Distributed Compressive Sensing Based Spectrum Sensing Method

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Abstract. For multi-antenna system, the difficulties of preforming spectrum sensing are high sampling rate and hardware cost. To alleviate these problems, we propose a novel utilization of distributed compressive sensing for the multi-antenna case. The multi-antenna signals first are sampled in terms of distributed compressive sensing, and then the time-domain signals are reconstructed. Finally, spectrum sensing is performed with help of energy-based sensing method. To evaluate the proposed method, we do the corresponding simulations. The simulation results proves the proposed method.

Keywords: Distributed compressive sensing · Spectrum sensing
Joint sparse model · Time-domain detection

1 Introduction

Spectrum sensing is the base of cognitive radio. At present, some known methods mainly conclude Energy-based algorithm, cyclostationary detection and eigenvalue-based algorithm [1, 2]. Generally speaking, these methods are applied in the individual antenna case. However, with the growing requirements of date rate and the improvement of wireless communication technologies, multi-antenna technologies have already been applied in many wireless communication systems. Subsequently, spectrum sensing under the multi-antenna circumstances become a problem to be solved. Currently, some multi-antenna based spectrum sensing methods were proposed, such as random matrix based methods and GLRT (generalized likelihood ratio test) methods [3–7]. For random matrix based methods, the signals sampled from multiple antennas are comprised of a random matrix, and then some parameters, such as eigenvalue, are extracted to perform spectrum sensing.

GLRT-based methods are a kind of technologies as solving the problem of multi-antenna spectrum sensing. In [4–6], some eigenvalues of sampled covariance matrix are used as test statistic. In literature [7], GLRT is exploited directly as test statistic, and the idea is evaluated in OFDM and MIMO system. It is well known that multi-antenna technology bring some advantages for the wireless communication. On the other hand, some disadvantages have also been introduced inevitably, such as too much

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data and high sampled frequency. Fortunately, compressed sensing provides a practical idea to deal with these difficulties. In 2006, compressed sensing is proposed [8], and then it has been fast applied to many fields, including the wireless communication, signal processing and image processing. In the view of compressed sensing, sample and compression are performed simultaneously, and the signal is sampled based on the signal sparsity but not the bandwidth used in the Nyquist sampling theorem, which can alleviate the computational complexity and hard cost. Meanwhile, in order to fully exploit the correlation of inter-signal and intra-signal, the framework of distributed compressed sensing is built on the base of the joint sparse model [9, 10], which bridge between multi-antenna based wireless communication and compressed sensing. More importantly, computational complexity is further reduced because of the correlation structure.

In this paper, we obtain the sampled signals in terms of distributed compressed sensing, which can reduce the hard cost and further decrease the subsequent computational complexity, and then the energy-based spectrum sensing is adopted. Because of the utilizing of the correlation of multiple antennas, the sparsity in single antenna case is extended to the multiple antenna case by virtue of joint sparse model. It follows that higher reconstruction probability is obtained with the constriction of the same sensing measurement.

2 The Description of the Proposed Method

2.1 Distributed Compressive Sensing

We suppose that the number of antennas is J, and the received signal ensemble can be expressed as $X = [x_1 \quad x_2 \quad \cdots \quad x_J]^T$, where $x_i \in R^N$. In the framework of distributed compressive sensing, the compressed measurements are written as

$$Y = \Phi X \tag{1}$$

where $Y = [y_1 \quad y_2 \quad \cdots \quad y_J]^T$, $\Phi = \begin{bmatrix} \Phi_1 & 0 & \cdots & 0 \\ 0 & \Phi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_J \end{bmatrix}$. For the individual signal, $y_i = \Phi_i x_i$, where $y_i \in R^M, \ \Phi_i \in R^{M \times N}$.

It is well known that the concept of common sparsity is built on the single signal. For multiple antennas, however, the multiple signals possess intra-signal and inter-signal correlation. Joint sparse models (JSM), called common/innovation component JSMS, were introduced to describe these characteristics, which includes three specific models, named JSM-1, JSM-2 and JSM-3. Therefore, in the framework of distributed compressive sensing, JSM is written uniformly as

$$X_j = Z_C + Z_j, \ j \in \{1, \ 2 \cdots J\} \tag{2}$$

where Z_C denotes the common component, and Z_j is the innovation component. Specifically, they can be sparsely represented as

$$\begin{aligned} Z_C &= \Psi_C \cdot \Theta_C, \quad \|\Theta_C\|_0 = K_C \\ Z_j &= \Psi_j \cdot \Theta_j, \quad \|\Theta_j\|_0 = K_j \end{aligned} \tag{3}$$

where $\|\cdot\|_0$ denotes the l_0-norm, e.g., the number of nonzero values of signal vector. In this setting, the signal ensemble X can be rewritten as

$$X = \Psi\Theta \tag{4}$$

where $\Psi = \begin{bmatrix} \Psi_C & \Theta_j & 0 & \cdots & 0 \\ \Psi_C & 0 & \Theta_j & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Psi_C & 0 & 0 & \cdots & \Theta_j \end{bmatrix}$, $\Theta = \begin{bmatrix} \Theta_C^T & \Theta_1^T & \Theta_2^T & \cdots & \Theta_J^T \end{bmatrix}^T$.

The different sparsity assumptions regarding the common and innovation component correspond to different models. When both of the common and innovation components are sparse, we call it JSM-1 model. When there exist no common components in the signal ensemble, we refer to it as JSM-2 model. In this model, each innovation component of signal ensemble is sparse, and all the signals possess the same sparse support but have different nonzero values in the same locations. A practical scenario well-modeled by JSM-2 model is MIMO communication system we often encounter in this paper. If the common component is not factorized sparsely, we name the model as JSM-3 model. It is widely recognized that the signal ensemble from multiple antennas of MIMO satisfy the condition of the common and innovation component. It follows that we restrict out attention on JSM-2 model. Currently, the recovery algorithms in the framework of JSM model are categorized into trivial pursuit and iterative greedy pursuit, such as DCS-SOMP arisen from conventional OMP algorithm.

2.2 The Proposed Algorithm

In order to interpret the proposed method, we first show the block diagram in Fig. 1. We can find from Fig. 1 that the proposed method consists of DCS, DCS-JOMP and energy-based detection algorithm. We will introduce them in the following section, respectively.

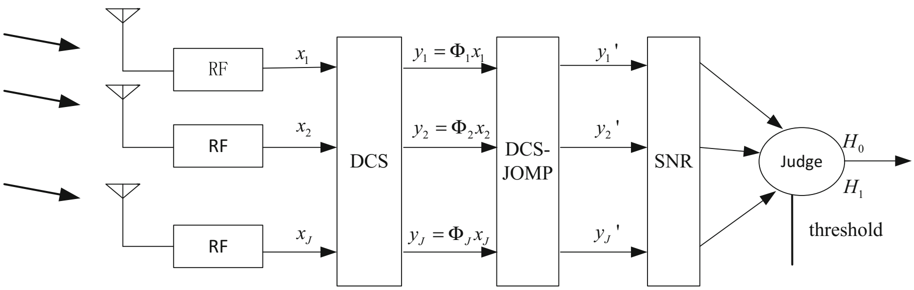

Fig. 1. The block diagram of the proposed method

For multi-antenna signals, the received signals fit with JSM-2 model. Therefore, distributed compressed sensing can be applied to sample the multi-antenna signals. Supposed that the sparsity of signal is K, the sampled signals in the framework of compressed sensing can be expressed as

$$\begin{cases} y_1 = \Phi_1 x_1 = \Phi_1 \Psi \theta_1 \\ y_2 = \Phi_2 x_2 = \Phi_2 \Psi \theta_2 \\ \cdots \\ y_J = \Phi_J x_J = \Phi_J \Psi \theta_J \end{cases} \tag{5}$$

where $x_i,\ i = 1, \cdots,\ J$ denotes the received signal from i^{th} antenna. $\Phi_i,\ i = 1, \cdots,\ J$ is measurement matrix, Ψ is the sparse basis, and θ_1 is sparse representation in the sparse basis.

Joint reconstruction of distributed compressed sensing (DCS-JOMP) is described as follows:

(1) Initialize. k is the times of iteration, Ω is the space spanned by coefficients vector to be reconstructed. $r_{j,k}$ is the residual error. Let $\Omega = []$, $r_{j,0} = y_j$.
(2) Judgment of the correlation. The column corresponding to the biggest correlation with $r_{j,k-1}$ is picked out from $\Phi_j \Psi$, i.e., $\xi_k = \arg \max\limits_{n \in \{1,2\ldots N\}} \sum\limits_{j=1}^{J} \left| \left\langle r_{j,k-1}, \phi_{j,n} \right\rangle \right|$. Then the space Ω is be updated to $\Omega = [\Omega\, \xi_k]$.
(3) Updating of residual base, $\Lambda_{j,k} = \Phi_{j,\Omega}$. Where $\Phi_{j,\Omega}$ is the group of the selected column of measurement matrix based on $\Omega = [\Omega\ \xi_k]$.
(4) Updating of the residual error. The sparse representation after the each iteration is denoted as $\theta_{j,k} = (\Lambda'_{j,k} \Lambda_{j,k})^{-1} \Lambda'_{j,k} y_j$, so the residual error is expressed as $r_{j,k} = y_j - \Lambda_{j,k} \theta_{j,k}$.
(5) Stopping the iteration. When $k > K$, we stop the iteration.

By exploiting DCS-JOMP algorithm, we obtain the time-domain signals. And then the error and noise are estimated to compute the SNR, further set the threshold. Finally, energy-based method is employed to perform spectrum sensing. Specific process is described in the following section.

For the conventional energy-based method, the test statistic is $Z = \sum\limits_{n=1}^{2TW} x^2(n)$. Where $2TW$ is the length of the received signals, T is the time interval, and W is the bandwidth. The received signal is $x(n) = s(n) + w(n)$.

For simplification, but without loss of generality, we normalize the received signal by the noise covariance, i.e., $w'(n) = w(n)/\sigma_w$, $s'(n) = s(n)/\sigma_w$. Therefore, the test statistic reduces to $Z = \sum\limits_{n=1}^{N} y'(n)^2$. In this situation, the binary hypothesis test can be expressed in the form

$$Z = \begin{cases} \sum_{n=1}^{2TW} w'(n)^2, & H_0 \\ \sum_{n=1}^{2TW} (s'(n) + w'(n))^2, & H_1 \end{cases} \tag{6}$$

By analyzing (6), we can conclude that the received signal follows the central chi-square distribution when no signal exists. Inversely, the received signal follows the non-central chi-square distribution with the non-central parameter

$$\delta = \sum_{n=1}^{2TW} s'(n)^2 = \sum_{n=1}^{2TW} \left(\frac{s(n)}{\sigma_w}\right)^2 = \frac{\sum_{n=1}^{2TW} s(n)^2}{\sigma_w^2} = \frac{2TWP_s}{P_n} = 2TW\gamma \tag{7}$$

Correspondingly, we can compute the detection probability and the false-alarm probability

$$P_d = P(Z > \lambda | H_1) = Q_u(\sqrt{\delta}, \sqrt{\lambda}) \tag{8}$$

$$P_f = P(Z > \lambda | H_0) = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} \tag{9}$$

where $\Gamma(.)$ is Gamma function, $\Gamma(.,.)$ is the incomplete gamma function, $Q_u(.,.)$ is the generalized Marcum Q function, the λ is the predetermined threshold. $u = TW$ is the production of time and bandwidth. Generally speaking, we refer to the false-alarm probability as constant, i.e., constant false-alarm probability, and then compute the decision threshold. Finally, substitution of threshold into (8) yields the detection probability.

3 Numerical Simulation and the Corresponding Analyzing

We first analyze the reconstruction error of compressed sensing and distributed compressed sensing for the various number of antennas. In the simulation, we assume that the signal is spare in the discrete cosine base, the length $N = 64$, the sparsity K is 4. The noise follows the Gaussian distribution, SNR = 10 dB. The times of Monte Carlo is 500. The reconstruction algorithm of compressed sensing and distributed compressed sensing are OMP algorithm and DCS-JOMP algorithm. The results are shown in Fig. 2.

It can be seen that the reconstruction error reduces with the increasing of the number of sensing measurements, which fit with the theoretical analysis. Additionally, for distributed compressed sensing, the reconstruction error is inversely proportional to the number of antennas. For example, for $M = 20$, the reconstruction error is 33.8% when compressed sensing is adopted, the reconstruction error is 10.6% and 7.4% for 2 antennas and 4 antennas when we exploit distributed compressed sensing.

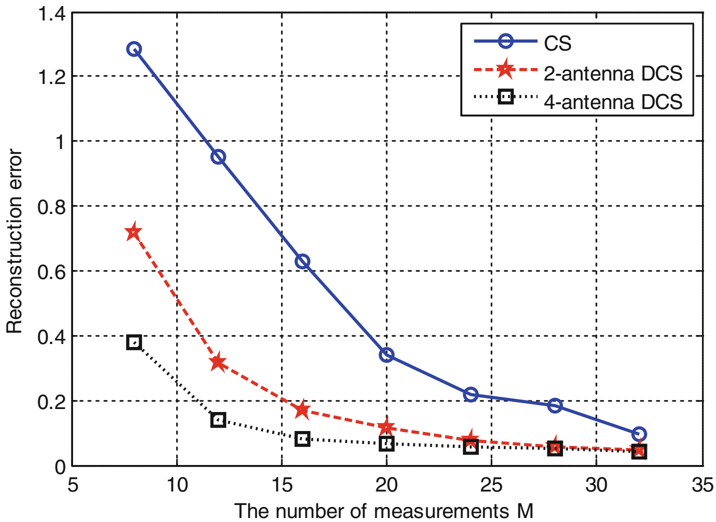

Fig. 2. The relationship between the number of measurements and the reconstruction error

To further evaluate the performance of the proposed method under the different antennas, we as before take 2 antennas and 4 antennas as the example. SNR is 3 dB. In the simulation, we use the detection probability under constant false-alarm probability to measure the performance of the proposed algorithm. The simulation results are illustrated in Fig. 3.

It is obviously observed that the detection probability of multi-antenna distributed compressed is higher than that of compressed sensing, and the detection probability varies with the number of antennas. For example, when $M = 20$, the detection probability is 82.1% when compressed sensing is adopted, the reconstruction error is 97.3% and 99.4% for 2 antennas and 4 antennas.

In the following, we evaluate the detection probability under the different SNR. The SNR varies from −15 dB to 10 dB. In addition, to compare with the conventional energy-based detection algorithm, its detection probability is also provided. In this simulation, the false-alarm probability is 0.05, the number of antenna J is 4. The number of sensing measurements is $M = 16$, and the sparsity is 4. We compute the threshold using (8), and then obtain the detection probability illustrated in Fig. 4.

It can been seen from Fig. 4 that the detection probability increases with the increasing of SNR. Generally, the performance of the conventional time-domain detection algorithm outperforms that of the proposed method. This is because that compressed sensing leads to the wastage of the signal energy. For example, when the detection probability reaches 100% for the conventional time-domain detection, SNR is 5 dB, and the sampled number is 64. For the proposed method, however, the number of antennas and sensing measurements are 4 and $M = 13$ respectively when the detection probability reaches 100%.

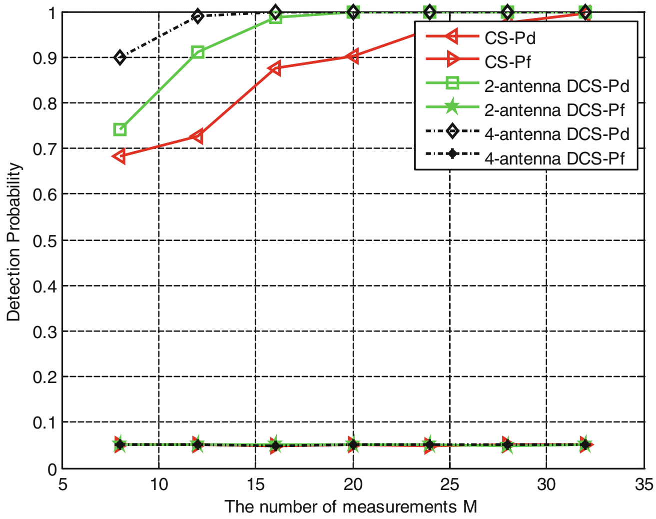

Fig. 3. The relationship between the number of measurements and the detection probability

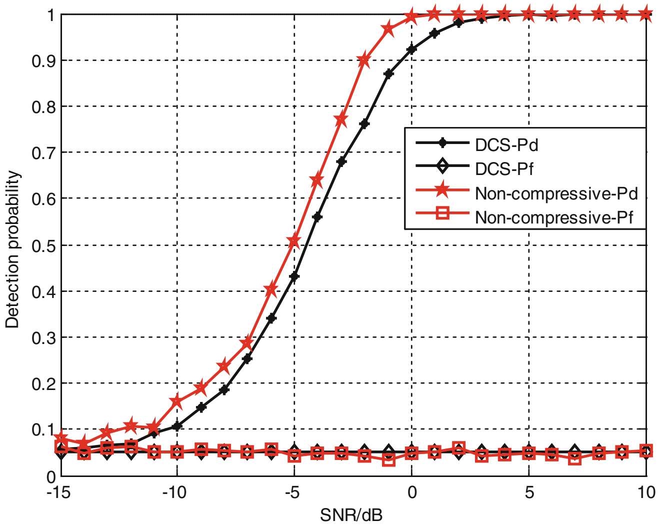

Fig. 4. The relationship between SNR and the detection probability

4 Conclusions

To solve the problem of high sampling rate and hardware cost, we exploit the intra-signal and inter-signal to sample the MIMO multi-antenna signals, which obviously decrease the sampling rate and hardware cost. Combining with energy-based sensing method, we proposed a novel spectrum sensing. The proposed method perform the nearly similar to the conventional time-domain spectrum sensing.

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