A New Class of Unequal Error Protection Rateless Codes with Equal Recovery Time Property

Shuang Wu[1], Zhenyong Wang[1,2(✉)], Dezhi Li[1], Gongliang Liu[1], and Qing Guo[1]

[1] School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin 150001, China
wooshuang@126.com, {zywang,lidezhi,liugl,qguo}@hit.edu.cn

[2] Shenzhen Academy of Aerospace Technology, Shenzhen 518057, China

Abstract. A new class of rateless codes which are able to provide unequal error protection (UEP) and equal recovery time (ERT) properties is proposed in this paper. Existing UEP-based LT codes have an important property termed unequal recovery time (URT), which means the data with different reliability requirements can be recovered with different overhead, and it is worth noting that the most important bits (MIB) also have better recovery time performance. The proposed codes can recover data with the same overhead and different error performance. We analyze the asymptotic and experimental error performance of the proposed codes, and give the comparison between the proposed and traditional codes, our results show that the new class of UEP rateless codes are useful for scenarios in which the data have different reliability and same timeliness requirements.

Keywords: Unequal error protection · Asymptotic analysis Finite-length analysis · Equal recovery time · Rateless codes

1 Introduction

LT codes, the first practical codes of the family named rateless codes, were invented by Luby [1]. The code rate of rateless codes are not fixed, in other words, the output symbols can be generated as many as needed.

Rateless codes could also provide an important property which named unequal error protection (UEP). The UEP rateless codes proposed by Rahnavard in [2,3] by distribute different selection probabilities to input symbols in different blocks in the encoding process, Sejdinovic etc. construct the UEP rateless codes by dividing the data into a series windows, and different window have different encoding times [4]. The mentioned schemes are all based on single source, Talari and Rahnavard also proposed a coding scheme by using distributed rateless codes to fit for two source one relay scenario and provide UEP property [5,6]. The UEP rateless codes also be used to solve some practical scenarios where the different data have different reliability requirements.

X. Gu et al. (Eds.): MLICOM 2017, Part I, LNICST 226, pp. 574–583, 2018.
https://doi.org/10.1007/978-3-319-73564-1_57

All the above mentioned UEP rateless codes have the same property which named unequal recovery time (URT). As for UEP rateless codes, the data which have better error performance, they always can be recovered faster, and the others are slower. In the ground transmission systems, as each entire encoding and decoding process only need a very short time slot duration, the URT property is negligible. Therefore, the URT property always be considered as by product, but for some scenarios where the duration time of each entire encoding and decoding process must be concerned, (for example, the deep space data transmission systems), the URT property may influence the user experience. Aiming to solve this problem, we proposed a new class of rateless codes which can provide UEP property and the recovery time of each block is nearly same. The proposed codes could provide UEP property, but as the data in different parts have different error protection level, these data could be recovered nearly at the same time, in other words, this cods could provide equal recovery time (ERT) property. For the proposed LT codes in this paper, the MIB and LIB parts would be recovered nearly at the same time, then the overall timeliness property would be better than the mentioned UEP/URT LT codes.

The paper is organized as follows. In Sect. 2, We review the related works, including the And-Or Tree analysis and the UEP cods [3]. The codes we proposed which could provide UEP and ERT properties are introduced in Sect. 3, a simple example and its asymptotic performance analysis are also given. Section 4 shows the comparison between the proposed UEP/ERT LT codes and the comparative UEP/URT LT codes by asymptotic and experimental results. And the conclusion of this paper is drawn in Sect. 5.

2 Related Works

In this section, we review the coding scheme proposed in [3] and analyze the UEP and URT properties of these codes.

For the UEP property, which means different parts of input symbols would be decoded with different error rates as there are same parts of output symbols are received. The URT property means that different parts of input symbols can be decoded with the same error rate as there are different parts of output symbols are received.

In [3], the authors interpret the URT as the UEP. As the encoding scheme which proposed in this paper, the input symbols in different blocks have different chosen probabilities when encoding an output symbol, the chosen probabilities for input symbols in each block are different, where the MIB symbols have higher chance to be selected to generate an output symbol than in LIB.

Consider a given LT code with parameters $\Omega(x)$, k, γ, where the k input symbols can be divided into a series of blocks $b_1, b_2, \ldots, b_i, \ldots$, and the number of input symbols in each block b_i is $\alpha_i k$, where $\sum_i \alpha_i = 1$. As the encoding scheme proposed in [3], input symbols in each block have their own selected probability when generating an output symbol, for the ith block, the selected probability is q_i, then the input degree distribution of block i which denotes by $\Lambda_i(x) = \sum_d \Lambda_{i,d} x^d$ can be calculated as

$$\Lambda_{i,d} = (\bar{d}_i^d e^{-\bar{d}_i})/d!, \tag{1}$$

where $\bar{d}_i = \frac{\gamma q_i \sum_d d\Omega_d}{\alpha_i}$. The input degree distribution of block i can be rewritten as

$$\Lambda_i(x) = e^{\bar{d}_i \gamma (x-1)}, \tag{2}$$

and the input edge distribution can be given as

$$\lambda_i(x) = e^{\bar{d}_i \gamma (x-1)} = e^{\frac{\gamma^2 q_i \sum_d d\Omega_d}{\alpha_i}(x-1)}. \tag{3}$$

For this encoding scheme, the output degree distributions of each block are all $\Omega(x)$. Therefore, we have the edge distribution $\omega(x) = \sum_d \omega_d x^d$, where $\omega_d = d\Omega_d$. The error rate of input symbols in each block can be calculated by

$$y_{l,i} = \lambda_i\Big(1 - \sum_d \omega_d \big(\sum_i q_i (1 - y_{l-1,i})\big)^{d-1}\Big). \tag{4}$$

As the output edge distributions are uniform, the error rate of input symbols in each block depends on their input edge distributions. As x in (3) is the probability of the output symbols which could transmit "1" to their neighbors, we have $0 \le x \le 1$. All the parameters except q_i are constant, so the value of $\lambda_i(x)$ monotonically decreases as q_i increases. Thus for the ith block, the error rate of input symbols is lower as q_i is larger, whatever the value of overhead γ. In other words, for a given error rate requirement, the input symbols in this block can be recovered with a lower overhead than the others. Therefore, in these codes, the UEP property can be interpret as URT.

3 Equal Recovery Time UEP Rateless Codes

In this section, we describe the proposed UEP rateless codes which provides the equal recovery time property.

The cause of UEP property can be shown by And-Or Tree analysis. For a given LT code is encoded uniformly at random, when the decoding process is finished, the probability of output symbols which could transmit "1" to its neighbor is p_l. Then for an input symbol with degree d, as there are d output neighbors, the error rate of this input symbol can be calculated as $e_d = (1-p_l)^d$.

It is not hard to find that, as $0 < p_l < 1$, e_d is monotonically decreases as d increases. Hence, the input symbols with higher average degree have better error performance than the others.

Then we consider the recovery time of each input symbol for the given LT code, as the definition of the BP decoding process for LT codes. Each input symbol can be recovered only if it is a neighbor of an output symbol with degree 1. Consider a moment in which the error rate of the input symbols is e_l, then for an output symbol with degree d, the probability this output symbol could recover one of its neighbors is $(1-e_l)^d$, as this probability is monotonically decreases as d increases, we could give the following hypothesis: The input symbol connected with output symbols with lower degrees have a higher chance to be recovered earlier.

3.1 Proposed Encoding Scheme

Consider a given LT code with parameters $\Omega(x)$, k, γ, where k input symbols can be divided into a series blocks $b_1, b_2, \dots, b_i, \dots$. The number of input symbols in each block b_i is $\alpha_i k$, where $\sum_i \alpha_i = 1$. To obtain UEP property, all the prior schemes use different selection probabilities or distribute degree distribution for input symbols in different block, these methods make the input symbols in different block have different input degree distribution and different average degree, so that the input symbols can perform different error rate. In these methods, the input symbols with higher average degrees have higher chance to be connected with output symbols with lower degree, and also provide URT property.

Aiming to obtain UEP LT codes without URT property, let $\mathrm{Q} = (q_{i,d})$ be a probability matrix with size $I \times D$, where element $q_{i,d}$ denotes the probability that the input neighbor belongs to the ith block of an output symbol with degree d, and $\sum_i q_{i,d} = 1$.

Consider the ith block, the total number of the edges connected with it E_i can be calculated as $E_i = \sum_d \gamma k d \Omega_d q_{i,d}$.

Then the input degree distribution $\Lambda_i(x)$ of the block i can be calculated as

$$\Lambda_{i,d} = \binom{E_i}{d} \left(\frac{1}{\alpha_i k}\right)^d \left(\frac{\alpha_i k - 1}{\alpha_i k}\right)^{(E_i - d)}, \tag{5}$$

if exist $\alpha_i k \to \infty$, and the average degree of input symbols is denoted by $\bar{d}_i$, where $\bar{d}_i = E_i / \alpha_i k$, Eq. (5) can be also rewritten as shows in (1) and the input edge distribution of this block can be calculated as

$$\lambda_i(x) = e^{\bar{d}_i \gamma (x-1)} = e^{\frac{\gamma^2 \sum_d d \Omega_d q_{i,d}}{\alpha_i}(x-1)}. \tag{6}$$

The expression of the output edge distribution of the ith block $\omega_i(x) = \sum_d \omega_{i,d} x^d$ as $\omega_{i,d} = (d\Omega_d q_{i,d}) / (\sum_d d\Omega_d q_{i,d})$, as $\lambda_i(x)$ and $\omega_i(x)$ are obtained, then we make the And-Or tree analysis for the given LT code. Following with the and or tree theorem which proposed in [9], for block i, the error rate of input symbols in this block can be calculated by $y_{l,i}$, and

$$\begin{aligned} y_{l,i} &= \lambda_i\Big(1 - \sum_d \omega_{i,d}\big(\sum_i q_{i,d}(1 - y_{l-1,i})\big)^{d-1}\Big) \\ &= e^{-\frac{\gamma^2 \sum_d d\Omega_d q_{i,d}}{\alpha_i}(\sum_d \omega_{i,d}(\sum_i q_{i,d}(1 - y_{l-1,i}))^{d-1})}. \end{aligned} \tag{7}$$

Considering the ith block and assuming the input symbol in this block have higher level reliable requirement, we should make the average degree of the input symbols in this block larger than others, which means $\bar{d}_i = \max\{\bar{d}_1, \bar{d}_2, \dots, \bar{d}_I\}$, as the input symbols also need to be recovered at the same time with the others, then the series $q_{i,d}$ should satisfy the following conditions: for a certain degree value $\hat{d}$, where $\hat{d}$ is not too small, if $d < \hat{d}$, $q_{i,d} = \alpha_i$, and if $d \geq \hat{d}$, $q_{i,d} > \alpha_i$, thus the average degree of the input symbols in the block i is larger than the others and for the lower degree output symbols, the probability of them choosing an input symbol in the ith block is equal to the others. Hence, the input symbols in the block i have better error performance than the others and the recovery time diversity is negligible.

3.2 A Special Case with 2 Blocks

For simplicity, consider a simple example with only 2 blocks, $\alpha_1 = 0.1$ and $\alpha_2 = 0.9$. The output degree distribution is $\Omega(x) = 0.007969x^1 + 0.493570x^2 + 0.166220x^3 + 0.072646x^4 + 0.082558x^5 + 0.056058x^8 + 0.037229x^9 + 0.055590x^{19} + 0.025023x^{64} + 0.003137x^{66}$, which is proposed in [15]. The input symbols in the first block are more important than which in the second block.

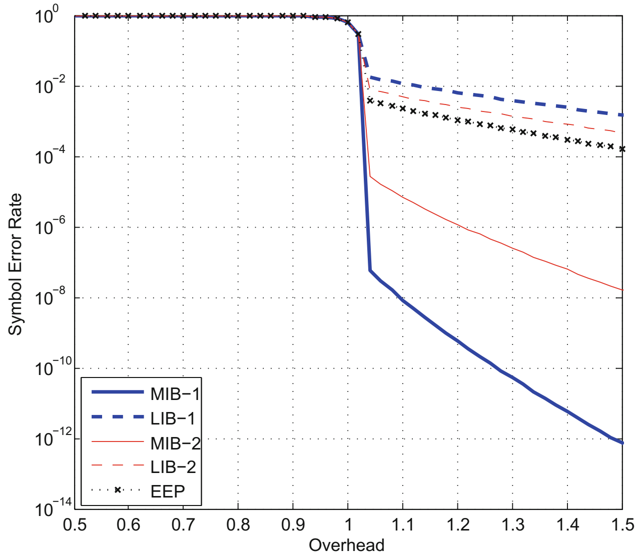

Fig. 1. Asymptotic UEP performance of the given LT code with 2 blocks.

As the And-Or tree iterative expression is given by (7), we can show the asymptotic performance of an given LT code to provide UEP and ERT property. For $1 \leq d \leq 66$, there exist $q_{i,d} = \alpha_i$, then the error rate of the symbols in each block are equal. The performance of this condition is shown as the curve "EEP" in Fig. 1. Then make $q_{1,64} = 0.5$ and $q_{1,66} = 1$, the asymptotic performance of the given code can be shown as the curves "MIB-1" and "LIB-1", the input symbols in block 1 are most important bits (MIB), and the others are less important bits (LIB), as the MIB have been distributed with higher average input degrees, then the error floor have better performance than the LIB, and the beginning of error floor for both MIB and LIB are nearly with the same overhead. Therefore the input symbols in blocks 1 and 2 with the UEP and the equal recovery time (ERT) properties. As the block with higher average degree have better error performance than the lower, then let $q_{1,64} = 0.4$ and $q_{1,66} = 0.8$, and the asymptotic performance of this case is shown as the curves “MIB-2” and “LIB-2”, it is easy to find the difference between the error floor region of case 1 is larger than case 2. Then we can give the following lemma.

Lemma 1. *The difference of the error performance between MIB and LIB would increase as the difference of the average input degree between MIB and LIB.*

3.3 Asymptotic Performance Analysis

Here we should analyze the relationship between the error rate and recovery time of the proposed coding scheme. Consider the given LT code, as there are $n = \gamma k$ output symbols that are received, then for an output symbol with degree d, assume the And-Or Tree iterative process could convergence at the Lth round, then the probability this output symbol could recovered an input symbol is $(\sum_i q_{i,d}(1-y_{L,i}))^{d-1}$, as $\sum_i q_{i,d} = 1$ and $1-y_{L,i} < 1$, then $\sum_i q_{i,d}(1-y_{L,i}) < 1$, for this reason, this probability monotonically decreases as d increases.

Then consider the error rate of the ith block $y_{L,i}$, as this probability is monotonically decreases as the overhead γ increases and exist $0 \leq y_{L,i} < 1$, assume there exists an overhead Γ, which is the beginning of the error floor, if $\gamma < \Gamma$, $1-y_{L,i} << 1$ and cannot be ignored. Therefore for an output with degree d, if d is very high, the probability this output symbol could recover an input symbol $(\sum_i q_{i,d}(1-y_{L,i}))^{d-1}$ is infinitesimal of higher order of probabilities for output symbols with lower degree. As a result, this probability could be ignored. In other words, nearly all the input symbols which have been recovered are recovered by output symbols with lower degree. As for each block, the probabilities input symbols covered by output symbols with lower degrees are the same. Thus if $\gamma < \Gamma$, we have the following $y_{L+1,1} \approx y_{L+1,2}$, which means as the overhead is less than Γ, the input symbols in both 2 blocks have the same error rate.

If $\gamma \geq \Gamma$, exist $1 - y_{L,i} \to 1$, then for the output symbols with high degree d, $(\sum_i q_{i,d}(1-y_{L,i}))^{d-1}$ is not very less and cannot be ignored, then as $q_{1,d} > q_{2,d}$, we have the following inequality $y_{L+1,1} < y_{L+1,2}$, which means that if the overhead is larger than Γ, the input symbols in first block have better error rate performance than the others in the second block.

4 Experimental Results

In this section, we will compare the performances of the comparative UEP LT codes [3] and the proposed codes.

For simplicity, we choose the number of input symbols to be $k = 1000$, then the mentioned degree distribution proposed in [15] and design for the codes with $k = 65536$ is not suitable. For this reason, we design a robust degree distribution for LT codes with $k = 1000$ and make some adjustments as following $\Omega(x) = 0.0782x + 0.4577x^2 + 0.1706x^3 + 0.0750x^4 + 0.0853x^5 + 0.0376x^8 + 0.0380x^9 + 0.0576x^{19}$.

For the comparative UEP LT codes, $q_1 = 0.2189$ and $q_2 = 0.7811$, for the proposed UEP LT codes, the select probability is shown in Table 1.

Table 1. Selection probabilities for 2 blocks when generate an output symbol with different degree

Degree	1	2	3	4	5	6	7	8
Block 1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.5
Block 2	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.5

Figures 2 and 3 show the asymptotic error rate performance of the comparative UEP LT codes and the proposed codes. For the comparative codes, the MIB symbols have better recovery time performance than the MIB symbols of the proposed codes, and the LIB symbols with worse recovery time performance than the LIB symbols of proposed codes. For the scenarios in which only MIB symbols have the requirement to be recovered, then the comparative codes have better recovery time performance than the proposed. But if all the symbols have to be recovered, then the proposed code have better recovery time performance than the comparative codes.

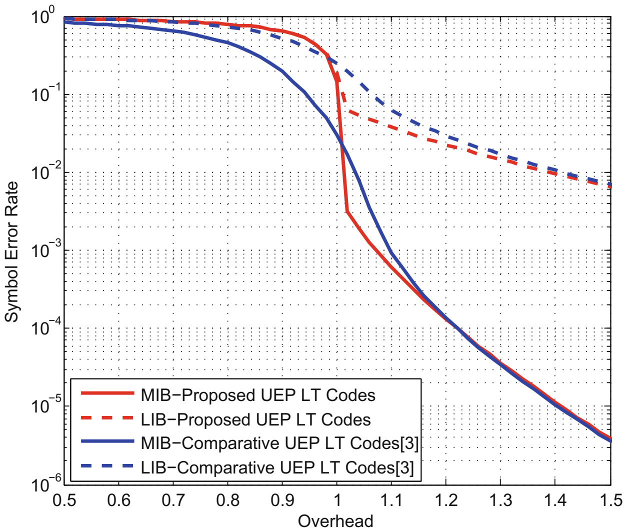

Fig. 2. Asymptotic UEP performance comparison of the given LT codes in log scale.

Figures 4 and 5 show the experimental error rate performance comparison between the comparative UEP LT codes and proposed UEP LT codes. As mentioned before, the number of input symbols is $k = 1000$ and the simulation

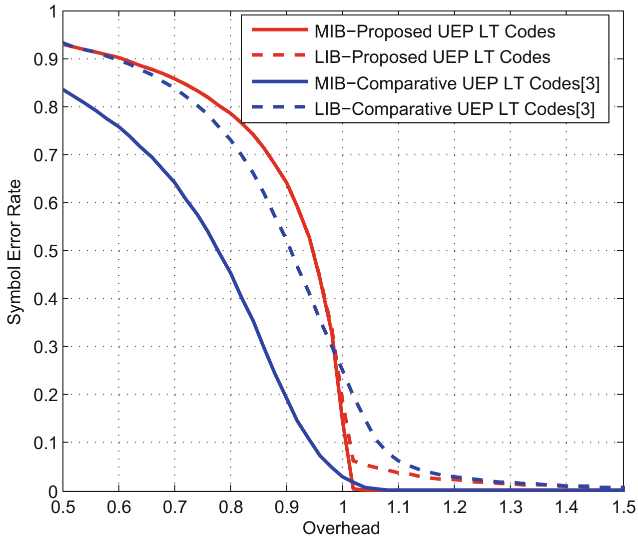

Fig. 3. Asymptotic UEP performance comparison of the given LT codes in linear scale.

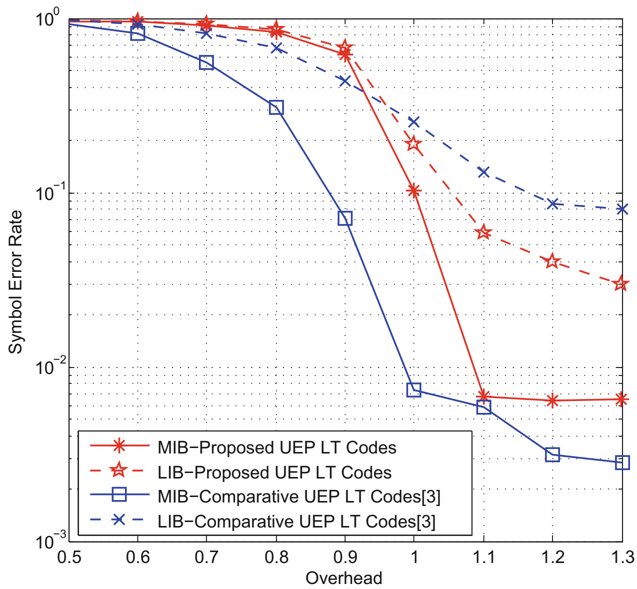

Fig. 4. Experimental UEP performance comparison of finite-length LT codes in log scale.

times is 1000. These two figures show the error performance of the comparative and proposed UEP LT codes in both the log and linear scales. The finite-length experimental results show the same as the asymptotic results, which means for

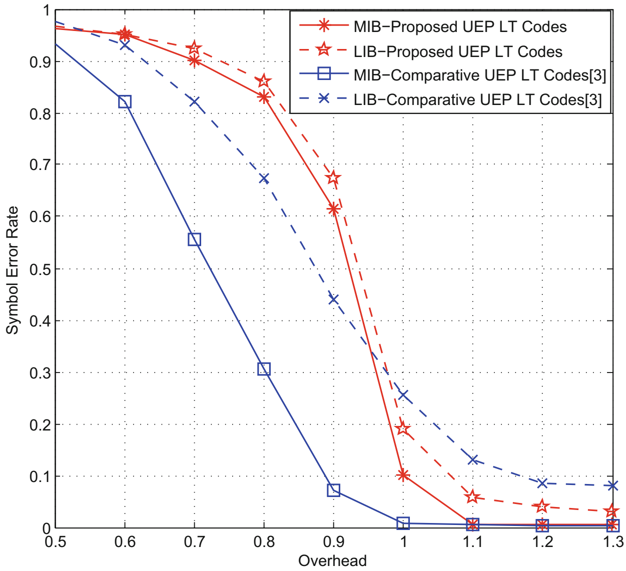

Fig. 5. Experimental UEP performance comparison of finite-length LT codes in linear scale.

the proposed codes the recovery time of both two blocks are nearly the same. But it is worth noting that there exists a difference between the asymptotic and finite-length experimental results, which is although the asymptotic results shows the error performance of comparative and proposed codes. The experimental error performance of these codes are not the same both for MIBs and LIBs. This is because the number of the input symbols is finite, although the overhead γ is large enough, the error rate cannot be considered as infinitesimal, and the probability that an output symbol with higher degree could recover one of its neighbor also cannot tend to 1. And the source of the errors between the asymptotic and finite-length experimental results is the calculated error of the And-Or Tree analysis.

5 Conclusion

In this paper, we proposed a new class of UEP LT codes which provide the ERT property. We derive the asymptotic error performance of the proposed codes, and we also analyzed the error performance by using experimental results. The results shows that we can use these codes to make the data with different reliability requirements to be recovered with the same overhead. In other words, these symbols could be recovered at the same time. For applications where the duration of each encoding and decoding process must be concerned, if all the data have timeliness requirements, the overall timeliness property of the proposed codes would better than the comparative ones. This means that the overall decoding overhead of the proposed codes would less than the comparative codes.

Acknowledgment. This work was supported by National Natural Science Foundation of China. (No. 61601147, No. 61571316, No. 61371100) and "the Fundamental Research Funds for the Central Universities" (Grant No. HIT. MKSTISP. 2016013).

References

1. Luby, M.: LT codes. In: Proceedings of the 43rd Symposium on Foundations of Computer Science, pp. 271–280 (2002)
2. Rahnavard, N., Fekri, F.: Finite-length unequal error protection rateless codes: design and analysis. In: Proceedings IEEE Global Telecommunication Conference, vol. 3, p. 5 (2005)
3. Rahnavard, N., Vellambi, B., Fekri, F.: Rateless codes with unequal error protection property. IEEE Trans. Inf. Theory **53**, 1521–1532 (2007)
4. Sejdinovic, D., Vukobratovic, D., Doufexi, A., Senk, V., Piechocki, R.J.: Expanding window fountain codes for unequal error protection. IEEE Trans. Commun. **57**, 2510–2516 (2009)
5. Talari, A., Rahnavard, N.: Distributed rateless codes with UEP property. In: Proceedings of the IEEE International Symposium on Information Theory, pp. 2453–2457 (2010)
6. Talari, A., Rahnavard, N.: Distributed unequal error protection rateless codes over erasure channels: a two-source scenario. IEEE Trans. Commun. **60**, 2084–2090 (2012)
7. Stefanovic, C., Vukobratovic, D., Chiti, F., Niccolai, L., Crnojevic, V., Fantacci, R.: Urban infrastructure-to-vehicle traffic data dissemination using UEP rateless codes. IEEE J. Sel. Areas Commun. **29**, 94–102 (2011)
8. Talari, A., Rahnavard, N.: A low-latency and error-resilient video-on-demand broadcasting protocol using UEP-rateless codes. In: Forty-Sixth Annual Allerton Conference, pp. 991–995, September 2008
9. Luby, M.G., Mitzenmacher, M., Shokrallahi, A.: Analysis of random processes via And-Or tree evaluation. In: Proceedings of the 9th Annual ACM-SIAM Symposium on Discrete Algorithms, pp. 364–373 (1998)
10. Yuan, X., Ping, L.: Quasi-systematic doped LT codes. IEEE J. Sel. Areas Commun. **27**, 866–875 (2009)
11. Yuan, X., Sun, R., Ping, L.: Simple capacity-achieving ensembles of rateless erasure-correcting codes. IEEE Trans. Commun. **58**, 110–117 (2010)
12. Kozat, U.C., Ramprashad, S.A.: Unequal error protection rateless codes for scalable information delivery in mobile networks. In: Proceedings of the IEEE INFOCOM, pp. 2316–2320 (2007)
13. Karp, R., Luby, M., Shokrollahi, A.: Finite length analysis of LT codes. In: Proceedings of International Symposium on Information Theory, p. 37 (2004)
14. Maatouk, G., Shokrollahi, A.: Analysis of the second moment of the LT decoder. IEEE Trans. Inf. Theory **58**(5), 2558–2569 (2012)
15. Shokrollahi, A.: Raptor codes. IEEE Trans. Inf. Theory **52**, 2551–2567 (2006)