

# Subcarrier Allocation-Based Simultaneous Wireless Information and Power Transfer for Multiuser OFDM Systems

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**Abstract.** Most of existing works on simultaneous wireless information and power transfer (SWIPT) for OFDM systems are studied based on power splitting or time splitting, which may lead to the time delay and the decreasing of sub-carrier utilization. In this paper, a multiuser orthogonal frequency division multiplexing (OFDM) system is proposed, which divides the sub-carriers into two parts, one for information decoding and the other one for energy harvesting. We investigate the optimization problem for maximizing the sum rate of users under the constraint of energy harvesting through optimizing the channel allocation and power allocation. By using the iterative algorithm, the optimal solution to the optimization problem can be achieved. The simulation results show that the proposed algorithm converges fast and outperforms the conventional algorithm.

**Keywords:** SWIPT · OFDM · Subcarrier allocation · Power allocation

## 1 Introduction

Orthogonal frequency division multiplexing (OFDM) is a viable air interface for providing ubiquitous communication services and high spectral efficiency, due to its ability to combat frequency selective multipath fading and flexibility in resource allocation. However, power-hungry circuitries and the limited energy supply in portable devices remain the bottlenecks in prolonging the lifetime of networks and guaranteeing quality of service (QoS). As a result, energy-efficient mobile communication has attracted considerable interest from both industry and academia [1–4]. Traditionally, energy has been harvested from natural renewable energy sources such as solar, wind, and geothermal heat, thereby reducing substantially the reliance on the energy supply from conventional energy sources. As a result, simultaneous wireless information and power transfer (SWIPT) is emerged. In [5], the concept of SWIPT is first put forward and the capacity-energy function is defined. Two classical models are put forward in paper [6, 7], including time switching (TS) model and power switching (PS) model. In TS model, the receiver switches into energy harvesting mode or

information mode within one transmission time. In PS model, the receiver splits the power into two parts with some ratio, one for information decoding and the other one for energy harvesting. SWIPT is combined with multiple-input-single-output (MISO) in [8], where a transmitter with multi-antenna transmits the same information to several banks of single antenna simultaneously. In [9], the optimization algorithm of power splitting based on down-link OFDMA is proposed by the iterative algorithm. A tradeoff between TS and PS is proposed in [10].

Different from the PS and TS models, we study a sub-carrier allocation-based SWIPT for multiuser OFDM systems without a splitter at the receiver. The sub-carriers of each user are separated into information decoding part and energy harvesting part. We address the problem of maximizing the sum rate of users while keeping enough harvested energy. The non-convex problem is solved by an iterative algorithm.

## 2 System Model

Consider a wireless OFDM down-link system consisting of one cognitive base station (CBS) and  $K$  users. Each user is only equipped with one antenna. Let  $K$  denote the sets of  $k$  users for  $k = \{1, 2, \dots, K\}$ . The OFDM bandwidth is assumed to be divided into  $N$  ( $N \geq K$ ) channels equally. The sub-carriers set is denoted with  $N$  for  $N = \{1, \dots, n\}$ . Each sub-carrier must be allocated to only one user. Parts of sub-carriers are used for energy harvesting, and the others are utilized for information decoding simultaneously. We suppose that the channel power gain on each sub-carrier is always constant in one transmission period time, which is given at the base station. Let  $h_{k,n}$  represent the gain of the  $k$ -th user on the  $n$ -th sub-carrier. Then the noise power of each sub-carrier can be corrupted by  $n_k$ , which is modeled as an additive white Gaussian noise (AWGN) random variable with zero mean and variance  $\sigma^2$ . The total transmission power is limited to power budget  $P$ . Therefore, the power allocated on the  $n$ -th sub-carrier is denoted as  $P_n$ . Let  $S^P$  represent the sub-carriers used for energy harvesting to power transfer. Accordingly, the other sub-carriers used for information decoding is denoted by  $S^I$ . Hence,  $S^I_k$  represents the sub-carriers used for information transfer on  $K$ -th user. One sub-carrier cannot be used for energy harvesting and transfer information simultaneously, so we have  $S^I \cap S^P = \emptyset$  and  $S^I \cup S^P = N$  (Fig. 1).

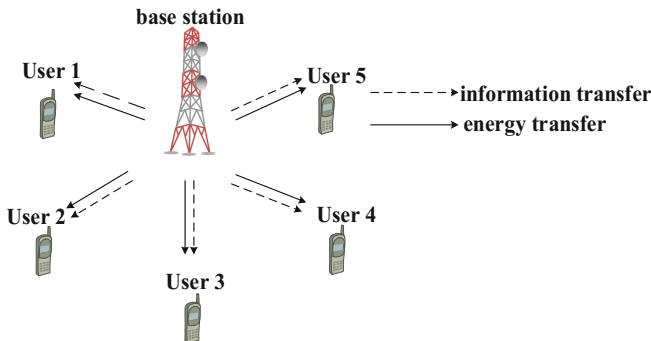


Fig. 1. System model.

### 3 Problem Formulation

Our aim is to maximize the sum rate of OFDM down-link under a restricted condition of the minimum harvested energy for each user. Let  $B_k$  represent to the minimum harvested energy of  $k$ -th user. Since one sub-carrier can only be allocated to one user, let  $\alpha_{n,k}$  be a binary channel allocation index. In other words,  $\alpha_{n,k} = 1$  means that the sub-carrier  $n$  is only allocated to the user  $k$  and  $\alpha_{n,k} = 0$  is determined on other terms. So it is written as

$$\sum_{k=1}^K \alpha_{k,n} = 1, \forall n \in N \tag{1}$$

The sum rate of system can be formulated as

$$\sum_{k=1}^K \sum_{n \in S^I} \alpha_{k,n} \log \left( 1 + \frac{h_{k,n} P_n}{\sigma^2} \right) \tag{2}$$

where  $n \in S^I$ . With energy harvesting efficiency  $\varepsilon$ , the harvested energy during one transmission block for user  $k$  is determined by

$$\sum_{n \in S^P} (\varepsilon h_{k,n} P_n + \sigma^2) \tag{3}$$

For  $\forall k \in K$ . Therefore, optimization model of maximum sum rate can be written as follows

$$\begin{aligned} & \max_{\alpha_{n,k}, S^I, P_n} \sum_{k=1}^K \sum_{n \in S^I} \alpha_{k,n} \log \left( 1 + \frac{h_{k,n} P_n}{\sigma^2} \right) \\ & \text{s.t. } \sum_{n \in N} P_n \leq P \\ & S^P \cup S^I = N \\ & S^P \cap S^I = \emptyset \\ & \sum_{k=1}^K \alpha_{k,n} = 1, \forall n \in N \\ & \alpha_{k,n} \in \{0, 1\}, \forall k \in K, n \in N \\ & P_n \geq 0, \forall n \in N \end{aligned} \tag{4}$$

### 4 Optimal Solution

Due to the non-convex problem, it is impossible to obtain the optimal solution directly. In this section a sub-optimal algorithm is proposed for solving this problem.

We firstly optimize  $\alpha_{k,n}$  with given  $P_n$  and  $S^I(S^P)$ , then optimize  $P_n$  with given  $\alpha_{k,n}$  and  $S^I(S^P)$ , and optimize  $S^I(S^P)$  with given  $\alpha_{k,n}$  and  $P_n$  at last. As mentioned above,  $P_n$  and  $S^I(S^P)$  are determined, so  $\alpha_{k,n}$  is optimized as follows

$$\begin{aligned}
 & \max_{\alpha_{n,k}} \sum_{k=1}^K \alpha_{k,n} \log \left( 1 + \frac{h_{k,n} P_n}{\sigma^2} \right), n \in S^I \\
 & \text{s.t. } \sum_{k=1}^K \alpha_{k,n} = 1, \forall n \in N \\
 & \alpha_{k,n} \in \{0, 1\}, \forall k \in K, n \in N
 \end{aligned} \tag{5}$$

The problem above is regarded as allocating the sub-carrier  $n$  to the assigned user for obtaining the maximum sum rate. In other words, the sub-carrier  $n$  ( $n \in S^I$ ) is allocated to the user  $k$  which can get the maximum  $h_{k,n} P_n$ , i.e.,  $\alpha_{k^*,n} = 1$ ,  $k^* = \arg \max_{k \in K} h_{k,n} P_n$  and  $\alpha_{k^*,n} = 0, \forall k \neq k^*, k \in K$ .

Secondly,  $P_n$  is optimized by  $\alpha_{k,n}$  and  $S^I(S^P)$ . In this proposition, the problem can be rewritten as

$$\begin{aligned}
 & \max_{P_n} \sum_{n \in S^I} \log \left( 1 + \frac{h_{k^*,n} P_n}{\sigma^2} \right) \\
 & \text{s.t. } \sum_{n \in S^P} (\varepsilon h_{k^*,n} P_n + \sigma^2) \geq B_k \\
 & \sum_{n \in N} P_n \leq P \\
 & P_n \geq 0, \forall n \in N
 \end{aligned} \tag{6}$$

Note that  $\alpha_{k^*,n} = 1, \alpha_{k,n} = 0, \forall k \neq k^*, k \in K$ . The converted problem is satisfied with convex model. Thus, the Lagrange dual decomposition is employed for solving this problem. The Lagrange dual function is given as follows:

$$g(\beta_1, \beta_2) = \max_{\{P_n\}} L(P_n) \tag{7}$$

where  $\beta_1, \beta_2$  are the Lagrange multipliers and they are determined by the sub-gradient method. Meanwhile,  $L(P_n)$  is expressed as:

$$\begin{aligned}
 L(P_n) = \sum_{n \in S^I} \log \left( 1 + \frac{h_{k^*,n} P_n}{\sigma^2} \right) + \beta_1 \{ \sum_{n \in S^P} (\varepsilon h_{k^*,n} P_n + \sigma^2) - B_k \} + \\
 \beta_2 (P - \sum_{n \in N} P_n)
 \end{aligned} \tag{8}$$

Then the dual problem can be simplified as follows:

$$\begin{aligned}
 & \min_{\beta_1, \beta_2} g(\beta_1, \beta_2) \\
 & \text{s.t. } \beta_1, \beta_2 \geq 0
 \end{aligned} \tag{9}$$

Because the dual problem is differentiable, it can be solved with the sub-gradient method. The sub-gradient is shown as follow:

$$\Delta \beta_1 = \sum_{n \in S^P} \varepsilon h_{k^*,n} P_n + \sigma^2 - B_k \tag{10}$$

$$\Delta \beta_2 = P - \sum_{n \in N} P_n \tag{11}$$

For given  $\beta_1, \beta_2$ , the optimal power  $P_n$  ( $n \in S^I$ ) is obtained by KKT conditions by using mathematical manipulation, as follows

$$P_n = \left( \frac{1}{\beta_2} - \frac{\sigma^2}{h_k} \right)^+ \tag{12}$$

where  $(\cdot)^+$  denotes  $\max(\cdot, 0)$ . Similarly, the allocated power  $P_n$  used for information transferring is determined as:

$$P_n = \begin{cases} P_{max} & h_{k,n}\varepsilon > \beta_2 \\ P_{min} & h_{k,n}\varepsilon \leq \beta_2 \end{cases} \tag{13}$$

where  $P_{max}$  and  $P_{min}$  represent the maximum and minimum power constraints on information decode respectively. According to  $P_n$  and  $\alpha_{n,k}$ ,  $S^l(S^p)$  can be obtained by substituting (11) and (12) into (8). Consequently, Lagrange dual function can be rewritten as

$$\begin{aligned} L(S^p) &= \sum_{k=1}^K \sum_{n \in N} \alpha_{n,k} \log \left( 1 + \frac{h_{k,n}P_n}{\sigma^2} \right) - \sum_{k=1}^K \sum_{n \in S^p} \alpha_{n,k} \log \left( 1 + \frac{h_{k,n}P_n}{\sigma^2} \right) \\ &\quad + \beta_1 \sum_{n \in S^p} (\varepsilon h_{k,n}P_n + \sigma^2) - \beta_1 B_k + \beta_2 P - \beta_2 \sum_{n \in N} P_n \\ &= \sum_{n \in S^p} \left\{ \beta_1 (\varepsilon h_{k,n}P_n + \sigma^2) - \sum_{k=1}^K \alpha_{n,k} \log \left( 1 + \frac{h_{k,n}P_n}{\sigma^2} \right) \right\} \\ &\quad + \sum_{n \in N} \left\{ \sum_{k=1}^K \alpha_{n,k} \log \left( 1 + \frac{h_{k,n}P_n}{\sigma^2} \right) - \beta_2 P_n \right\} - \beta_1 B_k + \beta_2 P \\ &= \sum_{n \in S^p} F_n + \sum_{n \in N} \left\{ \sum_{k=1}^K \alpha_{n,k} \log \left( 1 + \frac{h_{k,n}P_n}{\sigma^2} \right) - \beta_2 P_n \right\} \\ &\quad - \beta_1 B_k + \beta_2 P \end{aligned} \tag{14}$$

where

$$F_n = \beta_1 (\varepsilon h_{k,n}P_n + \sigma^2) - \sum_{k=1}^K \alpha_{n,k} \log \left( 1 + \frac{h_{k,n}P_n}{\sigma^2} \right) \tag{15}$$

Analyzing the formulate (13), only the first item on the right side is about to  $S^p$ . Thus, the optimal  $S^p$  can be achieved by maximum the item  $F_n$ , as follows

$$S^{p*} = \arg \max_{S^p} \sum_{n \in S^p} F_n^* \tag{16}$$

$S^{p*}$  can be easily gotten by substituting all the  $n$  into  $F_n$  to find the ones which are make  $F_n$  positive, then the rest of the set  $N$  are belongs to  $S^l$ . The proposed algorithm to solve the optimal problem is listed as the Algorithm 1.

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Algorithm 1. Proposed Algorithm for the Joint Optimization Problem

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1. Initialize  $\mathbf{P}_n(\mathbf{0})$ ,  $\alpha_{n,k^*}(\mathbf{0})$  and the error tolerance  $\varepsilon$ ;
  2. Given  $\mathbf{P}_n(\mathbf{0})$  and  $\alpha_{n,k^*}(\mathbf{0})$ , obtain  $\mathbf{S}^I(\mathbf{0})$  from (15);
  3.  $\mathbf{R}_k(\mathbf{0}) = \sum_{k=1}^K \alpha_{n,k}(\mathbf{0}) \log\left(\mathbf{1} + \frac{h_{k,n}P_n(\mathbf{0})}{\sigma^2}\right)$ ,  $\mathbf{n} \in \mathbf{S}^I$ ;
  4. Given  $\mathbf{P}_n(\mathbf{t})$  and  $\mathbf{S}^I(\mathbf{t})$ , set  $\alpha_{n,k^*}(\mathbf{t}) = \mathbf{1}$ ,  $k^* = \arg \max_{k \in K} h_{k,n}P_n$  and  $\alpha_{n,k^*}(\mathbf{t}) = \mathbf{0}, \forall k \neq k^*, k \in K$ ;
  5. Given  $\alpha_{n,k^*}(\mathbf{t} + \mathbf{1})$  and  $\mathbf{S}^I(\mathbf{t})$ , obtain  $\mathbf{P}_n(\mathbf{t} + \mathbf{1})$ ,  $\mathbf{n} \in N$  from (11) and (12);
  6. Given  $\alpha_{n,k^*}(\mathbf{t} + \mathbf{1})$  and  $\mathbf{P}_n(\mathbf{t} + \mathbf{1})$ , obtain  $\mathbf{S}^I(\mathbf{t} + \mathbf{1})$ ;
  7.  $\mathbf{R}_k(\mathbf{t} + \mathbf{1}) = \sum_{k=1}^K \alpha_{n,k}(\mathbf{t} + \mathbf{1}) \log\left(\mathbf{1} + \frac{h_{k,n}P_n(\mathbf{t} + \mathbf{1})}{\sigma^2}\right)$ ,  $\mathbf{n} \in \mathbf{S}^I$ ;
  8.  $t = t + 1$ ;
  9. Repeat (4) to (8) Until  $|\mathbf{R}_k(\mathbf{t} + \mathbf{1}) - \mathbf{R}_k(\mathbf{t})| \leq \varepsilon$ .
  10. Output optimal parameters.
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## 5 Simulation Results

In this section, the performance of the proposed algorithm based on multi-users SWIPT is demonstrated by simulation results.

We denote all the channels involved are following Rayleigh fading with unit mean. For simplicity, we assume that the minimum harvested energy limits for all the users are the same, i.e.,  $B_k = B$ . In addition, we set  $N = 16$ ,  $K = 5$ ,  $\sigma^2 = 1$ ,  $P_{\max} = P/N$ ,  $P_{\min} = 0$ , and  $\varepsilon = 1$ .

Figure 2 shows the convergence behavior of the proposed algorithm. It is seen that the proposed algorithm converges fast, which indicates that the proposed algorithm can be implemented practically. Figure 3 shows that the comparison between the proposed optimization algorithm and the conventional algorithm. It can be seen that the proposed algorithm performance better compared with conventional algorithm. The conventional allocated  $N$  sub-carriers to  $K$  users. By contrast, all the sub-carriers are used for information decoding and the consumed energy comes from the system. In the conventional algorithm, the water-filling approach is used for power allocation, this will cause some power waste. Moreover, all the sub-carriers are allocated to information decoding which results the energy consumption and less power can be used for information decoding. Figure 3 also shows that the sum rate of users increases as the sum transmit power  $P$  increases. This is because with the same target harvested energy, when the sum transmission power increases there will be more power allocated for information decoding.

Figure 4 shows that the total transmission power used for information decoding of user  $k$ . It can be seen that the user 5 is allocated the most power and user 2 is the least. That is because in our emulation, the user 5 has the best channel condition which can achieve higher sum rate.

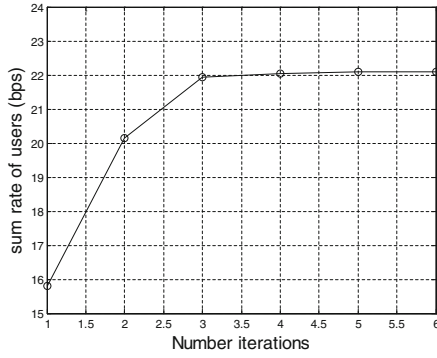


Fig. 2. Convergence behavior of the proposed algorithm.

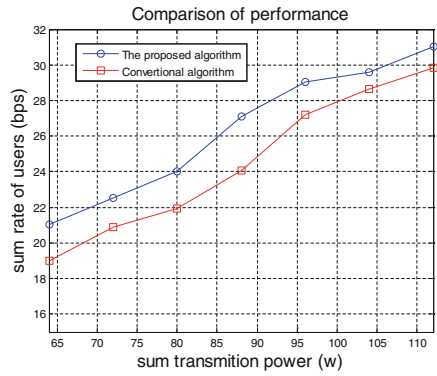


Fig. 3. The sum transmit power vs sum rate of users

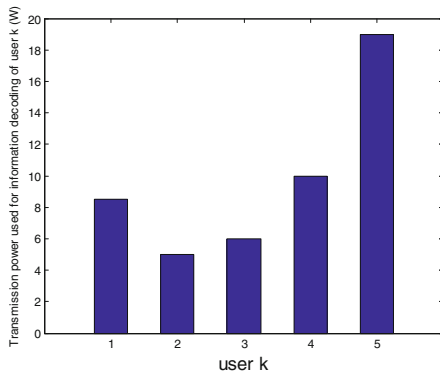


Fig. 4. The power used for information decoding of user  $k$

## 6 Conclusion

In this paper, we propose a joint optimization algorithm for SWIPT-based multi-user OFDM systems. Specifically, the OFDM sub-carriers of each user are divided into two parts, one for information decoding and the other one for energy harvesting. Therefore, we can obtain enough information rate without using time or power splitter at the receiver, on the premise of harvesting enough energy. The simulation results show that the proposed algorithm converges fast and outperforms the conventional algorithm.

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