{bmatrix} \mathbf{Q}_1(\theta) \\ \mathbf{D} \cdots \mathbf{D} \\ \mathbf{Q}_2(\theta) \end{bmatrix} = [\mathbf{w}_1^H, 1, \mathbf{w}_2^H] \begin{bmatrix} \mathbf{Q}_1(\theta) \\ \mathbf{D} \cdots \mathbf{D} \\ \mathbf{Q}_2(\theta) \end{bmatrix} = [\mathbf{0}, \dots, \mathbf{0}]_{1 \times (2M+1-N)N_1} \quad (20)$$

where  $\mathbf{w}_1$  is the first  $M$  rows of  $\mathbf{w}$ ,  $\mathbf{w}_2$  is the latter  $M$  rows of  $\mathbf{w}$ ,  $\mathbf{Q}_1(\theta)$  is the first  $M$  rows of  $\mathbf{Q}(\theta)$ ,  $\mathbf{Q}_2(\theta)$  is the latter  $M$  rows of  $\mathbf{Q}(\theta)$ , define  $\mathbf{G} = [\mathbf{D} \cdots \mathbf{D}]_{1 \times (2M+1-N)N_1}$ ,  $\mathbf{w}_1$  and  $\mathbf{w}_2$  will be acquired according to (20), that is

$$\hat{\mathbf{w}}_1 = -(\mathbf{G}(\mathbf{Q}_1(\theta))^{\#})^H \quad (21)$$

$$\hat{\mathbf{w}}_2 = -(\mathbf{G}(\mathbf{Q}_2(\theta))^{\#})^H \quad (22)$$

$\hat{\mathbf{w}}_1$  and  $\hat{\mathbf{w}}_2$  is the estimation of  $\mathbf{w}_1$  and  $\mathbf{w}_2$ ,  $(\cdot)^{\#}$  means solving pseudo-inverse, then we have

$$\hat{\mathbf{w}} = [\hat{\mathbf{w}}_1^T, 1, \hat{\mathbf{w}}_2^T]^T \quad (23)$$

Thus, estimation of gain-phase error can be calculated, and the number of sensors and the signals must satisfy  $2M + 1 > N_1 + N_2$ , the method is suitable for gain-phase error calculation in DOA estimation for mixed wideband signals, so we call it GPW for short.

## 4 Simulations

The structure of the array is illustrated as Fig. 1, consider two FS and two NS impinge on a uniform linear array with 7 omnidirectional sensors from  $(73^\circ, 85^\circ)$  and  $(40^\circ, 65^\circ)$  simultaneously, the frequency of the signals limited in 0.9 GHz–1.1 GHz. The band is divided into 9 frequency bins, here the gain and phase errors are generated in  $[0, 0.5]$  and  $[-20^\circ, 20^\circ]$  randomly respectively, 500 Monte-Carlo trials are repeated. SNR is 6 dB, number of snapshots is 30, Figs. 2 and 3 have shown gain and phase error estimation of different sensors at every frequency bin, where  $i$ th s-A means actual value of  $i$ th sensor, and  $i$ th s-E means the corresponding estimation, we can see from Figs. 2 and 3, GPW can estimate the gain and phase error of the array, especially when the frequency is near to the center point. As the center frequency corresponds to half of the wavelength, so it is more precise than the others.

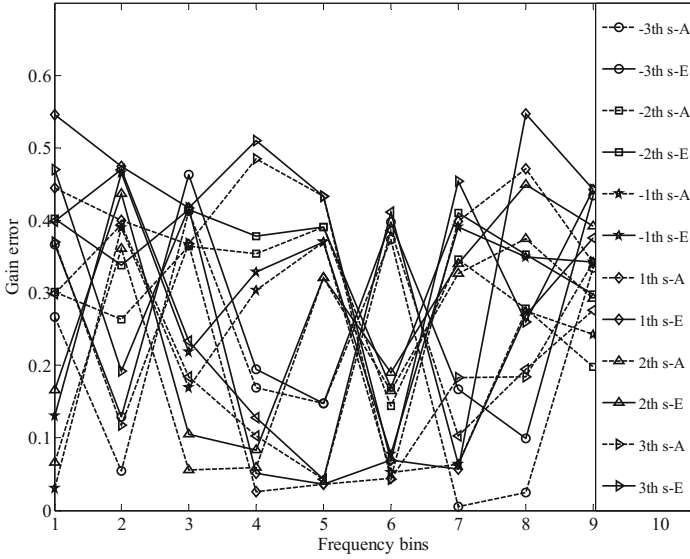


Fig. 2. Gain error estimation

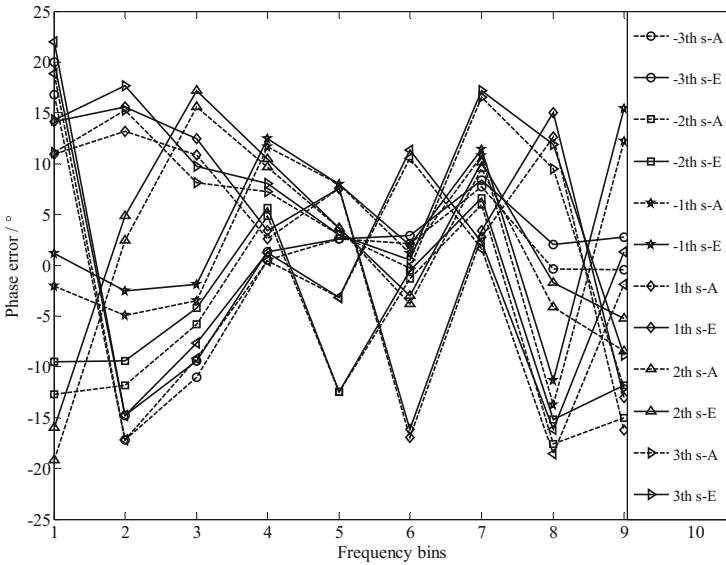


Fig. 3. Phase error estimation

## 5 Conclusion

A novel gain-phase error calculation approach in DOA estimation for mixed wideband signals is provided in this paper. First, the signals are transformed on the focusing frequency. Then peak searching is employed for determining the far-field sources.

Finally, Gain-phase error can be calculated according to the orthogonality of far-field signal subspace and noise subspace. However, it just applies to uniform linear array, we will be committed to study the technique for planar array in future.

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