An Efficient DOA Estimation and Network Sorting Algorithm for Multi-FH Signals

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Abstract. In order to use the spatial characteristic parameter of frequency hopping (FH) signal to realize FH sorting, an efficient FH signal DOA estimation algorithm is proposed in this paper. Firstly, the effective hop of signal is extracted from time and frequency domain and the spatial-time-frequency distribution matrix of this hop is established; then the SCMUSIC spatial spectrum is constructed using descending dimension method of noise sub-space based on MUSIC algorithm; finally we realize fast DOA estimation through half-spectrum searching so that FH sorting can be achieved using DOA information. Theoretical analysis and simulation results show that this algorithm has good effectiveness and estimation performance.

Keywords: Frequency-hopping (FH) · SCMUSIC spatial spectrum
Direction of arrival (DOA) · Morphological filtering · Network sorting

1 Introduction

Frequency-hopping signals have been widely used in military communication because of their characteristics of good security, strong anti-interference ability, low probability of interception and strong networking capability [1]. How to realize the correct network sorting for multiple frequency hopping signals with different frequency hopping parameters without prior knowledge is the essential issue of frequency hopping signal reconnaissance and countermeasure.

Signal direction of arrival (DOA) plays an important role in the separation of frequency hopping signals. In [2, 3], a novel space-time approach is developed for estimating the DOA of FH signals. However, it is only applicable to over-determined conditions; Electromagnetic vector antenna is used to estimate the DOA of frequency hopping signal in [4], but this approach can only deal with a limited number of frequency hopping signals; The concept of spatial-time-frequency was first proposed

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by Belouchrani in [5, 6], and then it was used in linear frequency modulation signal estimation and blind source signal separation; Chen [7] introduces the space-time-frequency analytical method into the FH signals by constructing the spatial-time-frequency distribution matrix of each hop and using the MUSIC algorithm, however this approach is very complicated; In [8, 9], root-music approach is proposed in order to replace the MUSIC algorithm, which reduced the complexity of the MUSIC algorithm, but the algorithm has high demand for the array and is not suitable for engineering applications; A spatial-polarimetric-time-frequency distributions and ESPRIT algorithm was proposed in [10, 11], to estimate DOA and polarization of FH signals, however the ESPRIT algorithm needs parameter matching, which increases the complexity of the algorithm.

Based on the above issues, the STFD&SCMUSIC algorithm is proposed in this paper to estimate DOA of multiple FH signals. Firstly, the effective hop of signal from time and frequency domain is extracted and the spatial-time-frequency distribution matrix of this hop is established; Then we introduce a conjugate noise subspace to construct the spatial spectral function of SCMUSIC algorithm based on the idea of noise subspace reduction, and realize the estimation of multi-frequency hopping signal; Finally, achieve FH signal network sorting according to the estimated DOA information using clustering algorithm; At the same time, the time-frequency map is amended via morphological filtering method in order to enhance performance of low SNR algorithm. The proposed method can not only adapt to different network information, but also can greatly reduce the complexity of the original algorithm.

2 Snapshots Model of FH Signal

Suppose that the hopping period of FH signal $\boldsymbol{s}_n(t)$ is T_n, there are K hops within time of Δt in total. ω_{nk} and φ_{nk} represent the carrier frequency and initial phase of K-th hop respectively and the time of initial hop is Δt_{0n}. Then the $\boldsymbol{s}_n(t)$ can be defined as:

$$s_n(t) = \upsilon_n(t) \sum_{k=0}^{K-1} \exp[j(\omega_{nk} t' + \varphi_{nk})] rect\left(\frac{t'}{T_n}\right) \tag{1}$$

Where $t' = t - (k-1)T_n - \Delta t_{0n}$, υ_n stands for the complex envelope of base band of signal $\boldsymbol{s}_n(t)$, *rect* is the unit rectangle pulse.

Assume that N FH signals impinge instantaneously onto an M-element array, the FH signal is not correlated with the noise between the array, the bandwidth of the receiver $B = f_{\max} - f_{\min}$, and the elements spacing $d < c/2f_{\max}$ (c denotes the speed of light), $f_{\max}$ and $f_{\min}$ denote the upper and lower limits of the receiver bandwidth respectively. Let the azimuth angle of the FH incident wave is θ and the wavelength is λ, then the steering vector of the array can be expressed as:

$$\boldsymbol{a}(\theta) = [1, e^{-j2\pi d \sin\theta/\lambda}, \cdots, e^{-j2\pi(M-1)d \sin\theta/\lambda}]^T \tag{2}$$

The array flow pattern matrix can be formulated as:

$$\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_N] \tag{3}$$

Assume the data vector is $\boldsymbol{X}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$, noise vector is $\boldsymbol{N}(t) = [n_1(t), n_2(t), \cdots n_M(t)]^T$, and FH source data vector is $\boldsymbol{S}(t) = [s_1(t), s_2(t), \cdots s_N(t)]^T$, so snapshot vector model for array can be defined as:

$$\boldsymbol{X}(t) = \boldsymbol{Y}(t) + \boldsymbol{N}(t) = \boldsymbol{AS}(t) + \boldsymbol{N}(t) \tag{4}$$

3 Construction of Space-Time-Frequency Matrix of FH Signal

The FH signal is a wide-band signal, and the carrier frequency of each hop jumps randomly, so the steering vector and manifold matrix of array jump with the carrier frequency, but it can be simplified as a narrow-band signal when studying one hop. So we establish the spatial-time-frequency distribution matrix of one effective hop of signal.

Cohen discrete time frequency distribution of signal $x_i(t)$ is expressed as:

$$\boldsymbol{D}_{x_i x_i}(t,f) = \sum_{\tau=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \varphi(l,\tau) x_i(t+l+\tau) x_i^*(t+l-\tau) e^{-j4\pi f \tau} \tag{5}$$

Where $\varphi(l,\tau)$ denotes kernel function. So the discrete cross-time-frequency distribution of signal $x_i(t)$ and $x_j(t)$ can be defined as:

$$\boldsymbol{D}_{x_i x_j}(t,f) = \sum_{\tau=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \varphi(l,\tau) x_i(t+l+\tau) x_j^*(t+l-\tau) e^{-j4\pi f \tau} \tag{6}$$

So the spatial-time-frequency distribution of signal $x(t)$ can be defined as:

$$\boldsymbol{D}_{XX}(t,f) = \sum_{\tau=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \varphi(l,\tau) X(t+l+\tau) X^H(t+l-\tau) e^{-j4\pi f \tau} \tag{7}$$

Where $[\boldsymbol{D}_{XX}(t,f)]_{ij} = \boldsymbol{D}_{x_i x_j}(t,f)$ $(i,j = 1, 2, \cdots, M)$ denotes the time frequency distribution between the output signals of each array. According to (4) and (7), the covariance matrix of time frequency domain can be written as:

$$E[\boldsymbol{D}_{XX}(t,f)] = \boldsymbol{A}\boldsymbol{D}_{SS}(t,f)\boldsymbol{A}^H + E[\boldsymbol{D}_{NN}(t,f)] \tag{8}$$

4 DOA Estimation and Network Sorting

4.1 Construction of SCMUSIC Spatial Spectrum

Suppose that the number of sources for each hop is L. According to eigenvalue decomposition of array covariance matrix, we can obtain the signal subspace $\boldsymbol{U}_S$ whose dimensional is L, and noise subspace $\boldsymbol{U}_N$ whose dimensional is $M-L$. The spatial spectral function $P_{MUSIC}(\theta)$ of MUSIC algorithm can be expressed as:

$$P_{MUSIC}(\theta) = \frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{U}_N\boldsymbol{U}_N^H\boldsymbol{a}(\theta)} \tag{9}$$

According to $P_{MUSIC}(\theta)$, seek the spectrum peak in θ domain, and the extreme value θ of $P_{MUSIC}(\theta)$ is the desired DOA, however, the MUSIC algorithm needs to seek the spectrum peak in the whole field, which makes the algorithm too complex to be realized. We can get (10) from the orthogonal subspace principle, i.e.

$$\boldsymbol{a}^H(\theta)\boldsymbol{U}_N = \boldsymbol{O} \tag{10}$$

According to the conjugate principle, (2) can be rewritten as:

$$\boldsymbol{a}^*(\theta) = [1, e^{j2\pi d\,\sin\theta/\lambda}, \cdots, e^{j2\pi(M-1)d\,\sin\theta/\lambda}]^T = \boldsymbol{a}(-\theta) \tag{11}$$

According to the relationship in formula (11), (13) can be rewritten as:

$$[\boldsymbol{a}^H(\theta)\boldsymbol{U}_N]^* = \boldsymbol{a}^T(\theta)\boldsymbol{U}_N^* = \boldsymbol{a}^H(-\theta)\boldsymbol{U}_N^* = \boldsymbol{O} \tag{12}$$

Where $\boldsymbol{U}_N^*$ denotes the conjugate of noise subspace $\boldsymbol{U}_N$. Suppose that the DOA of the signal source S is θ_s, from (12) we can see that there is a mirror source S' in the time-frequency domain whose DOA is $-\theta_s$, its steering vector is conjugated to steering vector of source S and orthogonal to $\boldsymbol{U}_N^*$. Therefore, $\boldsymbol{U}_N^*$ is introduced into the spatial spectrum of MUSIC algorithm, and the spatial spectrum function $P_{SCMUSIC}(\theta)$ of SCMUSIC can be defined as:

$$P_{SCMUSIC}(\theta) = \frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{U}_N\boldsymbol{U}_N^H\boldsymbol{U}_N^*\boldsymbol{U}_N^T\boldsymbol{a}(\theta)} \tag{13}$$

From (13), $P_{SCMUSIC}(-\theta)$ can be defined as:

$$P_{SCMUSIC}(-\theta) = \frac{1}{\boldsymbol{a}^H(-\theta)\boldsymbol{U}_N\boldsymbol{U}_N^H\boldsymbol{U}_N^*\boldsymbol{U}_N^T\boldsymbol{a}(-\theta)} = P_{SCMUSIC}(\theta) \tag{14}$$

We can see from (14) that $P_{SCMUSIC}(\theta)$ is an even function. If the noise subspace is $\boldsymbol{U}_N = [\boldsymbol{U}_{N_1}, \boldsymbol{U}_{N_2}, \cdots, \boldsymbol{U}_{N_{M-L}}]$, we can obtain

$$
\begin{aligned}
\boldsymbol{U}_N^H\boldsymbol{U}_N^* &= \begin{bmatrix} \boldsymbol{U}_{N_1}^* \\ \boldsymbol{U}_{N_2}^* \\ \vdots \\ \boldsymbol{U}_{N_{M-L}}^* \end{bmatrix} [\boldsymbol{U}_{N_1}^*, \boldsymbol{U}_{N_2}^*, \cdots, \boldsymbol{U}_{N_{M-L}}^*] \\
&= \begin{bmatrix} \boldsymbol{U}_{N_1}^*\boldsymbol{U}_{N_1}^* & \boldsymbol{U}_{N_1}^*\boldsymbol{U}_{N_2}^* & \cdots & \boldsymbol{U}_{N_1}^*\boldsymbol{U}_{N_{M-L}}^* \\ \boldsymbol{U}_{N_2}^*\boldsymbol{U}_{N_1}^* & \boldsymbol{U}_{N_2}^*\boldsymbol{U}_{N_2}^* & \cdots & \boldsymbol{U}_{N_2}^*\boldsymbol{U}_{N_{M-L}}^* \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{U}_{N_{M-L}}^*\boldsymbol{U}_{N_1}^* & \boldsymbol{U}_{N_{M-L}}^*\boldsymbol{U}_{N_2}^* & \cdots & \boldsymbol{U}_{N_{M-L}}^*\boldsymbol{U}_{N_{M-L}}^* \end{bmatrix}
\end{aligned} \tag{15}
$$

Substitute (15) into (13), we have:

$$
\begin{aligned}
P_{SCMUSIC}^{-1}(\theta) &= [\boldsymbol{a}^H(\theta)\boldsymbol{U}_N](\boldsymbol{U}_N^H\boldsymbol{U}_N^*)[\boldsymbol{U}_N^T\boldsymbol{a}(\theta)] \\
&= \begin{bmatrix} \boldsymbol{a}^H(\theta)\boldsymbol{U}_{N_1} & \boldsymbol{a}^H(\theta)\boldsymbol{U}_{N_2} & \cdots & \boldsymbol{\alpha}^H(\theta)\boldsymbol{U}_{N_{M-L}} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{N_1}^H\boldsymbol{U}_{N_1}^* & \boldsymbol{U}_{N_1}^H\boldsymbol{U}_{N_2}^* & \cdots & \boldsymbol{U}_{N_1}^H\boldsymbol{U}_{N_{M-L}}^* \\ \boldsymbol{U}_{N_2}^H\boldsymbol{U}_{N_1}^* & \boldsymbol{U}_{N_2}^H\boldsymbol{U}_{N_2}^* & \cdots & \boldsymbol{U}_{N_2}^H\boldsymbol{U}_{N_{M-L}}^* \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{U}_{N_{M-L}}^H\boldsymbol{U}_{N_1}^* & \boldsymbol{U}_{N_{M-L}}^H\boldsymbol{U}_{N_2}^* & \cdots & \boldsymbol{U}_{N_{M-L}}^H\boldsymbol{U}_{N_{M-L}}^* \end{bmatrix} \\
&\quad \begin{bmatrix} \boldsymbol{U}_{N_1}^T\boldsymbol{a}(\theta) \\ \boldsymbol{U}_{N_2}^T\boldsymbol{a}(\theta) \\ \vdots \\ \boldsymbol{U}_{N_{M-L}}^T\boldsymbol{a}(\theta) \end{bmatrix} = \sum_{i=1}^{M-L}\sum_{j=1}^{M-L} \left[\boldsymbol{a}^H(\theta)\boldsymbol{U}_{N_i}\right](\boldsymbol{U}_{N_i}^H\boldsymbol{U}_{N_j}^*)\left[\boldsymbol{U}_{N_j}^T\boldsymbol{a}(\theta)\right]
\end{aligned} \tag{16}
$$

From (10), (11) and (12), we can obtain:

$$
\begin{cases} \boldsymbol{a}^H(\theta_s)\boldsymbol{U}_{N_i} = \boldsymbol{O} \\ \boldsymbol{U}_{N_j}^T\boldsymbol{a}^H(-\theta_s) = [\boldsymbol{a}^H(\theta_s)\boldsymbol{U}_{N_i}]^T = \boldsymbol{O} \end{cases} \tag{17}
$$

Substitute (16) into (17), we have:

$$
P_{SCMUSIC}^{-1}(\pm\theta_s) = 0 \quad (s = 1, 2, \cdots L) \tag{18}
$$

Therefore, the spatial spectrum function $P_{SCMUSIC}(\theta)$ of the SCMUSIC algorithm takes the extremum at $\pm\theta_s$, so $P_{SCMUSIC}(\theta)$ is the symmetric spatial spectrum.

4.2 DOA Estimation and Network Sorting Based on SCMUSIC

When constructing the spatial spectral function $P_{SCMUSIC}(\theta)$, the noise subspace decreases with the L dimension, and the signal subspace increases with L dimension. Suppose that the new noise subspace is $\boldsymbol{U}_N'$, and new signal subspace is $\boldsymbol{U}_S'$. So $\boldsymbol{U}_N'$ is the intersection of the noise subspace $\boldsymbol{U}_N$ with its conjugate $\boldsymbol{U}_N^*$, and $\boldsymbol{U}_S'$ is the union of the signal subspace $\boldsymbol{U}_S$ with its conjugate $\boldsymbol{U}_S^*$.

Let $\boldsymbol{R} = \boldsymbol{I} - \boldsymbol{U}_N\boldsymbol{U}_N^H\boldsymbol{U}_N^*\boldsymbol{U}_N^T$, the zero space of $\boldsymbol{R}$ is $\boldsymbol{R}^{\circ}$, (i.e. $\boldsymbol{R}^{\circ} = \{\boldsymbol{x}|\boldsymbol{R}\boldsymbol{x} = \boldsymbol{O}, \boldsymbol{x} \in C^M\}$), the new noise subspace $\boldsymbol{U}_N'$ is the same as the zero space $\boldsymbol{R}^{\circ}$ [12], i.e. $\boldsymbol{U}_N' = \boldsymbol{R}^{\circ}$.

We can obtain the Singular value decomposition for $\boldsymbol{R}$:

$$\boldsymbol{R} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}^H \tag{19}$$

Where $\boldsymbol{\Lambda} = diag(\sigma_1, \sigma_2, \cdots, \sigma_M)$ is a diagonal matrix, and $\boldsymbol{V} = [\boldsymbol{V}_{2L}\ \tilde{\boldsymbol{V}}_{M-2L}]$, $\boldsymbol{V}_{2L}$ is composed of $2L$ nonzero singular value of $\boldsymbol{V}$, $\tilde{\boldsymbol{V}}_{M-2L}$ is composed of $M-2L$ zero singular value of $\boldsymbol{V}$. We can see that $\tilde{\boldsymbol{V}}_{M-2L}$ is the standard orthogonal basis of $\boldsymbol{R}^{\circ}$, the spatial spectrum function $P_{SCMUSIC}(\theta)$ can be rewritten as:

$$P_{SCMUSIC}(\theta) = \frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{U}_N\boldsymbol{U}_N^H\boldsymbol{U}_N^*\boldsymbol{U}_N^T\boldsymbol{a}(\theta)} = \frac{1}{\boldsymbol{a}^H(\theta)\tilde{\boldsymbol{V}}_{M-2L}\tilde{\boldsymbol{V}}_{M-2L}^H\boldsymbol{a}(\theta)} \tag{20}$$

According to (20), the half-spectral peak search is carried out in the θ domain to obtain the $\pm\theta_s$, which makes the extremum of $P_{SCMUSIC}(\theta)$. If $\|\boldsymbol{a}^H(\theta_s)\boldsymbol{U}_N\| \approx 0$, then θ_s is the required DOA, otherwise $-\theta_s$ is the required DOA. Therefore, the complexity of the full spectrum peak search in the MUSIC algorithm is reduced to half of that, which greatly reduces the complexity of the algorithm. Through the clustering analysis of the obtained DOA, the network sorting of multi-FH signal can be achieved.

5 Simulation and Analysis

There are three frequency hopping signal FH1, FH2, FH3 in the space, the incident angle is $\theta_1 = 20°$, $\theta_2 = 40°$, $\theta_3 = 60°$ respectively, the hopping period is 10 μs, the carrier frequency jumps between 0–0.5, the sampling rate is 100 MHz, the number of receiving array is 4 and the number of snapshots is 3000.

100 Monte Carlo experiments were performed, the root mean square error of DOA, and estimated success rate were used as the performance criterion. The root mean square error (RMSE) of DOA is defined as:

$$RMSE = \sqrt{\frac{1}{L}\sum_{i=1}^{L}(\tilde{\theta}_i - \theta)^2} \tag{21}$$

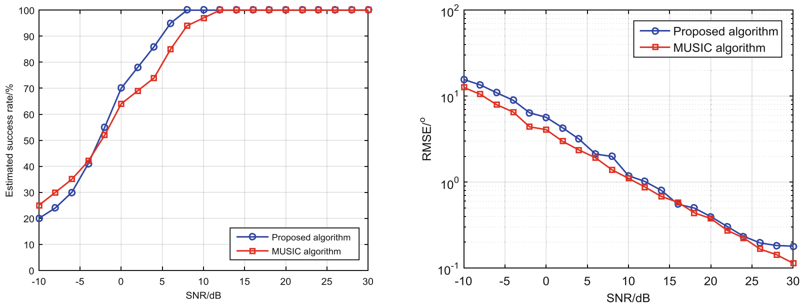
Where L denotes the source number, $\tilde{\theta}$ and θ denote the estimated and true values of the DOA respectively. The estimated success rate η is defined as:

$$\eta = N_1/N \tag{22}$$

Where N_1 denotes the number of successful experiments with DOA estimated deviations less than 2°, and N denotes the total number of experiments.

5.1 Experiment 1

Figure 1 shows the performance comparison of DOA estimation in proposed algorithm and MUSIC algorithm when SNR increase from −10 dB to 30 dB.



(a) (b)

Fig. 1. Performance comparison of DOA estimation. (a) The success rate of experiment 1, (b) the RMSE of experiment 1

It can be seen from Fig. 1(a) that with the increase of SNR, the η of the proposed algorithm and the MUSIC algorithm are gradually increased; the RMSE are both gradually decreased; when SNR is greater than −2 dB, the η of the proposed algorithm is larger than the MUSIC algorithm, and when the SNR reaches 8 dB, the η of the proposed algorithm is close to 100% while the MUSIC algorithm needs to reach about 12 dB.

It can be seen from Fig. 1(b) that with the increase of SNR, the RMSE of the proposed algorithm and the MUSIC algorithm are gradually decreased; the RMSE of the proposed algorithm is slightly larger than MUSIC algorithm in general; When the SNR is greater than 15 dB, the RMSE of the proposed algorithm is close to the MUSIC algorithm.

5.2 Experiment 2

The time required for the DOA estimation of the two algorithms SNR increase from −8 dB to 20 dB is shown in Table 1.

Table 1. Comparison of two algorithms for DOA estimation time required (s)

Algorithm type	−8 dB	−4 dB	0 dB	4 dB	8 dB	12 dB	16 dB	20 dB
Proposed algorithm	6.413	6.214	6.804	6.384	6.501	6.320	6.449	6.423
MUSIC algorithm	12.908	12.840	12.857	12.807	12.856	12.874	12.783	13.196

It can be seen from Table 1 that the time required for the DOA estimation of the proposed algorithm is about 6.43 s, while the MUSIC needs 12.85 s. Therefore, the complexity of the MUSIC algorithm can be reduced to half of that.

6 Conclusion

The DOA information of the frequency hopping signals can be effectively used to complete multi-FH signal network sorting. The STFD&SCMUSIC algorithm is deduced and explained in this paper to estimate the DOA information of multiple FH signals, and the networking sorting is achieved through the clustering of the estimated DOA. Theoretical analysis and simulation results show that the proposed algorithm can reduce the computational complexity of the traditional MUSIC algorithm by 50% while the RMSE is equivalent to it and the estimated success rate is higher than it.

References

1. Sha, Z.C.: Online hop timing detection and frequency estimation of multiple FH signals. ETRI J. **35**(5), 748–756 (2013)
2. Liu, X., Sidiropoulos, N.D., Swami, A.: Blind high-resolution localization and tracking of multiple frequency hopped signals. J. IEEE Trans. Signal Process. **50**(4), 889–901 (2002)
3. Liu, X., Li, J., Ma, X.: An EM algorithm for blind hop timing estimation of multiple FH signals using an array system with bandwidth mismatch. J. IEEE Trans. Veh. Technol. **56**(5), 2545–2554 (2007)
4. Wong, K.T.: Blind beamforming/geolocation for wide band-FFHs with unknown hop-sequences. J IEEE Trans. Aerosp. Electr. Syst. **37**(1), 65–76 (2001)
5. Belouchrani, A., Amin, M.G.: Time-frequency MUSIC. J. IEEE Signal Process. Lett. **6**(5), 109–110 (1999)
6. Belouchrani, A., Amin, M.G.: Blind source separation based on time-frequency signal representations. J. IEEE Trans. Signal Process. **46**(11), 2888–2897 (1998)
7. Chen, L.H., Wang, Y.M., Zhang, E.Y.: Directions of arrival estimation for multicomponent frequency-hopping/direction sequence spread spectrum signals based on spatial time-frequency analysis. J. Signal Process. **25**(8), 1309–1313 (2009)
8. Chen, L.H.: Directions of arrival estimation for multicomponent frequency-hopping signals based on spatial time-frequency analysis. J. Syst. Eng. Electr. **33**(12), 2587–2592 (2011)
9. Zhang, D.W., Guo, Y., Qi, Z.S., et al.: Joint estimation algorithm of direction of arrival and polarization for multiple frequency-hopping signals. J. Electr. Inf. Technol. **37**(7), 1695–1701 (2015). (in Chinese)
10. Zhang, D., Guo, Y., Qi, Z., et al.: A joint estimation algorithm of multiple parameters for frequency hopping signals using spatial polarimetric time frequency distributions. J. Xi'an Jiaotong Univ. **49**(8), 17–23 (2015). (in Chinese)
11. Zhang, D., Guo, Y., Qi, Z., et al.: A joint estimation of 2D-DOA and polarization estimation for multiple frequency hopping signals. J. Xi'an Jiaotong Univ. **49**(8), 17–23 (2015)
12. Yan, F.G., Liu, S., Jin, M., et al.: Fast DOA estimation based on MUSIC symmetrical compressed spectrum. J. Syst. Eng. Electr. **34**(11), 2198–2202 (2012)