Study on Correlation Properties of Complementary Codes and the Design Constraints of Complementary Coded CDMA Systems

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Abstract. Complementary codes (CCs) are a kind of two-dimensional spreading codes with ideal correlation properties to resolve the interference-limited problem of traditional CDMA systems. This paper proves the ideal correlation properties of CCs with non-integral chip delay under the definition of aperiodic correlation functions. The comparisons of CCs with traditional spreading codes on auto- and cross-correlation properties under different definitions of correlation functions will also present to verify the correctness of the proof work and to show that a CC-CDMA system is able to achieve MPI- and MAI-free communication owning to the proved ideal aperiodic correlation properties.

Keywords: Complementary codes · CDMA · Correlation properties
Multiple access interference · Multi-path interference

1 Introduction

Owning to better anti-interference ability, higher frequency efficiency, higher security and lower radiation, Code Division Multiple Access (CDMA) with spread spectrum technique has been widely applied in wireless communication systems in the last 50 years, since its origins in the military field and navigation systems. Till now, CDMA is still the preferred multiple access technique in satellite communications, although it has lost competitiveness compared with Frequency Division Multiple Access (FDMA) in cellular systems [1,2].

Now, we are interested in exploring reasons for the decline and walk-off of CDMA from a technical perspective. It is well known that all existing CDMA-based 2-3G standards are interference-limited, particularly in the presence of multiple access interference (MAI) and multi-path interference (MPI). It has to be admitted that the immediate cause is the unsatisfactory properties of

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the spreading sequences, while the primary cause is the uncoupled design of spreading codes with the systems and environment of communication.

The study on spreading codes for CDMA applications is a traditional research topic and many candidate codes have been found in the literature, however, they were generated and applied to the systems only based on the knowledge of seemly acceptable properties in their periodic auto- and cross-correlation functions. Due to the poor properties of spreading codes, a great deal of auxiliary sub-systems or techniques should be added to CDMA systems, such as the power control and multiuser detection, to mitigate the problems associated with the spreading codes, such as near-far effect, MAI and MPI, etc.

In order to bring CDMA back on track and to speed up the evolution of CDMA technologies, a possible solution has been proposed with the help of a new spreading technique based on complementary codes (CCs)[3]. Different from all traditional spreading sequences, the orthogonality of CCs is established based on a "flock" of element sequences jointly. As a result, ideal auto- and cross-correlation properties are realizable at the same time, while it never happens for any traditional spread sequences as proved by the Welch bound [4] and Sarwate bound [5].

In the work [6], we have present a survey on the history of CCs. However, a deeply studies on the correlation properties of CCs, especially with realistic communication environment has not been presented. Taking complete CCs [3] as a classic example, this paper proves the ideal correlation properties of CCs with non-integral chip delay under the definition of aperiodic correlation functions. Comparisons of CCs and traditional spreading codes on auto- and cross correlation properties are also presented in this paper to verify correctness of the proof work. Finally, an analysis on the detecting process of a complementary coded CDMA (CC-CDMA) system is presented with the design constraints of CC-CDMA systems concluded at the end of this paper.

2 Definitions and Code Construction

2.1 Definitions of CCs

Different from all traditional spreading sequences, the orthogonality of CCs is established based on a "flock" of element sequences jointly. A family of CCs, denoting as $\mathcal{C}(K, M, N)$, contents K CCs each with M element sequences. Due to its two-dimensional feature, let $\mathbf{C}^{(k)} = \{\mathbf{c}_m^{(k)}\}_{m=1}^{M}$ be a CC with M element sequences $\mathbf{c}_m^{(k)} = [c_{m,1}^{(k)}, c_{m,2}^{(k)}, \cdots, c_{m,N}^{(k)}]$. M is called flock size (which determines the number of element sequences used by the same user), and N is the code length. In this way, MN is the "congregated length" of a CC, and it determines the processing gain of the corresponding CC-CDMA system. For the CDMA application, K CCs are needed as signature codes for K users.

2.2 Construction of Complete Complementary Codes

Complete Complementary Codes (CCCs) [3] is one of the most popular CCs and this section gives the construction method of CCCs to facilitate the following proof and simulation work.

Let $\mathbf{A} = [a_{i,j}]$, $\mathbf{B} = [b_{i,j}]$, $\mathbf{D} = [d_{i,j}]$ be three $N \times N$ orthogonal matrices with $|a_{i,j}| = |b_{i,j}| = |d_{i,j}| = 1$, where $i, j \in \{1, 2, \cdots, N\}$. $\mathbf{a}_i = [a_{i,1}, a_{i,2}, \cdots, a_{i,N}]$ denotes i-th row of $\mathbf{A}$.

Step 1. Construct N sequences with length N^2, as

$$\mathbf{E}^{(k)} = [b_{k,1}\mathbf{a}_1, b_{k,2}\mathbf{a}_2, \cdots, b_{k,N}\mathbf{a}_N,] = [e_1^{(k)}, e_2^{(k)}, \cdots, e_{N^2}^{(k)}], \quad k = 1, 2, \cdots, N \tag{1}$$

Step 2. Construct m-th element sequence of k-th CCs in a family of CCCs using the above N sequences with matrix $\mathbf{D}$, as

$$\begin{aligned} \mathbf{c}_m^{(k)} &= \big[d_{m,1}e_1^{(k)}, d_{m,2}e_2^{(k)}, \cdots, d_{m,N}e_N^{(k)}, d_{m,1}e_{N+1}^{(k)}, d_{m,2}e_{N+2}^{(k)}, \cdots, d_{m,N}e_{2N}^{(k)}, \\ &\quad \cdots \quad d_{m,1}e_{N^2-N+1}^{(k)}, d_{m,2}e_{N^2-N+2}^{(k)}, \cdots, d_{m,N}e_{N^2}^{(k)}] \\ &= [c_{m,1}^{(k)}, c_{m,2}^{(k)}, \cdots, c_{m,N^2}^{(k)}\big], \qquad k, m = 1, 2, \cdots, N \end{aligned} \tag{2}$$

The above construction method of a family CCCs $\mathcal{C}(N, N, N^2)$ can be visually described in Fig. 1.

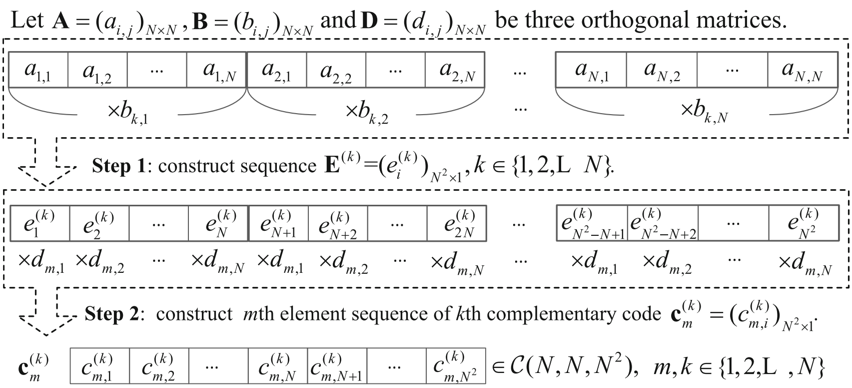


Fig. 1. The construction method of complete complementary codes.

3 Proof of Ideal Correlation Properties

3.1 Definitions of Complementary Correlation

Correlation properties of spreading codes are the key feature to effect the system performance of CDMA systems. Correlation function is usually used to describe

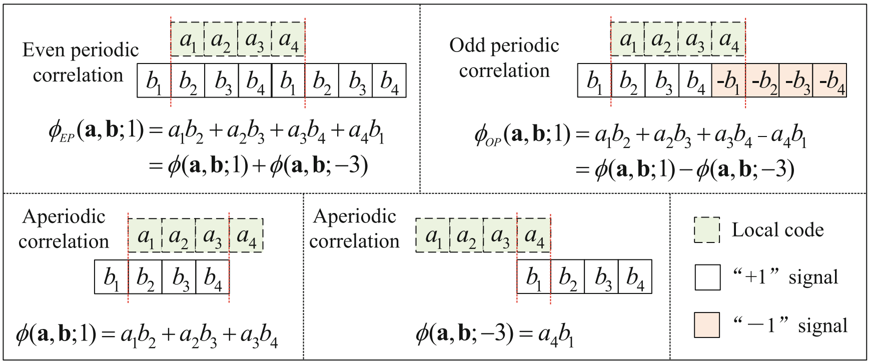


Fig. 2. Even periodic, odd periodic and aperiodic correlation functions and their relationships.

the correlation properties and three familiar definitions of correlation functions, even periodic, odd periodic and aperiodic correlation functions and their relationships, are visually described in Fig. 2.

When a and b is the same sequence, it is called auto-correlation function which is desired to be a delta function for a CDMA system to eliminate MPI, otherwise, it is called cross-correlation function which is desired to be a zero function for a CDMA system to eliminate MAI.

As can be seen from Fig. 2, the even periodic correlation function only describes the correlation properties when the adjacent bits have the same phase, while the odd periodic correlation function only describes the correlation properties when the adjacent bits have the positive phase. In fact, the phase of adjacent bits is random. Therefore, neither of them two is able to guarantee a MPI-free or MAI-free CDMA system, even though auto-correlation is a delta function and cross-correlation function is a zero function under the definitions of even or odd correlation functions. However, it can be easily proved that ideal aperiodic correlation properties are sufficient and necessary condition for both ideal even and odd correlation properties. Although it is more difficult to achieve ideal aperiodic correlation properties, but it is able to guarantee both MAI- and MPI-free in a CDMA system with any combination of adjacent bits.

Therefore, in this paper, the correlation properties of CCs are characterized by the complementary aperiodic correlation function which is calculated as the sum of the aperiodic correlation functions of all element sequences with the same delay δ, or

$$\rho(\mathbf{C}^{(k_1)}, \mathbf{C}^{(k_2)}; \delta) = \sum_{m=1}^{M} \phi(\mathbf{c}_m^{(k_1)}, \mathbf{c}_m^{(k_2)}; \delta) = \begin{cases} MN, & \delta = 0, k_1 = k_2 \\ 0, & \text{elsewhere} \end{cases} \tag{3}$$

where $\mathbf{C}^{(k_1)}, \mathbf{C}^{(k_2)} \in \mathcal{C}(K, M, N)$, $k_1, k_2 \in \{1, 2, \cdots, K\}$, and $\phi\left(\mathbf{c}_m^{(k_1)}, \mathbf{c}_m^{(k_2)}; \delta\right)$ is the aperiodic correlation function of $\mathbf{c}_m^{(k_1)}$ and $\mathbf{c}_m^{(k_2)}$. The ideal aperiodic correlation properties are described behind the second equal sign in (3).

3.2 Ideal Aperiodic Correlation Properties

The ideal aperiodic correlation properties of CCCs, as defined and constructed in the above sections, will be proved as followed.

As the construction method of CCCs, n-th chip of m-th element sequence of k-th CC can be expressed as

$$c_{m,n}^{(k)} = a_{x,y} b_{k,x} d_{m,y}, \quad k, m \in \{1, 2, \cdots, N\}, n \in \{1, 2, \cdots, N^2\} \tag{4}$$

where, $x = \lceil \frac{n}{N} \rceil$,$y =< n >_N + N\delta(< n >_N)$. The operator $< \cdot >_x$ means to calculate x-mod,$\lceil x \rceil$ denotes the ceil of x and $\delta(t)$ denotes a delta function.

According to (3), when $\delta \geq 0$, the complementary aperiodic correlation function of any two CCs in a family CCCs can be expressed as:

$$\rho(\mathbf{C}^{(k)}, \mathbf{C}^{(g)}; \delta) = \sum_{m=1}^{M} \sum_{n=1}^{N^2-\delta} c_{m,n}^{(k)} c_{m,n+\delta}^{(g)} = \sum_{m=1}^{M} \sum_{n=1}^{N^2-\delta} a_{x,y} b_{k,x} d_{m,y} a_{x',y'} b_{k,x'} d_{m,y'} \tag{5}$$

where $k, g \in \{1, 2, \cdots, N\}$, $x' = \lceil \frac{n+\delta}{N} \rceil y' =< n + \delta >_N + N\delta(< n + \delta >_N)$.

It is easy to prove that when $i \neq i'$, $\sum_{j=1}^{N} a_{i,j} a_{i',j} = \sum_{j=1}^{N} b_{i,j} b_{i',j} = \sum_{j=1}^{N} d_{i,j} d_{i',j} = 0$.

Now we prove the ideal aperiodic correlation properties of CCCs in three cases:

(1) when $\delta \neq qN$ and $\delta \neq 0$, $q \in Z^+$, $y \neq y'$, we get

$$\rho(\mathbf{C}^{(k)}, \mathbf{C}^{(g)}; \delta) = \sum_{n=1}^{N^2-\delta} a_{x,y} a_{x',y'} b_{k,x} b_{g,x'} \sum_{m=1}^{M} d_{m,y} d_{m,y'} = 0 \tag{6}$$

(2) when $\delta = qN$, $y = y'$ and $x' = x + q$, we get

$$\begin{aligned} \rho(\mathbf{C}^{(k)}, \mathbf{C}^{(g)}; \delta) &= \sum_{n=1}^{N^2-\delta} a_{x,y} a_{x',y} b_{k,x} b_{g,x'} \sum_{m=1}^{M} d_{m,y} d_{m,y} \\ &= N \sum_{x=1}^{N-q} \sum_{y=1}^{N} a_{x,y} a_{x+q,y} b_{k,x} b_{g,x'} \\ &= N \sum_{x=1}^{N-q} b_{k,x} b_{g,x+q} \sum_{y=1}^{N} a_{x,y} a_{x+q,y} \\ &= 0 \end{aligned} \tag{7}$$

(3) when $\delta = 0$, $y = y'$ and $x' = x$, we get

$$\begin{aligned} \rho(\mathbf{C}^{(k)}, \mathbf{C}^{(g)}; \delta) &= \sum_{n=1}^{N^2} a_{x,y} a_{x,y} b_{k,x} b_{g,x} \sum_{m=1}^{M} d_{m,y} d_{m,y} \\ &= N \sum_{x=1}^{N} b_{k,x} b_{g,x} \sum_{y=1}^{N} a_{x,y} a_{x,y} \end{aligned}$$

$$= N^2 \sum_{x=1}^{N} b_{k,x} b_{g,x}$$
$$= \begin{cases} N^3 & k = g \\ 0 & k \neq g \end{cases} \quad (8)$$

When $\delta < 0$, it is easy to prove that above conclusion is tenable. In conclusion, the CCCs constructed in Sect. 2.2 satisfies the ideal aperiodic correlation properties, as

$$\rho(\mathbf{C}^{(k)}, \mathbf{C}^{(g)}; \delta) = \begin{cases} N^3 & k = g, \delta = 0 \\ 0 & \text{elsewhere} \end{cases} \quad (9)$$

3.3 Ideal Correlation Properties with Non-integral Chip Delay

In practical CDMA systems, there exists non-integral chip delay between signals from multiple users or multiple paths. In this section, we will prove that the ideal aperiodic correlation properties of CCs still guarantee the interference-free communication even with non-integer chip-shift, taking the situation in Fig. 3 as an example.

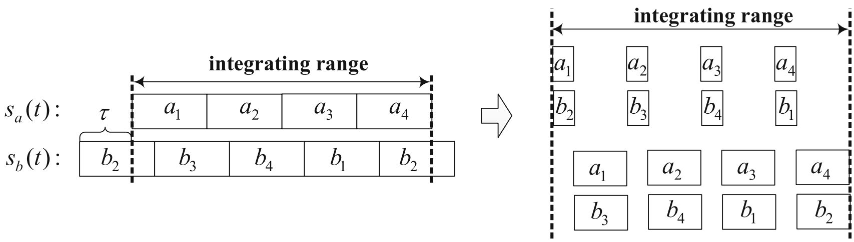


Fig. 3. The process of correlation with non-integral chip delay.

As shown in Fig. 3 the signal $s_a(t)$ and $s_b(t)$ are spread by the sequences $\mathbf{a} = [a_1, a_2, a_3, a_4]$ and $\mathbf{b} = [b_1, b_2, b_3, b_4]$ respectively. There exists chip delay τ between $s_a(t)$ and $s_b(t)$ due to multiple path transmission or asynchronous multi-user communication. When $\tau \neq qT_c$, $q \in Z^+$, T_c is the chip interval, we get

$$\int_0^{4T_c} s_a(t)s_b(t)dt = (T_c - \tau)(a_1b_2 + a_2b_3 + a_3b_4 + a_4b_1)$$
$$+ \tau(a_1b_3 + a_2b_4 + a_3b_1 + a_4b_2)$$
$$= (T_c - \tau)\phi_{EP}(\mathbf{a}, \mathbf{b}; 1) + \tau\phi_{EP}(\mathbf{a}, \mathbf{b}; 2)$$
$$= (T_c - \tau)[\phi(\mathbf{a}, \mathbf{b}; 1) + \phi(\mathbf{b}, \mathbf{a}; 3)] + \tau[\phi(\mathbf{a}, \mathbf{b}; 2) + \phi(\mathbf{b}, \mathbf{a}; 2)] \quad (10)$$

As shown in Fig. 3 and (10), correlation function with any non-integral chip delay equals to two correlation functions with integral chip delay. Therefore, the correlation properties of CCs with non-integral chip delay is still ideal owning to its ideal correlation properties with any integral chip delay.

4 Comparison on Correlation Properties with of Traditional Spreading Codes

In this section, taking a family of CCCs $\mathcal{C}(4, 4, 16)$ as an example, the simulated correlation properties of CCs are shown to verify correctness of the proof work. The congregated length of $\mathcal{C}(4, 4, 16)$ is 64, therefore, the correlation properties of Gold sequences with length 63 and Walsh codes with length 64 are also simulated.

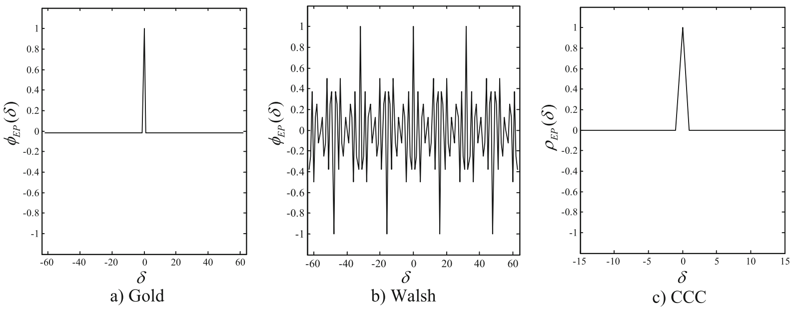


Fig. 4. Even periodic auto-correlation properties of different spreading codes.

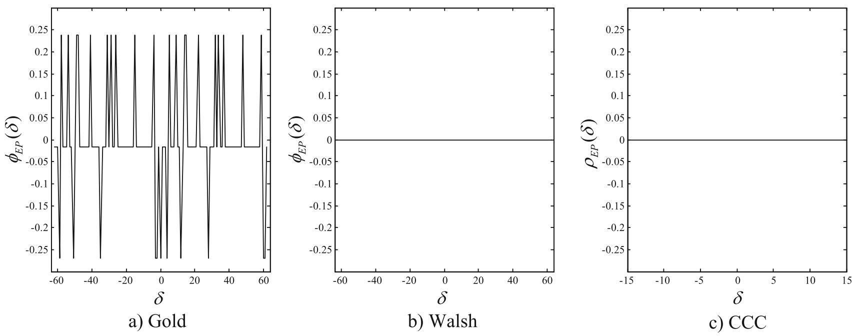


Fig. 5. Even periodic cross-correlation properties of different spreading codes.

The auto- and cross-correlation properties of the three spread codes under different definition of correlation functions: even periodic, odd periodic and aperiodic correlation functions are shown in Figs. 4, 5, 6, 7, 8 and 9 respectively. As can be seen from the simulated results, CCs are able to achieve ideal correlation properties (the auto-correlation is a delta function and the cross-correlation is a zero function) under all the three definitions. Gold code just achieves approximate

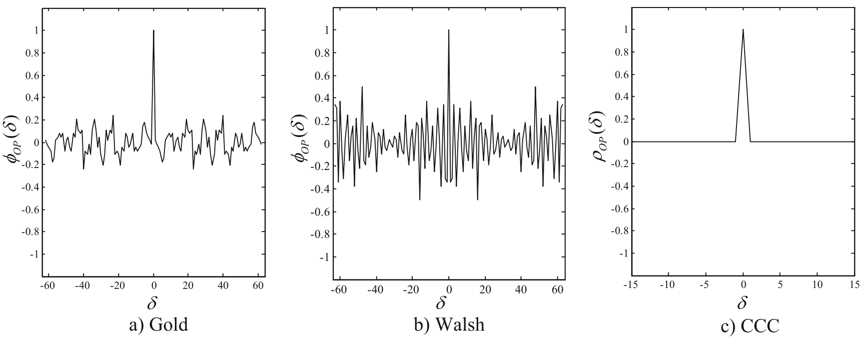


Fig. 6. Odd periodic auto-correlation properties of different spreading codes.

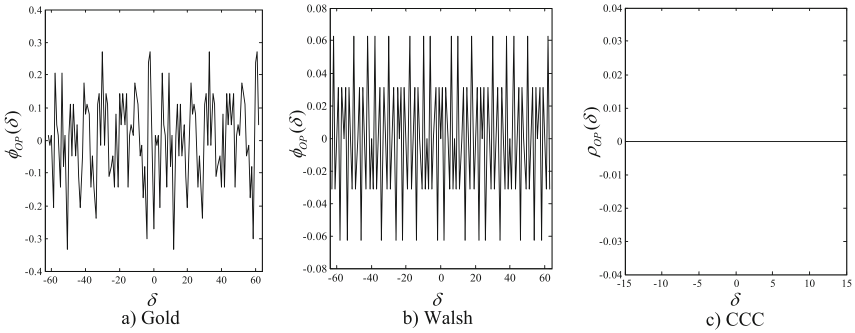


Fig. 7. Odd periodic cross-correlation properties of different spreading codes.

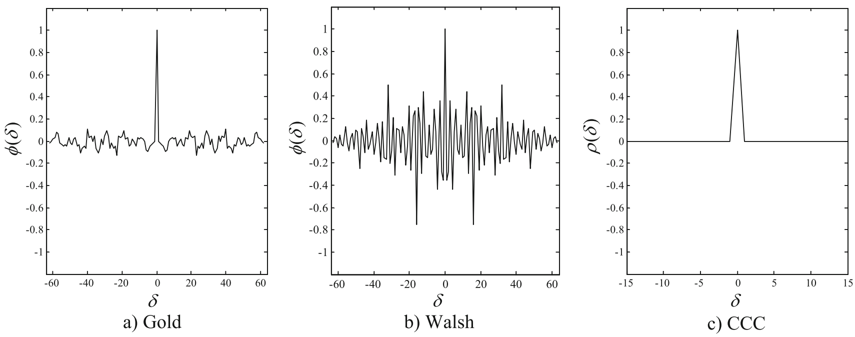
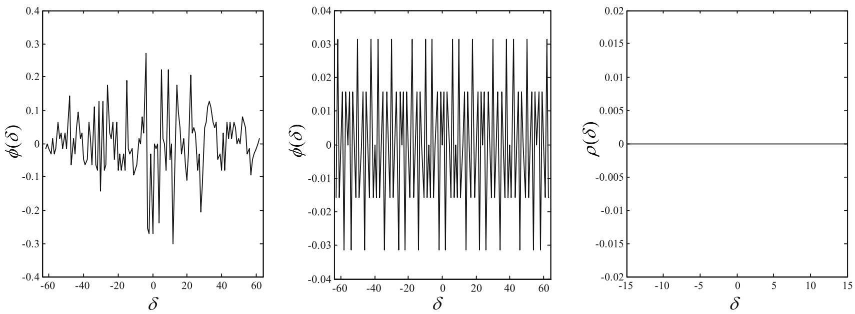


Fig. 8. Aperiodic auto-correlation properties of different spreading codes.

ideal auto-correlation property with even periodic correlation definition and Walsh code just achieves approximate ideal cross-correlation property with even periodic correlation definition. Therefore, a CDMA system with Gold code as its spreading sequence performs better under MPI, while it with Walsh code performs better under MPI. However, opposite phase between adjacent bits is



a) Gold b) Walsh c) CCC

Fig. 9. Aperiodic cross-correlation properties of different spreading codes.

usual. In this situation, the non-zero sidelobe in odd periodic auto-correlation property of Gold, as shown in Fig. 6(a), will result in MPI, while the non-zero sidelobe in odd periodic cross-correlation property of Gold, as shown in Fig. 7(b), will result in MAI.

5 Conclusions and Discussions

This paper proves the ideal correlation properties of CCs with non-integral chip delay under the definition of aperiodic correlation functions. The above comparisons of CCs with traditional spreading codes on auto- and cross-correlation properties under different definitions of correlation functions verify the correctness of the proof work and show that a CC-CDMA system is able to achieve MPI- and MAI-free communication owning to the proved ideal aperiodic correlation properties.

However, due to the two-dimensional nature of CCs, the implementation of CC-CDMA system is a challenging work. In a direct sequence (DS) CC-CDMA system, each user will be allocated a particular CC from a code set as its signature code, and a user should spread its data with M element sequences of CC, respectively. In order to realize the spreading and de-spreading processes as definition of aperiodic correlation function, a CC-CDMA system must satisfies the following four conditions:

(1) M streams of spread signals of one user are required to be transmitted in M independent subchannels and separated at a receiver;

(2) each stream of spread signal should be de-spread using the right element sequence of the CC allocated to the user at a receiver;

(3) each stream of spread signal should be synchronized and therefore they have the same chip-delay;

(4) the de-spread signals with M element sequences should combined with equal gains.

Therefore, it's challenging to design and implement a CC-CDMA system. The work [7] has present a comprehensive survey of existing literature in the area of CC-CDMA system and it divided the existing CC-CDMA solutions into two categories: time division multiplex and frequency division multiplex CC-CDMA systems, according to the kinds of independent sub-channels. However, both of the two CC-CDMA system architecture have its problem on implementation complexity or spread and spectrum efficiency. Therefore, as for the future works, we will pursue to optimize the system deign of CC-CDMA systems.

References

1. Zhou, Y., Jiang, T., Huang, C., et al.: Peak-to-average power ratio reduction for OFDM/OQAM signals via alternative-signal method. IEEE Trans. Veh. Technol. **63**(1), 494–499 (2014)
2. Nguyen, H.C., de Carvalho, E., Prasad, R.: Multi-user interference cancellation schemes for carrier frequency offset compensation in uplink OFDMA. IEEE Trans. Wirel. Commun. **13**(3), 1164–1171 (2014)
3. Suehiro, N., Hatori, M.: N-shift cross-orthogonal sequences. IEEE Trans. Inf. Theory **34**(1), 143–146 (1988)
4. Welch, L.: Lower bounds on the maximum cross correlation of signals (corresp.). IEEE Trans. Inf. Theory **20**(3), 397–399 (1974)
5. Sarwate, D.V., Pursley, M.B.: Crosscorrelation properties of pseudorandom and related sequences. Proc. IEEE **68**(5), 593–619 (1980)
6. Sun, S.-Y., Chen, H.-H., Meng, W.: A survey of complementary coded wireless communications. IEEE Commun. Surv. Tutor. **17**(1), 52–69 (2015)
7. Sun, S., Han, S., Yu, Q., Meng, W., Li, C.: A survey of two kinds of complementary coded CDMA wireless communications. In: 2014 IEEE Global Communications Conference, pp. 468–472 (2014)