Lattice Reduction Aided Linear Detection for Generalized Spatial Modulation

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Abstract. For reducing the complexity of equalization, linear equalization can be adopted for generalized spatial modulation (GSM) which is a special case of multiple-input-and-multiple-output (MIMO). However, because of its inferior performance, linear equalization may be infeasible for practical GSM systems which has large number of antennas and constellation. On the other hand, lattice-reduction (LR) is an effective method to improve the performance of linear equalization. The lattice reduction can't be utilized by GSM directly, because signals on some antennas don't exist. For tackling this problem, we propose a compatible 8-QAM constellation scheme integrating LR-aided linear equalization with GSM effectively. Next, we prove that LR-aided linear equalizers collect the same diversity order as that exploited by the ML detector under Rayleigh fading channels, and implement some simulations. Simulation results show the superior of the proposed 8-QAM over traditional 4-QAM and 8-QAM under Rayleigh fading channel. Moreover, our scheme obtains the full receive diversity under correlated channel.

Keywords: Generalized spatial modulation · Lattice reduction
Linear detection · 8-QAM

1 Introduction

Spatial modulation (SM) has been recognized as a promising MIMO transmission technology to develop energy-efficient, low-complexity solutions that satisfy target throughput requirements in future mobile networks [1]. Comparing with traditional MIMO scheme, SM only activates one antenna according to the information bits per channel use. So only one RF chain is needed, corresponding results are that the hardware implementation cost reduces, inter-channel interference is avoided, and inter-antenna problem doesn't exist. These merits make the SM become the research focus of MIMO.

The spectral efficiency of SM is increased logarithmically with the number of transmit antenna, which cannot satisfy the demand for higher throughput under the fixed transmit antenna number. For tackling this problem, generalized spatial modulation (GSM) activating more than one antenna during each transmission is introduced [2–4]. Now, there exists two types of GSM, One is that all active antennas send the same modulated symbol simultaneously [2, 3], and the other is that different active

antennas send different modulated symbol independently [4]. For convenience, we call the former single symbol generalized spatial modulation (SS-GSM), the latter multiple symbol generalized spatial modulation (MS-GSM). It is worth to notice that GSM eliminates the demand that the transmit antenna number must be a power of two. Moreover, SS-GSM completely keep the advantages of single RF chain obviously, and MS-GSM have a higher spectral efficiency. However, ICI and IAS problems still exist in MS-GSM.

For MIMO receiver, maximum likelihood (ML) detection can obtain the best bit-error rate (BER). However, the huge computation complexity make ML infeasible in practical system. At the same time, linear equalization has an inferior performance while low complexity. Considering the important factors impacting the linear receiver is column coherence of channel matrix, and lattice reduction can reduce the dependency among column effectively, some researchers propose LR-aided linear equalization for Vertical Bell Laboratories Layered Space-Time (V-BLAST) architecture [5]. However, LR is not suitable for GSM, because only a portion of antennas are activating during each transmission, and receiver cannot obtain the consecutive integer lattice point in I-Q constellation diagram. For tacking this problem, we propose a compatible 8-QAM constellation scheme for SS-GSM and MS-GSM, which meets the requirement of lattice reduction. Sequentially, we prove that LR-aided linear equalizers can collect the same diversity order as that exploited by the ML detector and the simulations are implemented to corroborate our theoretical claims.

The remainder of this paper is organized as follows. Section 2 describes system and signal models. Section 3 extends the proposed equalizations to GSM and proposes a compatible 8-QAM constellation. Section 4 gives performance analyze. Section 5 shows the simulation results, followed by concluding remarks in Sect. 6.

2 GSM System Model

Consider a GSM system with N_t transmit antennas and N_r receive antennas. For each symbol time, only N_a transmit antennas is activated to send symbols, where $2 \leq N_a \leq N_t$. Obviously, there are $C_{N_t}^{N_a}$ kinds of possible active antennas combinations, among them, at most N kinds of combinations can be used to transmit $\log_2 N$ bits information, where $N = 2^{\lfloor \log C_{N_t}^{N_a} \rfloor}$ and $\lfloor \bullet \rfloor$ is the floor function. In the context of this paper, we call the N kinds of combinations a spatial constellation and any one of them a spatial symbol. The transmitted symbol vector can be written as

$$\mathbf{X} = \mathbf{E}_i \mathbf{s}, \tag{1}$$

where $\mathbf{E}_i$ is a $N_t \times N_a$ matrix which contains N_a columns chosen from the $N_t \times N_t$ identity matrix and is the matrix form expression for a spatial symbol. Definitely, the spatial symbol can also be represented in combination indices form as $I_j = \{j_1, j_2, \cdots, j_{N_a}\}$, where every element j_k, $k \in [1, 2, \cdots, N_a]$ denotes the antenna index which is arranged in ascending order for convenience. As a result, the spatial constellation in combination indices form can be written as $\mathcal{I} = \{I_1, I_2, \cdots, I_N\}$.

The ordinals of columns in $\mathbf{E}_i$ are chosen according to the antenna indices in each spatial symbol, so there are totally N kinds of $\mathbf{E}_i$ that make up of the spatial constellation in matrix form ε, where $\varepsilon = \{E_1, E_2, \cdots, E_N\}$. $\mathbf{s} = [s_1, \cdots, s_{N_a}]^T$ is the power-normalized $N_a \times 1$ signal symbol vector namely $\mathrm{E}[\mathbf{ss}^H] = \mathbf{I}_{N_a}$. For SS-GSM, the N_a signal symbols are the same, namely $s_1 = s_2 = \cdots = s_{N_a} \in \mathcal{S}$, where $\mathcal{S}$ is the conventional QAM constellation. As to MS-GSM, the N_a signal symbols are independent, which means $\mathbf{s} \in \mathcal{S} \times \mathcal{S} \cdots \mathcal{S} = \mathcal{S}^{N_a}$. Definitely for M-QAM, in each symbol time, SS-GSM can transmit $\log_2 N + \log_2 M$ bits whereas MS-GSM can transmit $\log_2 N + N_a \log_2 M$ bits.

Assume that the channel is flat-fading quasi-static, where the channel matrix is invariant during a frame and changes independently among frames, and each entry of the channel matrix $\mathbf{H}$ follows an independent and identically distributed (i.i.d) complex Gaussian distribution $\mathcal{CN}(0, 1)$ with mean 0 and variance 1. The received signal vector can be represent as

$$\mathbf{y} = \mathbf{H}\mathbf{E}_i\mathbf{s} + \mathbf{n} = \mathbf{H}\mathbf{X} + \mathbf{n}, \tag{2}$$

where $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the additive noise vector following complex Gaussian distribution with mean zero and covariance matrix $\mathrm{E}[\mathbf{nn}^H] = \sigma^2 \mathbf{I}_{N_r}$. Let $\mathbf{H}_{\mathrm{E}_i} = \mathbf{H}\mathbf{E}_i$ and it is easy to know that $\mathbf{H}_{\mathrm{E}_i} \in \mathcal{H} = \{\mathbf{H}_{\mathrm{E}_1}, \mathbf{H}_{\mathrm{E}_2}, \cdots, \mathbf{H}_{\mathrm{E}_N}\}$ is sub-channel matrix including N_a columns chosen from the channel matrix $\mathbf{H}$. Equation (2) can be rewritten as:

$$\mathbf{y} = \mathbf{H}_{\mathrm{E}_i}\mathbf{s} + \mathbf{n}. \tag{3}$$

At the receiver, the ML detector provides the optimal performance by exhaustively searching through spatial constellation and signal constellation. The output for SS-GSM can be written as

$$\left(\hat{\mathbf{E}}_i, \hat{\mathbf{s}}\right) = \arg \min_{\mathbf{H}_{\mathrm{E}_i} \in \mathcal{H}, s_1 = s_2 = \cdots s_{N_a} \in \mathcal{S}} \left\|\mathbf{y} - \mathbf{H}_{\mathrm{E}_i}\mathbf{s}\right\|_2^2, \tag{4}$$

and

$$\left(\hat{\mathbf{E}}_i, \hat{\mathbf{s}}\right) = \arg \min_{\mathbf{H}_{\mathrm{E}_i} \in \mathcal{H}, \mathbf{s} \in \mathcal{S}^{N_a}} \left\|\mathbf{y} - \mathbf{H}_{\mathrm{E}_i}\mathbf{s}\right\|_2^2 \tag{5}$$

for MS-GSM.

3   LR-Aided Linear Equalization for GSM MIMO System

3.1   Lattice Reduction

Considering that the problems involved in wireless communications are mostly complex valued, we only introduce the complex lattice and complex lattice reduction here [6, 7]. A complex-valued lattice of rank m in the n-dimension complex space $\mathbb{C}^n$ is defined as

$$\mathcal{L} \triangleq \left\{ x | x = \sum_{l=1}^{m} z_l b_l, z_l \in \mathbb{Z}_j \right\}, \tag{6}$$

where $b_l \in \mathbb{C}^n$ are complex basis vectors and $\mathbb{Z}_j = \mathbb{Z} + j\mathbb{Z}$ is the set of complex integers that are also called Gausses integers. The complex basis vectors can be arranged into an $n \times m$ complex matrix $\mathbf{B}$ which can be simply called the basis of the lattice, thus any element $\mathbf{x}$ in the lattice can be represent as $\mathbf{x} = \mathbf{Bz}$. It can be show that there exist infinitely bases which can be chosen for a specific lattice. For different bases, the same lattice can have different orthogonality defect, which defined as

$$\xi(\mathbf{B}) = 1 - \frac{\det\left(\mathbf{B}^H \mathbf{B}\right)}{\prod_{n=1}^{N_t} \|\mathbf{b}_n\|^2}. \tag{7}$$

There is $0 \le \xi(\mathbf{B}) \le 1$ for all $\mathbf{B}$. $\xi(\mathbf{B}) = 1$ if $\mathbf{B}$ is singular and $\xi(\mathbf{B}) = 0$ if the columns of $\mathbf{B}$ are orthogonal to each other. Lattice reduction algorithms are to reduce the orthogonality defect of a lattice by column swaps and size reductions, while hold the same lattice. The implementation process of these algorithms is using a unimodular matrix to multiply the original basis, the corresponding equation is given as:

$$\mathcal{L}(\mathbf{B}_{LR}) = \mathcal{L}(\mathbf{B}) \Leftrightarrow \mathbf{B}_{LR} = \mathbf{BT}, \tag{8}$$

where all elements of $\mathbf{T}$ are Gausses integers, $\det(\mathbf{T}) = \pm 1$, and $\xi(\mathbf{B}_{LR}) < \xi(\mathbf{B})$.

A powerful and famous reduction criterion for arbitrary lattice dimensions is introduced by Lenstra et al. [8], and the algorithm they proposed is known as the LLL (or L^3) algorithm. Since the signal and channel matrix are complex valued, we use complex LLL algorithm (CLLL) provided in [9].

3.2   LR-Aided Linear Equalization

With the channel matrix after lattice reduction $\mathbf{H}_{LR} = \mathbf{HT}$, the received signal can be written as

$$\mathbf{y} = \mathbf{HX} + \mathbf{n} = \mathbf{HTT}^{-1}\mathbf{X} + \mathbf{n} = \mathbf{H}_{LR}\mathbf{Z} + \mathbf{n}. \tag{9}$$

The idea of LR-aided linear equalization is to apply linear equalization to $\mathbf{Z}$ instead of $\mathbf{X}$ and calculate $\mathbf{X}$ by $\mathbf{X} = \mathbf{TZ}$. The estimation of $\mathbf{Z}$ are obtained as:

$$\hat{\mathbf{Z}}_{ZF} = \mathcal{Q}_l\left(\left(\left(\mathbf{H}_{LR}^H \mathbf{H}_{LR}\right)^{-1}\mathbf{H}_{LR}^H\right)\mathbf{y}\right), \tag{10}$$

$$\hat{\mathbf{Z}}_{MMSE} = \mathcal{Q}_l\left(\left(\left(\mathbf{H}_{LR}^H \mathbf{H}_{LR} + \sigma^2 \mathbf{I}_{N_t}\right)^{-1}\mathbf{H}_{LR}^H\right)\mathbf{y}\right), \tag{11}$$

where $\mathcal{Q}_l(\bullet)$ denotes the component-wise rounding operation. The estimation of $\mathbf{X}$ is given as:

$$\hat{\mathbf{X}}_{\text{ZF}} = \mathcal{Q}_2\big(\mathbf{T}\hat{\mathbf{Z}}_{\text{ZF}}\big) = \mathcal{Q}_2\Big(\mathbf{T}\mathcal{Q}_1\Big(\big(\big(\mathbf{H}_{\text{LR}}^{\text{H}}\mathbf{H}_{\text{LR}}\big)^{-1}\mathbf{H}_{\text{LR}}^{\text{H}}\big)\mathbf{y}\Big)\Big), \tag{12}$$

$$\hat{\mathbf{X}}_{\text{MMSE}} = \mathcal{Q}_2\big(\mathbf{T}\hat{\mathbf{Z}}_{\text{MMSE}}\big) = \mathcal{Q}_2\Big(\mathbf{T}\mathcal{Q}_1\Big(\big(\big(\mathbf{H}_{\text{LR}}^{\text{H}}\mathbf{H}_{\text{LR}} + \sigma^2\mathbf{I}_{N_t}\big)^{-1}\mathbf{H}_{\text{LR}}^{\text{H}}\big)\mathbf{y}\Big)\Big), \tag{13}$$

where $\mathcal{Q}_2(\bullet)$ restricts the symbols to lie in the constellation. If we extend the original channel matrix and received signal vector as

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma\mathbf{I}_{N_t} \end{bmatrix}, \tilde{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N_t \times 1} \end{bmatrix}, \tag{14}$$

the result of MMSE can also be written in the ZF form:

$$\hat{\mathbf{X}}_{\text{MMSE}} = \mathcal{Q}\Big(\big(\tilde{\mathbf{H}}^{\text{H}}\tilde{\mathbf{H}}\big)^{-1}\tilde{\mathbf{H}}^{\text{H}}\tilde{\mathbf{y}}\Big), \tag{15}$$

For V-BLAST MIMO system, formula (12) and (13) represent the whole detection. However, for SM system, the spatial symbol and signal symbol must be separated according to estimated vector $\hat{\mathbf{X}}_{\text{ZF}}$ or $\hat{\mathbf{X}}_{\text{MMSE}}$. For the brevity, we use $\hat{\mathbf{X}}$ to stand for $\hat{\mathbf{X}}_{\text{ZF}}$ or $\hat{\mathbf{X}}_{\text{MMSE}}$.

Considering the entries corresponding to inactive antennas in estimated vector should approach zero, we choose the largest N_a entries in $\hat{\mathbf{X}}$: $\big(\hat{x}_{i_1}, \hat{x}_{i_2}, \cdots, \hat{x}_{i_{N_a}}\big), |\hat{x}_{i_k}| \geq |\hat{x}_m|$, $i_k \in \tilde{I} = \{i_1, i_2, \cdots, i_{N_a}\}, m \in (U - \tilde{I}), U = \{1, \cdots, N_t\}$ and arrange the corresponding antennas indices combination in ascending order: $i_1 < i_2 < \cdots < i_{N_a}$. The estimation of spatial symbol is given as

$$\hat{I}_j = \mathcal{Q}_3\big(\tilde{I}\big) = \mathcal{Q}_3\big(\{i_1, i_2, \cdots, i_{N_a}\}\big), \tag{16}$$

where $\mathcal{Q}_3(\bullet)$ forces the $\hat{I}_j$ to lie in the spatial constellation. After estimating the spatial symbol, choose the entry of $\hat{\mathbf{X}}$ corresponding to the first active antenna index as the estimation of signal symbol for SS-GSM:

$$\hat{s}_1 = \hat{s}_2 = \cdots = \hat{s}_{N_a} = \hat{x}_{i_1}, \tag{17}$$

As to MS-GSM, choose the entries of $\hat{\mathbf{X}}$ corresponding to the entire antenna index of the spatial symbol $\hat{I}_j$ as the estimation of signal symbol:

$$\hat{\mathbf{s}} = \big[\hat{x}_{i_1}, \hat{x}_{i_2}, \cdots, \hat{x}_{i_{N_a}}\big]. \tag{18}$$

Considering that the MMSE equalizer is equal to the ZF equalizer in the form of the extended system (15), we summarize the main steps of the LR-aided ZF equalization to represent the LR-aided linear equalization detector for GSM systems in Table 1.

Table 1. The main steps of LR-aided ZF equalization detector for MIMO GSM system.

S1	At the receiver, perform the CLLL algorithm to get the more orthogonal channel matrix and the unimodular matrix: $[\mathbf{H}_{LR}, \mathbf{T}] = \mathrm{CLLL}(\mathbf{H})$
S2	In formula (10), use ZF equalization to equalize the received signal vector, and obtain $\hat{\mathbf{Z}}$
S3	In formula (12), recover the transmitted symbol vector $\mathbf{X}$
S4	In formula (16), estimate the spatial symbol $\hat{I}_j$ from $\hat{\mathbf{X}}$
S5	According to $\hat{I}_j$, estimate the signal symbol $\hat{s}$ for SS-GSM or MS-GSM. in formula (17) or (18)

3.3   A Compatible 8-QAM Constellation for GSM

Based on the theory of lattice, $\mathbf{X}$ should stem from the infinite integer space [10]. Considering the necessary condition of the lattice reduction is consecutive integers lattice, in [10, 11], square M-QAM constellation is translated into a finite consecutive integers set by proper scaling and shifting for V-BLAST MIMO system. However, the general expression of GSM which includes zero entries in transmitted vector don't satisfy this condition. For example, a 4-QAM constellation in GSM as shown in Fig. 1, which includes the original point, and scaling and shifting operation cannot turn points of constellation into consecutive integers. So the lattice reduction isn't suitable for GSM. For tacking this problem, we propose an 8-QAM constellation which can satisfy the complex consecutive integer requirement for $\mathbf{X}$, as depicted in Fig. 2. The proposed 8-QAM constellation is made up of two orthogonal real integers set: $\Lambda \times \Lambda$, where $\Lambda = \{-1, 0, 1\}$ (Fig. 3).
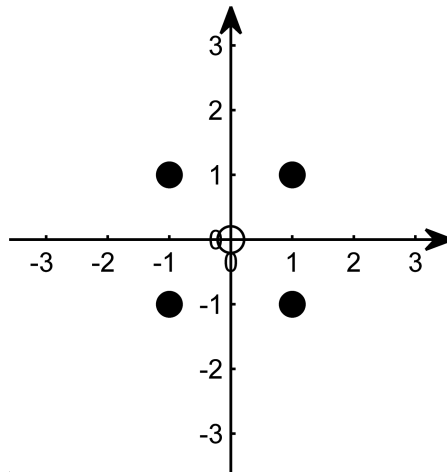
Fig. 1. A 4-QAM constellation in GSM.
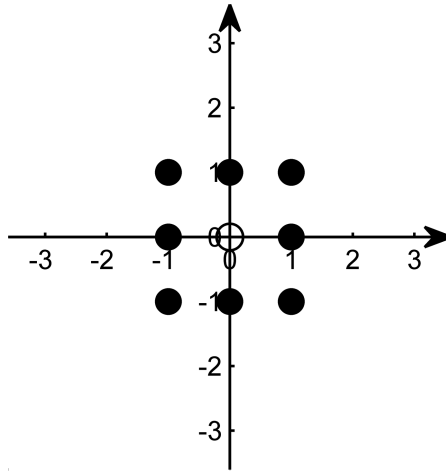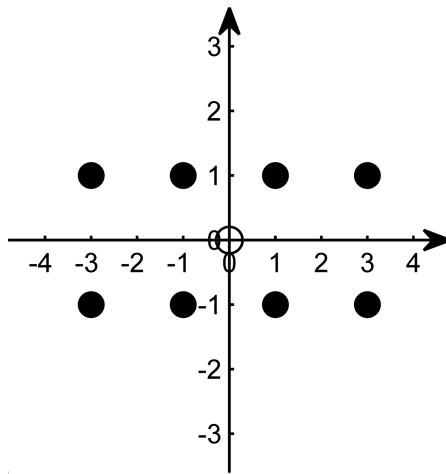
Fig. 2. Proposed 8-QAM constellation.

Fig. 3. Contrasting 8-QAM constellation

4 Performance Analyze

The LR-aided linear detectors for MIMO V-BLAST system can collect the same diversity order as that exploited by the ML detector [13], based on this, the diversity order of LR-aided linear detectors for MIMO GSM system is analyzed here.

Proposition 1: Given the signal model in (1), system model in (2), the proposed 8-QAM constellation in Sect. 4.2, channels consisting of i.i.d complex Gaussian distributed entries with zero mean and unit variance, the diversity order collected by LR-aided ZF equalization for GSM is N_r.

Proof: Substitute (9) for **y** in (10), the equalization result in step S2 of Table 1 can be rewritten as:

$$\hat{\mathbf{Z}}_{ZF} = \mathcal{Q}_1\left(\mathbf{H}_{LR}^\dagger \mathbf{y}\right) = \mathcal{Q}_1\left(\mathbf{H}_{LR}^\dagger \mathbf{H}_{LR}\mathbf{Z} + \mathbf{H}_{LR}^\dagger \mathbf{n}\right) = \mathbf{Z} + \mathcal{Q}_1\left(\mathbf{H}_{LR}^\dagger \mathbf{n}\right). \tag{19}$$

As a result, the estimation of transmitted symbol vector can be expressed as:

$$\hat{\mathbf{X}}_{ZF} = \mathcal{Q}_2\left(\mathbf{T}\hat{\mathbf{Z}}_{ZF}\right) = \mathcal{Q}_2\left(\mathbf{T}\left(\mathbf{Z} + \mathcal{Q}_1\left(\mathbf{H}_{LR}^\dagger \mathbf{n}\right)\right)\right) = \mathbf{X} + \mathcal{Q}_2\left(\mathbf{T}\mathcal{Q}_1\left(\mathbf{H}_{LR}^\dagger \mathbf{n}\right)\right). \tag{20}$$

Definitely, there will be $\hat{\mathbf{X}}_{ZF} = \mathbf{X}$ if $\mathcal{Q}_1\left(\mathbf{H}_{LR}^\dagger \mathbf{n}\right) = 0$. The correct estimation of $\mathbf{X}$ will lead to correct estimate of I_j and $\mathbf{s}$ according to the method described in Sect. 4.1. Therefore, the symbol error probability for a given $\mathbf{H}$ is upper-bounded by

$$P_{e|\mathbf{H}} \leq 1 - P\left(\mathcal{Q}_1\left(\mathbf{H}_{LR}^\dagger \mathbf{n}\right) = 0|\mathbf{H}\right) \tag{21}$$

$\tilde{\mathbf{H}}^\dagger$ can be represented as $[\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_{N_t}]^T$, where $\mathbf{a}_i^T$, $i \in [1, N_t]$ is the ith row of $\tilde{\mathbf{H}}^\dagger$. The upper bound can be rewritten as

$$P_{e|\mathbf{H}} \leq P\left(\max_{1 \leq i \leq N_t}|\mathbf{a}_i^T \mathbf{n}| \geq \frac{1}{2}|\mathbf{H}\right). \tag{22}$$

According to Lemma 1 in [12], we obtain the inequality:

$$\max_{1 \leq i \leq N_t}\left\|\mathbf{a}_i^T\right\| \leq \frac{1}{\sqrt{1 - \xi(\mathbf{H}_{LR})} \cdot \min\limits_{1 \leq i \leq N_t}\left\|\tilde{\mathbf{h}}_i\right\|}, \tag{23}$$

where $\tilde{\mathbf{h}}_i, i \in [1, N_t]$ is the i th column of $\mathbf{H}_{LR}$. Consider that

$$\max_{1 \leq i \leq N_t}\left|\mathbf{a}_i^T \mathbf{n}\right| \leq \max_{1 \leq i \leq N_t}\left\|\mathbf{a}_i^T\right\| \cdot \|\mathbf{n}\| \leq \frac{\|\mathbf{n}\|}{\sqrt{1 - \xi(\mathbf{H}_{LR})} \cdot \min\limits_{1 \leq i \leq N_t}\left\|\tilde{\mathbf{h}}_i\right\|}, \tag{24}$$

$P_{e|\mathbf{H}}$ is further bounded by

$$P_{e|\mathbf{H}} \leq P\left(\frac{\|\mathbf{n}\|}{\sqrt{1 - \xi(\mathbf{H}_{LR})} \cdot \min\limits_{1 \leq i \leq N_t}\left\|\tilde{\mathbf{h}}_i\right\|} \geq \frac{1}{2}|\mathbf{H}\right) \tag{25}$$

Since $\mathbf{H}_{LR}$ is derived from $\mathbf{H}$ by using the CLLL algorithm with parameter δ, and $\mathbf{H}$ is full rank with probability one, we can obtain the following inequality according to Lemma 1 in [9]:

$$\sqrt{1 - \xi(\mathbf{H}_{\text{LR}})} \geq 2^{\frac{N_{\text{col}}}{2}} \left(\frac{2}{2\delta - 1} \right)^{-\frac{N_{\text{col}}(N_{\text{col}}+1)}{4}} := c_{\delta}, \tag{26}$$

where N_{col} is the number of columns of $\mathbf{H}$. Use $\mathbf{h}_{\text{min}}$ to stand for the minimum non-zero norm vector in the lattice generated by $\mathbf{H}$. Because $\mathbf{H}_{\text{LR}}$ spans the same lattice as $\mathbf{H}$, we have

$$\|\mathbf{h}_{\text{min}}\| \leq \min_{1 \leq i \leq N_{\text{t}}} \left\| \tilde{\mathbf{h}}_i \right\|. \tag{27}$$

Substitute (26) and (27) into (25), and the (25) can be simplified as:

$$P_{\text{e}|\mathbf{H}} \leq P \left(\|\mathbf{n}\| \geq \frac{c_{\delta} \|\mathbf{h}_{\text{min}}\|}{2} \Big| \mathbf{H} \right). \tag{28}$$

Average (28) with respect to the random matrix $\mathbf{H}$, we have

$$\begin{aligned}
P_{\text{e}} = E_{\mathbf{H}} \left[P_{\text{e}|\mathbf{H}} \right] &\leq E_{\mathbf{H}} \left[P \left(\|\mathbf{n}\|^2 \geq \frac{c_{\delta}^2 \|\mathbf{h}_{\text{min}}\|^2}{4} \Big| \mathbf{H} \right) \right] \\
&= E_{\mathbf{n}} \left[P \left(\|\mathbf{h}_{\text{min}}\|^2 \leq \frac{4\|\mathbf{n}\|^2}{c_{\delta}^2} \right) \Big| \mathbf{n} \right].
\end{aligned} \tag{29}$$

$\|\mathbf{n}\|^2$ is a central Chi-square random variable with $2N_{\text{r}}$ degrees of freedom and mean $N_{\text{r}}\sigma^2$ since $\mathbf{n}$ is an $N_{\text{r}} \times 1$ complex white noise vector with zero mean and covariance matrix $\sigma^2 \mathbf{I}_{N_{\text{r}}}$. According to Lemma 2 in [12], we have

$$P \left(\|\mathbf{h}_{\text{min}}\|^2 \leq \varepsilon \right) \leq c_{N_{\text{r}} N_{\text{t}}} \varepsilon^{N_{\text{r}}}, \tag{30}$$

where $c_{N_{\text{r}} N_{\text{t}}}$ is a finite constant depending on N_{r} and N_{t}. Then P_{e} can be further bounded as:

$$P_{\text{e}} \leq E_{\mathbf{n}} \left[c_{N_{\text{r}} N_{\text{t}}} \left(\frac{4}{c_{\delta}^2} \right)^{N_{\text{r}}} \|\mathbf{n}\|^{2N_{\text{r}}} \right]. \tag{31}$$

Calculate the N_{r} th moment of Chi-square random variable $\|\mathbf{n}\|^2$, we get the final result:

$$P_{\text{e}} \leq c_{N_{\text{r}} N_{\text{t}}} \left(\frac{4}{c_{\delta}^2} \right)^{N_{\text{r}}} \frac{(2N_{\text{r}} - 1)!}{(N_{\text{r}} - 1)!} \left(\frac{1}{\sigma^2} \right)^{-N_{\text{r}}}. \tag{32}$$

Therefore, the diversity order of the LR-aided ZF detector for GSM is greater than or equal to N_{r}. Since the maximum diversity order for the GSM MIMO system is N_{r}, the LR-aided ZF detector for GSM collects diversity N_{r}. The result is easy to extend for LR-aided MMSE detector. ∎

5   Simulation Results

In this section, we will testify three conclusion by Monte Carlo method with MATLAB: (1) under the condition that GSM system adopts the LR-aided linear detector, compared with the contrasting 8-QAM constellation depicted in Fig. 5 and the 4-QAM constellation depicted in Fig. 2, the proposed 8-QAM constellation has inherent performance superiority; (2) the proposed detector obtains full receive diversity order; (3) simulation results with Kronecker Model show that the proposed method still can collect receive diversity on correlated channel. Except (3), all simulations are implemented under the channel described in Sect. 2.

Figures 4 and 5 show the simulation results of BER performance among three kinds of signal constellation using MMSE or LR-aided MMSE detector for SS-GSM and MS-GSM system. The antenna configuration for these two system is $N_t = 4, N_a = 2, N_r = 6$. It is easy to know that the spectral efficiencies are $R_{SS-4-QAM} = 4$ bit/s/Hz, $R_{SS-8-QAM} = 5$ bit/s/Hz   for   SS-GSM   system   and $R_{MS-4-QAM} = 6$ bit/s/Hz, $R_{MS-8-QAM} = 8$ bit/s/Hz for MS-GSM system. We can see that the simulation results of SS-GSM is nearly the same as those of MS-GSM. While under the condition that the spectral efficiency of three schemes are identical, the proposed 8-QAM have superior BER at lower Signal Noise Ratio (SNR) compared with the contrasting 8-QAM: the improvement stage starts at 10 dB for the proposed 8-QAM while lattice reduction takes effect from 15 dB for the contrasting 8-QAM. Although the BER improvement of the proposed 8-QAM adopting lattice reduction is merely 0.5 dB, the spectral efficiency of 8-QAM is apparently higher than 4-QAM. In summary, the proposed 8-QAM constellation is superior to the contrasting 8-QAM and the contrasting 4-QAM when using the LR-aided MMSE detector.
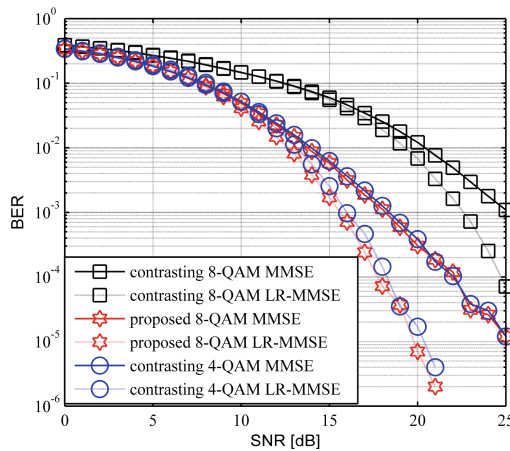

Fig. 4. BER performance of MMSE and LR-MMSE detector among three kinds of constellations for SS-GSM MIMO system.
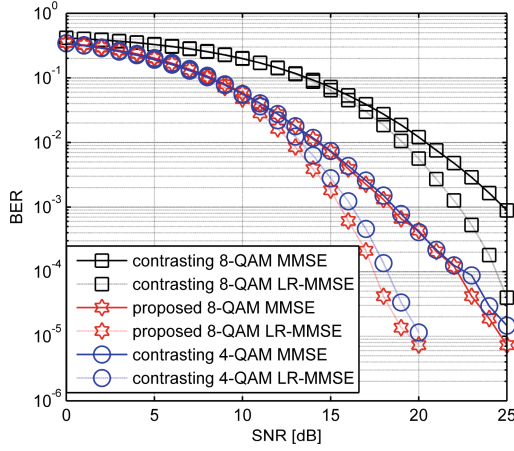
Fig. 5. BER performance of MMSE and LR-MMSE detector among three kinds of constellations for MS-GSM MIMO system.

Figures 6 and 7 compare the performance of LR-aided linear detector with the joint ML detector for SS-GSM and MS-GSM system while using the proposed 8-QAM constellation. The antenna configuration is $N_t = 4, N_a = 2, N_r = 6$. As we can see from the figures, regardless of the SS-GSM or MS-GSM system, the advantage lattice reduction presenting appears when SNR is higher than 10 dB and finally collect receive diversity. Compared with SS-GSM MIMO system, LR-aided linear detector works better on MS-GSM: the performance gap between the LR-aided linear detector and the optimal detector is about 6 dB for SS-GSM while nearly 3 dB for MS-GSM. It also should be noted that lattice reduction faster the unification of ZF and MMSE equalization at high SNR [11].
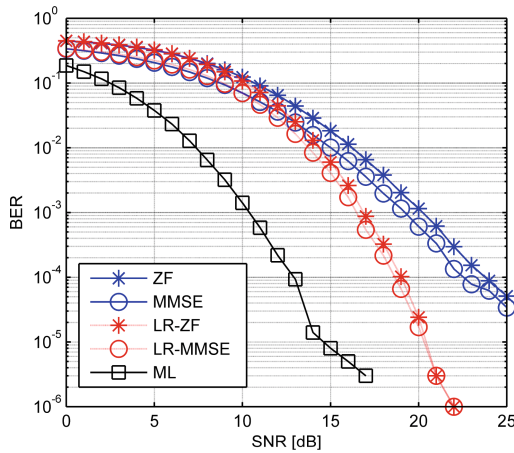

Fig. 6. BER performance of linear equalization detector, LR-aided equalization detector and ML detector employing the proposed 8-QAM constellation for SS-GSM MIMO system.
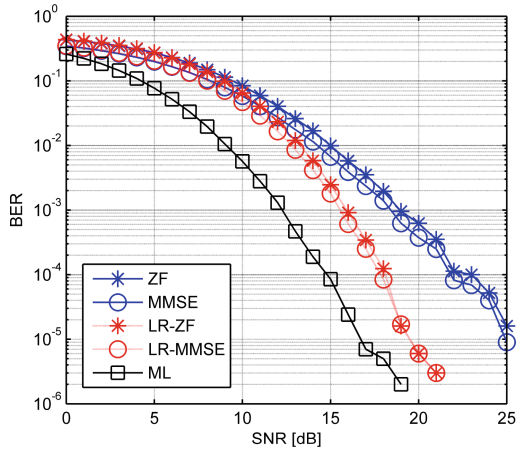
Fig. 7. BER performance of linear equalization detector, LR-aided equalization detector and ML detector employing the proposed 8-QAM constellation for MS-GSM MIMO system.

Figure 8 compares the BER performance under uncorrelated fading channel and correlated channel generated according to Kronecker Model. The antenna configuration of the MS-GSM system adopting the proposed 8-QAM constellation is $N_t = 5, N_a = 2, N_r = 6$. The correlation coefficient among the transmitting antennas and receiving antennas is 0.2, namely $r_{transmiter} = r_{receiver} = 0.2$. As we can see from the Fig. 8, the LR-aided MMSE detector can effectively alleviate the impact of channel correlation: the SNR gap between uncorrelated and correlated channel is nearly 8 dB for MMSE detector while merely 3 dB for LR-aided MMSE detector. Moreover, the LR-aided MMSE detector still can collect the receive diversity under the correlated channel.
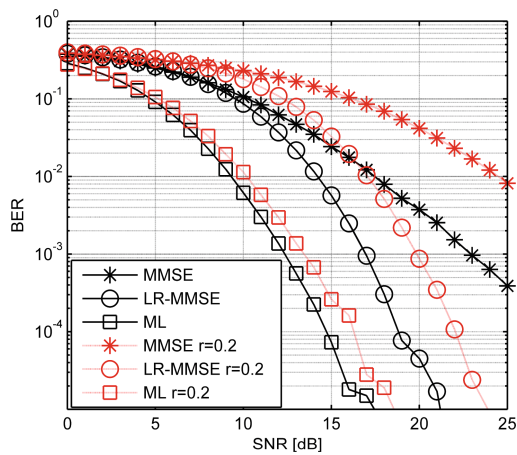
Fig. 8. BER performance under uncorrelated fading channel and correlated channel that employ the Kronecker Model.

6   Conclusion

In this paper, we introduce the LR-aided linear detector into GSM modulation and prove the method can collect the full receive diversity. Considering the characteristic of the transmit vector of GSM system, a compatible 8-QAM constellation is proposed to meet the LR requirement that the coefficient of the lattice base should be consecutive integers. Compared with the contrasting 4-QAM and 8-QAM, simulation results verify the superiority of the proposed 8-QAM on BER performance. At last, we also find that the LR-aided detector is more robust when dealing with the correlated channel.

In this paper, there are still some inadequacies to be study further. First, the theoretical analysis only proves that the LR-aided linear detector can collect the full receive diversity and does not further reveal how to affect the BER of GSM. Second, the performance gap between the LR-aided linear detector and ML detector is considerable, which need more effort to further promote the performance of linear detector for GSM system under the constraints of low complexity requirement.

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