

Low-Complexity Signal Detection Scheme Based on LLR for Uplink Massive MIMO Channel

Xifeng Chen, Liming Zheng, and Gang Wang^(✉)

Communication Research Center, Harbin Institute of Technology,
Harbin 150001, China
chenxf_hit@163.com, {zheng, gwang51}@hit.edu.cn

Abstract. This paper proposes low-complexity detection algorithms for Massive MIMO system: Multiple Dominant Eigenvector Detection Algorithm (MDEDA) and Antenna Selection Scheme (ASS). Both the schemes calculate the log likelihood ratios (LLRs). Based on the Single Dominant Eigenvector Detection (SDEDA), MDEDA searches transmitted signal candidates in multiple dominant eigenvector directions. For one thing, combined multiple eigenvectors, MDEDA attains better BER performance, for another, it greatly reduces the number of transmitted signal candidates. The ASS contains Single Antenna Selection Scheme (SASS) and Multiple Antenna Selection Scheme (MASS), focus on channel error modeling, the ASS assumes the signal of some antennas corresponding to the constellation points in order to minimize the channel error. SASS searches all transmit antennas, nevertheless, MASS chooses multiple antennas based on the eigenvalue. Finally, SASS gains better BER performance but more complexity. Finally, SASS provides an excellent trade-off between performance and complexity.

Keywords: Massive MIMO · Signal detection · Dominant eigenvector search · Antenna selection · LLR

1 Introduction

Massive MIMO is one of the promising technologies for next-generation wireless communication system with a large number of antennas at the base-station (BS) serving a large number of users concurrently and within the same frequency band [1–3]. The price to pay are the increased complexity of signal processing with the increase of the number of antennas. The optimal signal detection for the system is the maximum likelihood detection (MLD) [4] which can achieve the minimum bit error rate (BER). However, MLD requires a prohibitively large amount of computational complexity that exponentially increases with both the number of data streams and that of constellations.

Linear detection can decrease the complexity greatly, especially when the number of BS antennas is much larger than the number of the uplink users (i.e., the low system loading factors), linear detectors like the minimum mean square error (MMSE) detector are appropriate in terms of both complexity and performance [5]. Unfortunately, for the massive MIMO system whose number of BS antennas and number of the uplink user

are approximate, a single linear detection may result in more loss of performance. The system is exactly our object of study. Hence, the balance of complexity and performance of detection schemes in massive MIMO system have attracted lots of attention.

To reduce such complexity, an iterative receiver based on the turbo principle has been proposed [6]. The iterative receiver can improve reliability of signal detection by exchanging log likelihood ratio (LLR) of coded bits between soft demodulator and soft channel decoder parts. The method in [7] first employs a low complexity in order to find the transmitted signal candidate that maximizes the log likelihood function, that is the maximum likelihood sequence (MLS). Then, the method applies another low complexity algorithm in order to find the transmitted signal candidate that maximize the log likelihood function under a constraint that a coded bit be inverse to that of the estimated MLS, which is referred to as inverse-bit MLS (IB-MLS). Thus, this conventional method needs to apply the low-complexity algorithm for all the coded bits so as to find IB-MLS, which requires high complexity. A one-dimensional algorithm, named plural projection method (PM) was proposed in [8], which can simultaneously find MLS and IB-MLS in the direction of significance eigenvector with MMSE detection as starting point. However, one-dimension search algorithm suffers a severe degradation in BER performance over spatially correlated MIMO channels, because multiple dominant directions of eigenvector are likely to appear [9].

This paper proposes a low-complexity algorithm that can find MLS and IB-MLS in multi-dimensional direction of eigenvector. Based on channel error modeling, this paper also proposes a stream search scheme. Computer simulations demonstrate that the proposed scheme can maintain excellent receiver performance while reducing the complexity drastically.

2 System Model

Consider an uplink massive MIMO system with N_T transmit antennas and N_R receive antennas. Then the associate massive MIMO transmission can be model as

$$y = Hs + n \quad (1)$$

where H is a $N_R \times N_T$ complex channel matrix and is assumed to be flat Rayleigh fading channel and known perfectly at the receiver. At the transmitter side, the information bit s is generated in the source and is first encoded by a convolutional encoder and then mapped to symbols of different constellation points. The mapped complex symbols are divided into N_T separate independent parallel streams with a transmitted signal vector $s = [s_1, s_2, \dots, s_{N_T}]^T \in \vartheta^{N_T}$, where ϑ stands for the complex constellation and $|\vartheta| = 2^Q = M$ with M stands for the modulation order, (e.g., for QPSK, $M = 4$), as a result, each transmit vector s is associated with $N_T \times Q$ binary values $x_{i,b} \in \{0, 1\}$, $i = 1, \dots, N_T$, $b = 1, \dots, Q$, corresponding to the b th bit of symbols of s_i , y is a $N_R \times 1$ received signal vector $y = [y_1, \dots, y_{N_R}]^T$, and n is a $N_R \times 1$ vector of independent zero-mean complex Gaussian distributed noise vector with variance $\sigma^2 = N_0$ per complex entry (Fig. 1).

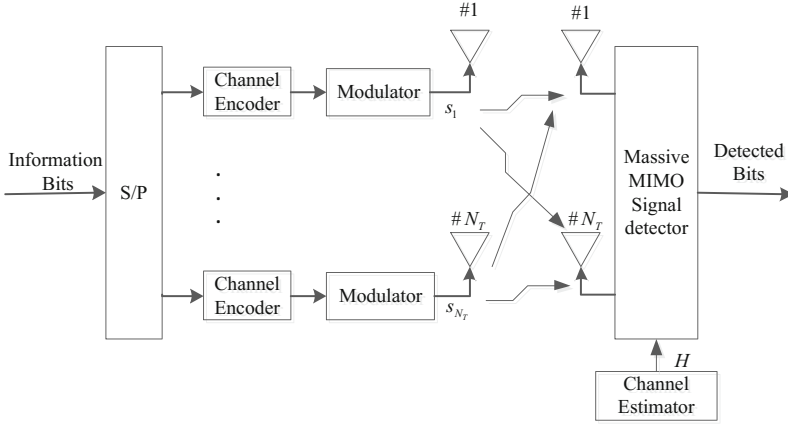


Fig. 1. Massive MIMO system

3 Search in Direction of Dominant Eigenvector Based on MMSE

3.1 Analysis of MMSE Detection

The MMSE detection multiplies y by the weigh matrix and the resultant \hat{x} is given by

$$\hat{x} = PH^H y, \tag{2}$$

$$P = (H^H H + \sigma^2 I_{N_T})^{-1}, \tag{3}$$

where P is the inverse matrix to be considered and I_{N_T} is the N_T -by- N_T identity matrix. The derivation assumed that $\langle ss^H \rangle = I_{N_T}$, and the detected signal is equal to a hard decision of \hat{x} .

According to (2) and (3), we have

$$\langle (s - \hat{x})(s - \hat{x})^H \rangle = \sigma^2 P, \tag{4}$$

then, the difference between s and \hat{x} can be expressed as

$$s - \hat{x} = P^{1/2} \tilde{n}, \tag{5}$$

where \tilde{n} is a N_T -by-1 zero-mean complex Gaussian distributed noise vector of with variance $\sigma^2 = N_0$.

Next, since $H^H H$ is an Hermite matrix and is assumed to be positive definite, the eigenvalue deposition of P^{-1} yields

$$P = (H^H H + \sigma^2 I_{N_T})^{-1} = V D V^H, \quad (6)$$

where V is an N_T -by- N_T unitary matrix and is given by

$$V = [v_1, v_2, \dots, v_{N_T}], \quad (7)$$

$v_k, k \in [1, N_T]$ is the k -th N_T -by-1 normalized eigenvector. D is an N_T -by- N_T diagonal matrix and is given by

$$D = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_{N_T}], \quad (8)$$

where $\lambda_k (> 0)$ is the eigenvalue of the k -th eigenvector v_k . Without loss of generality, $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_{N_T}$ is assumed.

According to (7) and (8),

$$P^{1/2} = V D^{1/2} V^H, \quad (9)$$

$$P^{1/2} \tilde{n} = \sum_{k=1}^{N_T} \sqrt{\lambda_k} v_k^H n v_k. \quad (10)$$

Finally, (5) and (10) imply that the decision errors by the MMSE detection are likely to occur in the direction of v_{k_0} when λ_{k_0} is very large. The direction coincide with eigenvector of P having dominant eigenvalues.

3.2 Conventional Single Dominant Eigenvector Detection Algorithm (SDEDA)

With \hat{x} as a starting point, the one-dimensional search, performs one-dimensional search in the direction of v_p to find MLS and IB-MLS. Suppose that a hard decision of $x_{k,m,p} (1 \leq k \leq N_T, 1 \leq m \leq M, 1 \leq p \leq N_p)$, where N_p is the number of dominant eigenvalue and $1 \leq N_p \leq N_T$.

$$x_{k,m,p} = \hat{x} + \mu(k, m) v_p, \quad (11)$$

$\mu(k, m)$ is a complex number which determines the distance between $x_{k,m,p}$ and \hat{x} . $\mu(k, m)$ is obtained so that the k -th element of the hard decision of $x_{k,m,p}$ can be equal to one of constellations $a(m) (1 \leq m \leq M)$, and is given by

$$\mu(k, m) = \rho \frac{\eta_{k,m}}{(v_p)_k}, \quad (12)$$

$$\eta_{k,m} = a(m) - (\hat{x})_k \quad (13)$$

In the case of rectangular QAM, ρ is set as

$$\rho = \begin{cases} 1.0 & \text{for } a(m) = \text{Dec}[(\hat{x})_k] \\ & \text{for } a(m) \neq \text{Dec}[(\hat{x})_k] \\ 1 + \frac{\xi d_{\min}}{2|\text{Re}(\eta_{k,m})|} & |\text{Re}(\eta_{k,m})| > |\text{Im}(\eta_{k,m})| \\ & \text{for } a(m) \neq \text{Dec}[(\hat{x})_k] \\ 1 + \frac{\xi d_{\min}}{2|\text{Im}(\eta_{k,m})|} & |\text{Re}(\eta_{k,m})| \leq |\text{Im}(\eta_{k,m})| \end{cases} \quad (14)$$

where $\text{Dec}[\cdot]$ denotes the hard decision operation and ξ is a real number to satisfy $|\xi| \leq 1$. d_{\min} is the minimum distance between the constellations; $d_{\min} = \sqrt{2}$ for QPSK modulation.

MLS and IB-MLS are selected from $\text{Dec}[x_{k,m,p}]$ plus $\text{Dec}[\hat{x}]$ on the basis of the matrix. Since the number of $x_{k,m,p}$ is less than or equal to $N_T M N_p$, the number of the hard decisions called transmitted signal candidates is at most $N_T M N_p + 1$.

3.3 Proposed Multiple Dominant Eigenvector Detection Algorithm (MDEDA)

Transmitted signal may get performance degradation in several directions. So compared with the one-dimensional search scheme above, the multi-dimension search scheme searches transmitted signal in multiply dominant directions of eigenvector. The detail is as following.

Compared with (11), transmitted signal candidates are given by

$$x_{k,m} = \hat{x} + \sum_{p=1}^{N_p} \mu_p(k, m) v_p, \quad (15)$$

where $\mu_p(k, m)$ is step size at the p -th dominant direction of eigenvector.

Let us assume that the k -th element of the candidate is equal to $a(m)$, where $m(1 \leq m \leq M)$ is an integer and $a(m)$ is one of the constellation point. So we have

$$\sum_{p=1}^{N_p} \mu_p(k, m) (v_p)_k = a(m) - (\hat{x})_k, \quad (16)$$

where $(\cdot)_k$ denotes the k -th element of a vector. The equation can be rewritten in a vector format as

$$a(m) - (\hat{x})_k = \tilde{v}_k^H \mu, \quad (17)$$

$$\tilde{v}_k^H = [(v_1)_k, (v_2)_k, \dots, (v_{N_p})_k], \quad (18)$$

$$\mu^H = [\mu_1(k, m), \mu_2(k, m), \dots, \mu_{N_p}(k, m)], \quad (19)$$

where \tilde{v}_k^H and μ are N_p -by-1 vectors.

Log likelihood function can be transformed into

$$\begin{aligned} L(s) &= \|(y - H\hat{x}) - H(s - \hat{x})\|^2 \\ &= L(\hat{x}) + \sigma^2 \left(\|\hat{x}\|^2 - \|s\|^2 \right) + (s - \hat{x})^H P^{-1} (s - \hat{x}). \end{aligned} \quad (20)$$

When SNR is high, the second term can be neglected. Substituting (6) and (8) into (20) result in

$$L(x_{k,m}) \approx L(\hat{x}) + \sum_{p=1}^{N_p} (\lambda_p)^{-1} \left| v_p^H (s - \hat{x}) \right|^2. \quad (21)$$

The equation can be rewritten in a vector format as

$$L(x_{k,m}) \approx L(\hat{x}) + \|\tilde{D}^{-1/2} \mu\|^2, \quad (22)$$

$$\tilde{D}^{-1} = \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_{N_p}]. \quad (23)$$

The proposed algorithm performs the maximum likelihood estimation of μ for obtaining candidate of s . The minimization of $L(x_{k,m})$ under the constraint of (16) can be solved by the method of Lagrange multiplier. Thus, the estimation becomes equivalent to finding μ that minimizes the following cost function $f(\mu)$:

$$f(\mu) = \mu^H \tilde{D}^{-1} \mu + \omega [a(m) - \hat{x}_k - \tilde{v}_k^H \mu] + \omega^* [a^*(m) - \hat{x}_k^* - \mu^H \tilde{v}_k], \quad (24)$$

where ω is the complex Lagrange multiplier. By calculation, the desired step size μ is obtained as

$$\mu = [a(m) - (\hat{x})_k] \tilde{D} (\tilde{v}_k^H \tilde{D} \tilde{v}_k)^{-1} \tilde{v}_k. \quad (25)$$

MLS and IB-MLS are selected from the set C , whose element is $\text{Dec}[x_{k,m}]$ plus $\text{Dec}[\hat{x}]$ on the basis of the matrix. Since the number of $x_{k,m}$ is less than or equal to $N_T M$, the number of the hard decisions called transmitted signal candidates is at most $N_T M + 1$. Finally, calculate the LLR [9] of these candidates.

4 Antenna Selection Scheme (ASA) Based on Decision Errors Modeling

According to (4), we may as well assume $d = s - \hat{x}$ and e follows complex Gaussian distribution, thus we have

$$\langle dd^H \rangle = \sigma^2 P, \quad (26)$$

$$p[d] = \frac{1}{(\pi\sigma^2)^{N_T} \det P} \exp\left(-\frac{d^H P^{-1} d}{\sigma^2}\right), \quad (27)$$

Next, we propose the single antenna selection scheme (SASS), which chooses the k -th ($1 \leq k \leq N_T$) antenna, assume that s_k is equal to a modulation constellation point $b(m)$ ($1 \leq m \leq M$), the decision error of the k -th antenna is

$$d_k = b(m) - \hat{x}_k = d(m, k). \quad (28)$$

Under the constraint of (28), apply the Lagrange multipliers in terms of decision error e :

$$L[d] = d^H P^{-1} d + \alpha (R_k^H d - d(m, k)) + \alpha^* (d^H R_k - d^*(m, k)), \quad (29)$$

where α is a complex Lagrange multiplier, and R_k is an N_T -by-1 unit vector of which the k -th element is 1 and the others are 0.

Finally,

$$\alpha^* = -(R_k^H P R_k)^{-1} d(m, k), \quad (30)$$

$$\hat{d} = (R_k^H P R_k)^{-1} d(m, k) P R_k = \frac{P_k}{P_{kk}} d(m, k), \quad (31)$$

where P_k and P_{kk} are the k -th column vector and the (k, k) -th. Let $\hat{s}(m, k)$ denotes detected signal s . So $\hat{s}(m, k)$ can be given by

$$\hat{s}(m, k) = \hat{x} + \frac{P_k}{P_{kk}} [b(m) - \hat{x}_k], \quad (32)$$

When $\hat{s}(m, k)$ are obtained with all combinations of m and k , the number of candidate is $1 + N_T M$, then the final detected signal \hat{s} is selected as the one according to log likelihood ratio.

In the scheme above, we choose just one antenna, to obtain more performance gain, we extend the number of antenna to plural l , which has $C_{N_T}^l$ antenna selection in total. There is no doubt that the number of candidate vectors is increased greatly, which results in high computation complexity. So select just one set of antenna based on some principle is essential, which is named as Multiple Antenna Selection Scheme (MASS).

The antennas of which transmission performance is degraded owing to the MMSE should be selected. Combined with Multi-Dimensional Search scheme above, we can choose l antennas of which eigenvalue is relatively small. Then the number of candidate transmitted vector is $1 + M^l$, which increases exponentially with the number of antennas we choose. So if we choose too many antennas, there's no doubt that the complexity is unacceptable.

5 Simulation Results and Analysis

Computer simulations were conducted to verify performance of the proposed algorithms. The simulation conditions are listed in Table 1, in the following, we simulated six kinds of detection schemes, including SASS, MDEDA, SDEDA, MASS, MMSE-OSIC, MMSE-PIC.

Table 1. Simulation conditions

Number of transmit antennas N_T	32
Number of transmit antennas N_R	32
Number of dominant eigenvector $N_P = l$	2
Modulation	QPSK
Channel coding	Convolution code
Decoding	LLR
Range of SNR	0–20 dB
Channel model	Rayleigh fading ($\sqrt{\frac{\pi}{2}}, 2 - \frac{\pi}{2}$)

Observing from Figs. 2 and 3, BER of SASS and MDEDA is superior to that of other schemes. Furthermore, SASS outperforms MASS, and MDEDA is better than SDEDA, corresponding to the theory above. MMSE-OSIC and MMSE-PIC have poor detection performance, and the former's complexity increases with transmitted

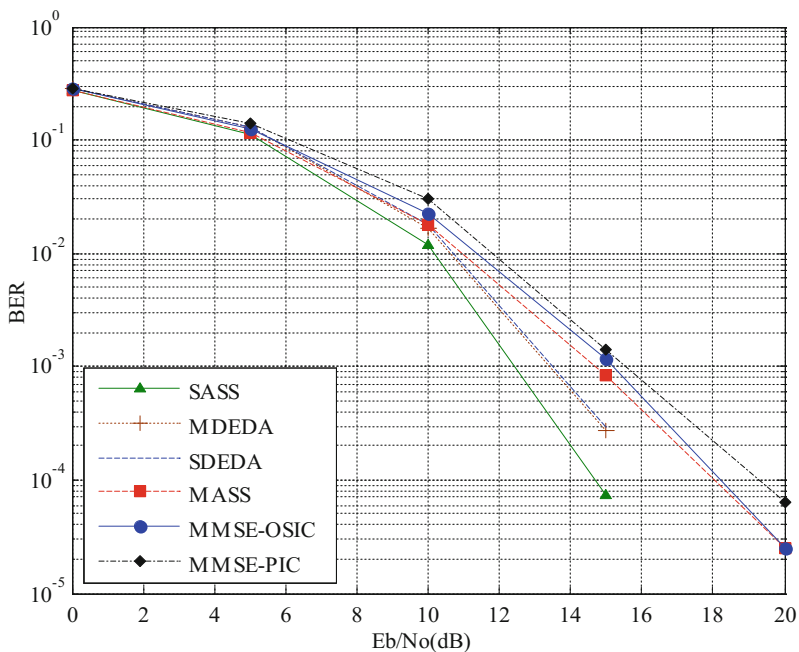


Fig. 2. Average BER with $N_T = N_R = 32$

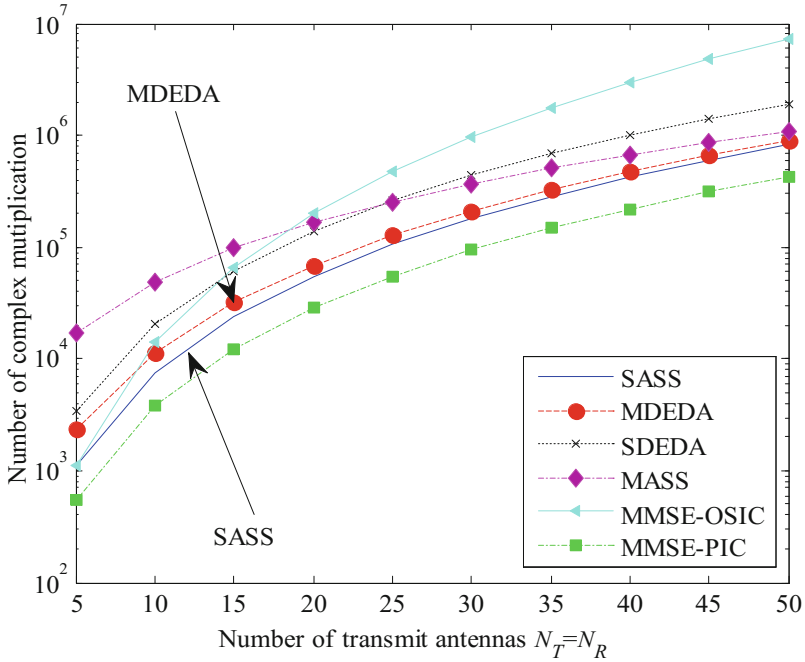


Fig. 3. Computational complexity with $N_T = N_R = 32$

antennas rapidly, however, MMSE-PIC has the least complexity to adapt to system of which detection performance requirements are not high. Fortunately, SASS and MDEDA get a superior trade-off between performance and complexity. In the condition of Fig. 2, MLD has complexity of 2.0×10^{22} , SASS reduce the complexity to about 10^{-18} of that of MLD. Compared with the other schemes, SASS achieves low-complexity detection algorithm and ensures the BER performance.

6 Conclusion

This paper has proposed low-complexity signal detection algorithms for Massive MIMO system, including MDEDA, SASS and MASS. MDEDA combined the effect of several eigenvector, thus attaining better BER performance and less complexity. Focusing on error modeling, SASS and MASS are proposed. SASS searched all transmit antennas, and MASS just choose several antennas. SASS got less complexity and superior BER performance, compared with MDEDA. In the system of Massive MIMO, SASS obtained a superior trade-off between performance and complexity.

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