

A Non-line-of-Sight Localization Method Based on the Algorithm Residual Error Minimization

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Abstract. Wireless localization has become a key technology location based services, and the non-line-of-sight (NLOS) propagation is one of the most important error source in the localization. Therefore, this paper defines a novel algorithm residual error (ARE) in NOLS environment, and estimates the position of mobile station (MS) by minimizing this ARE, where the quadratic programming is employed to solve the minimization problem. The simulation results show that the proposed algorithm produces significant performance improvements in NLOS environments.

Keywords: Wireless localization · Non-line-of-sight error
Algorithm residuals · Quadratic programming

1 Introduction

The wireless localization technology is one of the key techniques in the future internet of things, and therefore has attracted widely attentions. For example, in early 1990's, the FCC announced emergency call standard which requires a localization accuracy within 125 m [1]. So far, the localization parameters usually utilized the time-of-arrival (TOA/TDOA), angle-of-arrival (AOA) and received-signal-strength (RSS) or other information [2–5], and the positioning algorithms might include CHAN algorithm, Taylor series method, FANG algorithm, Friedlander algorithm, spherical interpolation algorithm (SI) and SX algorithm [6–10]. However, in non-line of sight (NLOS) environments, these previous algorithms could not achieve good performance, since the NLOS error in a real-world cellular network may approach 500–700 m. Meanwhile, the NLOS error cannot be statistically modeled. Therefore, the NLOS error suppression had become one of the key issues to the practical localization applications.

There are three kinds of NLOS mitigation methods. The first attempted to accurately model the NLOS environment, followed a position estimator exploiting this model [11, 12]. However, it is difficult in practice to obtain an accurate model to describe the complicated NLOS propagating environments. Thus, this kind of method was difficult to be widely used. The second kind of algorithm identified the NLOS base

stations (BS), and then employed only the LOS BSs to estimate the MS position [13, 14]. Such algorithms required a certain number of LOS BSs, but the NLOS BS identification performance could not be controlled, resulting in the positioning performance degradation sometimes. The third class of algorithm tried to weight the ranging measurements or intermediate estimations, and the weights were usually derived from the geometric and algebraic relationship between the BSs and the MSs [15–17]. The advantage of this kind of algorithm was that the MS could always be positioned, while its disadvantage was the limited estimation accuracy.

In order to tackle the above issues, this paper defines a novel residual error, i.e., the ARE, and then an optimization model is constructed. In detail, the optimization objective function is defined as the residual error of two conventional algorithms, and the constraints come from the relationships between measurements and corresponding true distances. Finally, a quadratic programming is employed to solve the optimization problem and achieve the position estimation. Computer simulations show that the proposed algorithm is superior to conventional localization algorithms in NLOS environments.

2 Range Based and Range-Inverse Based Localizations

Let (x_i, y_i) and (x, y) denote the coordinate of the i -th BS and MS, we have the BS-MS distance as

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (1)$$

After some mathematical transformations, we rewrite (1) as

$$r_i^2 - K_i = -2x_i x - 2y_i y + R \quad (2)$$

where $K_i = x_i^2 + y_i^2$ and $R = x^2 + y^2$. Equation (2) can be written in the matrix form, i.e.

$$\mathbf{P} = \mathbf{A}\mathbf{X} \quad (3)$$

where $\mathbf{P} = \begin{bmatrix} r_1^2 - K_1 \\ r_2^2 - K_2 \\ \vdots \\ r_N^2 - K_N \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 2x_1 & 2y_1 & -1 \\ 2x_2 & 2y_2 & -1 \\ \vdots & \vdots & \vdots \\ 2x_N & 2y_N & -1 \end{bmatrix}$, $\mathbf{X} = \begin{bmatrix} x \\ y \\ R \end{bmatrix}$.

It is easy to derive the least squares (LS) solution from (3)

$$\hat{\mathbf{X}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{P} \quad (4)$$

Next, we define the reciprocal of r_i , i.e.

$$R_i = \frac{1}{r_i} = \frac{1}{\sqrt{(x - x_i)^2 + (y - y_i)^2}} \quad (5)$$

Squaring both sides of (5), we have

$$R_i^2 = \frac{1}{K_i - 2x_i x - 2y_i y + R} \quad (6)$$

After some maths operations, we have

$$R_i^2 K_i - 1 = (2x_i x - 2y_i y - R) R_i^2 \quad (7)$$

Similarly, we can turn (7) into a matrix form, namely

$$\mathbf{Y} = \mathbf{C}\mathbf{X} \quad (8)$$

where $\mathbf{Y} = \begin{bmatrix} R_1^2 K_1 - 1 \\ R_2^2 K_2 - 1 \\ \vdots \\ R_N^2 K_N - 1 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 2x_1 R_1^2, & 2y_1 R_1^2, & -R_1^2 \\ 2x_2 R_1^2, & 2y_2 R_1^2, & -R_1^2 \\ \vdots & \vdots & \vdots \\ 2x_N R_1^2, & 2y_N R_1^2, & -R_1^2 \end{bmatrix}$, $\mathbf{X} = \begin{bmatrix} x \\ y \\ R \end{bmatrix}$.

Thus, the LS solution can be found as

$$\hat{\mathbf{X}} = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T \mathbf{Y} \quad (9)$$

3 The ARE Based Localization Algorithm

As said in Sect. 1, this section will detailed introduce the ARE based localization by utilizing the quadratic programming model, where the objective function, the constraints and the final optimization problem are investigated next.

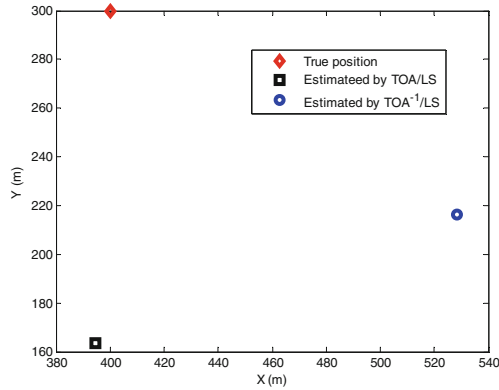


Fig. 1. Positioning results for different algorithms

3.1 The Object Function

In the NLOS environment, the above two position estimates will be different, which indicates that the residual error of different positioning algorithms is reasonable. Figure 1 shows that when the BS number is five, the above two position estimates, i.e., $\hat{\mathbf{X}}_1 = \hat{\mathbf{X}}$ and $\hat{\mathbf{X}}_2 = \hat{\mathbf{X}}$, will deviate from each other significantly. Accordingly, we can define an object function based on the ARE as

$$F(\mathbf{v}) = \text{norm}(\mathbf{P}(\hat{\mathbf{X}}_1 - \hat{\mathbf{X}}_2))^2 \quad (10)$$

where $\text{norm}(\bullet)$ represent the l_2 -norm and $\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Moreover, the relation between the range inverse and its true value can be written as

$$R_i^0 = \frac{1}{\alpha_i} R_i \quad (11)$$

where α_i represents the scaling factor. Then, formula (7) can be rewritten as

$$R_i^2 K_i - \alpha_i^2 = (2x_i x - 2y_i y - R) R_i^2 \quad (12)$$

From (12), we have the LS solution as

$$\hat{\mathbf{X}}_2 = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T (\mathbf{C} - \mathbf{v}) \quad (13)$$

where $\mathbf{B} = [R_1^2 K_1, R_2^2 K_2, \dots, R_N^2 K_N]^T$, $\mathbf{v} = [\alpha_1^2, \alpha_2^2, \dots, \alpha_N^2]^T$. Similarly, the range based position estimation can be rewritten as

$$\hat{\mathbf{X}}_1 = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{Y} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T (\mathbf{T} \mathbf{v} - \mathbf{Y}') \quad (14)$$

where $\mathbf{T} = \text{diag}\{r_1^2, r_2^2, \dots, r_N^2\}$, $\mathbf{Y}' = [K_1, K_2, \dots, K_N]^T$. Finally, the objective function (10) can be rewritten as

$$F(\mathbf{v}) = \text{norm}(\mathbf{P}((\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T (\mathbf{T} \mathbf{v} - \mathbf{Y}') - (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T (\mathbf{B} - \mathbf{v}))) \quad (15)$$

Next, we can turn the location estimation into an optimization problem, i.e.,

$$\text{minimize } F(\mathbf{v}) \quad (16)$$

3.2 The Constraints

The constraint is the rule that object parameters need to follow, and the optimization algorithms is to meet these constraints and find an expected value of the objective function to achieve the optimal solution. The proposed algorithm present in this paper has two main constraints, the first one derived from [15].

At first we should ensure the lower bound of vector \mathbf{v}

$$\mathbf{V}_{\min} = [\alpha_{1,\min}^2, \alpha_{2,\min}^2, \dots, \alpha_{N,\min}^2] \tag{17}$$

where $\alpha_{i,\min} = \max\left\{\frac{L_{i,j}-d_j}{d_i} \mid j \neq i, j \in [1, N], i \in [1, N]\right\}$. Note $L_{i,j}, i \neq j$ refers the distance between the i -th BS and j -th BS, and $\max\{\bullet\}$ denotes the maximum element of a vector (or set). Finally the first constraint can be expressed as $\mathbf{V}_{\min} \leq \mathbf{v} \leq \mathbf{V}_{\max}$ with $\mathbf{V}_{\max} = [1, 1, \dots, 1]^T$.

The second constraint comes from a fact that in the NLOS environment, the distance between MS and BS must be smaller than the measured distance. Hence, the MS must lie in the public areas, namely the feasible region. This constraint can be written as

$$\tilde{\mathbf{R}} \leq \mathbf{D}_{meas} \tag{18}$$

where $\tilde{\mathbf{R}} = \begin{bmatrix} \text{norm}(\mathbf{X} - BS_1) \\ \text{norm}(\mathbf{X} - BS_2) \\ \vdots \\ \text{norm}(\mathbf{X} - BS_N) \end{bmatrix}$, $\mathbf{D}_{meas} = \begin{bmatrix} r_1^2 \\ r_2^2 \\ \vdots \\ r_N^2 \end{bmatrix}$.

3.3 The Optimization Problem

According to Sects. 3.1 and 3.2, we can put the NLOS weight search into an optimization problem as follows

$$\begin{cases} \min & F(\mathbf{v}) \\ \text{subject to} & \\ & \mathbf{V}_{\min} \leq \mathbf{v} \leq \mathbf{V}_{\max} \\ & \tilde{\mathbf{R}} \leq \mathbf{D}_{meas} \end{cases} \tag{19}$$

Equation (19) can be solved by quadratic programming [18], and by substituting obtained vector into (10), we can obtain the optimal MS position estimate.

4 Simulation and Analysis

This paper exploits the classical BS topology as $(0, 0)$, $(\sqrt{3}r, 0)$, $(\frac{\sqrt{3}r}{2}, \frac{3}{2}r)$, $(-\frac{\sqrt{3}r}{2}, \frac{3}{2}r)$ and $(-\sqrt{3}r, 0)$, where r denotes the radius of a cellular cell, 1000 m in our study. In simulations, the measured noise will be modeled as a zero-mean Gaussian noise with its standard deviation of 10 m if unspecified. By contrast, the NLOS error cannot be accurately modeled, thus it is assumed as a uniformly distributed random variable ranging from 0 to MAX [19]. In addition, there are four algorithms compared in simulations, including the proposed algorithm, the CLS algorithm [20], the LLOP algorithm [21] and the TS-WLS algorithm [19].

4.1 Effects of NLOS Error

Figure 2 shows the NLOS error effect on the accuracy of tested algorithms, in which the MS is located in [400, 400]. From Fig. 2, we clearly see that all algorithms will produce higher RMSE with rising NLOS errors. Although the proposed algorithm differs from the CLS algorithm trivially for MAX less than 300 m, the performance advantage of the proposed method is obviously for a larger NLOS error scenario, i.e., $MAX > 300$ m.

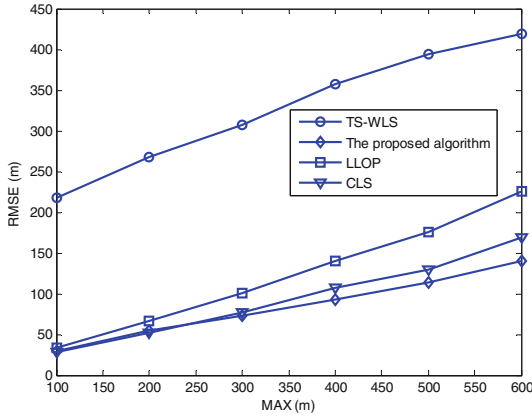


Fig. 2. NLOS error effect

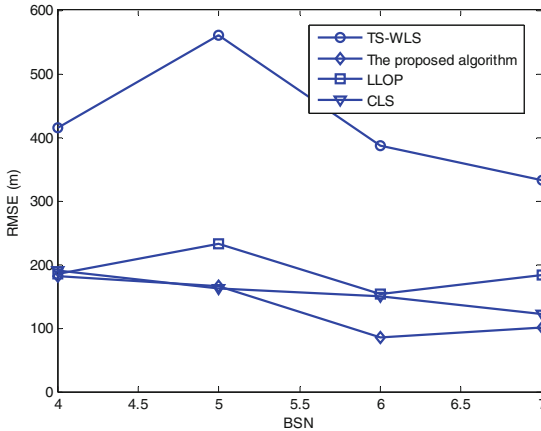


Fig. 3. BS number effect

4.2 Effects of the BS Number

Figure 3 shows the effects of BS number under the typical seven-BS topology, where the maximum value of NLOS error is 400 m and the MS randomly distributed within the cellular cell. From it, we explicitly find that the increase of BS number has improved the accuracy of all algorithms. It is also easy to see that for the proposed algorithm and CLS algorithm, they produce similar performance so long as the BS number is less than five, while the proposed algorithm significantly outperforms the CLS method with a higher BS number. From Figs. 2 and 3, the performance order of above algorithms must be, the proposed algorithm > CLS > LLOP > TS-WLS.

4.3 Effects of the LOS-BS Number

Figure 4 shows the effects of different LOS-BS numbers. As can be seen from this figure, the increasing LOS-BS number will increase the accuracy of the proposed algorithm. For instance, when the LOS-BS number is 1, the probability of accuracy of 120 m is 85%, but when the number reaches 2, the probability is 92%.

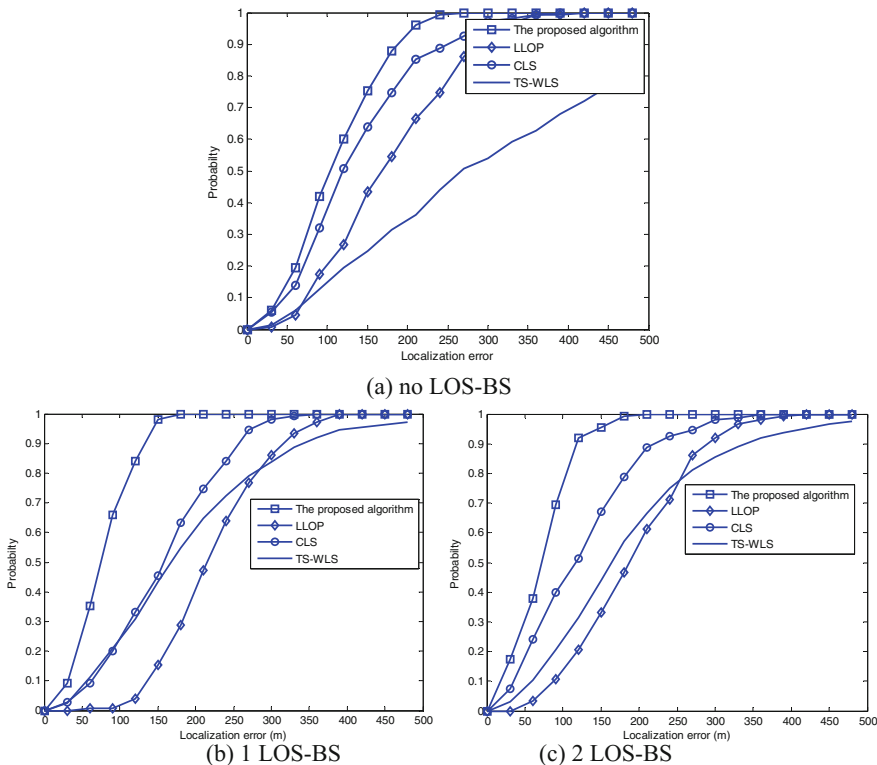


Fig. 4. Effect of LOS-BS number

In summary, the proposed algorithm is superior to the traditional location algorithm on accuracy, and the increase of the BS number will make accuracy of the proposed algorithm increase significantly. Simultaneously, since the LOS-BS will narrow the scope of feasible region, it also improves the accuracy of the proposed algorithm.

5 Conclusions

The NLOS error is a key and difficult point in wireless localization. Therefore it is important to study the localization under the NLOS corrupts. In this paper, we propose a new concept of residual error based on the positioning difference of different localization algorithms, and then we employ the optimization theory to reach a NLOS suppression localization, in which the estimation model is transferred into an optimum weights search. The quadratic programming is exploited to solve it and significantly improves the performance. Simulations prove that the proposed method is superior to some conventional algorithms.

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