Dynamic Characteristic Analysis for Complexity of Continuous Chaotic Systems Based on the Algorithms of SE Complexity and C_0 Complexity

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Abstract. In this paper, SE algorithm and C_0 algorithm were described in detail. The complexity characteristics of Lü chaotic system, Chua chaotic memristive system, Bao hyperchaotic system, Chen hyperchaotic system are analyzed based on SE algorithm and C_0 algorithm. We have compared with the dynamical characteristics of four systems by using the conventional dynamic analysis methods and the methods of complexity, the comparative results demonstrate that SE complexity and C_0 complexity can reflect the complexity of continuous chaotic systems accurately and effectually. Through the contrast for the complexity characteristics of two continuous chaotic systems and two continuous hyperchaotic systems, we can obtain that the varying trend of SE complexity and C_0 complexity have much well coherence, and it provides a dynamical analytical method for the research of chaos theory.

Keywords: SE complexity · C_0 complexity · Dynamic analysis
Chaotic system · Hyperchaotic system

1 Introduction

Chaos theory is a nonlinear dynamical science which has been thriving over the past decades. The application of chaos theory is more and more widely, especially in the information security areas [1–3]. The complexity is an ability of the chaotic system can generates random sequences, the value of complexity depend on the random degree with the sequences. Thus, the scientific community has been paying more and more attention to the algorithm of complexity in recent years.

The algorithms of complexity are generally divided into the algorithms based on behavioral complexity (FuzzyEn algorithm [4–6] and SCM algorithm [7, 8]) and the algorithms based on structural complexity (SE algorithm and C_0 algorithm). The larger the complexity of time series, the greater randomness, the more difficult the sequences are restored to the original sequences.

During mid-20th century, Kolmogorov et al. have expounded the concept of complexity, and at the same time they have put forward the Kolmogorov algorithm of complexity [9]. However, it is only a rough research. At that time, because of the

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limitations with science and technology, it can also not verify the correctness by using the computer. Until 1976, Lempel and Ziv presented the Limpel-Ziv algorithm [10] in their papers, and this algorithm is also the sublimation of the Kolmogorov algorithm. It is widely used in the fields of bio-medicine [11], weather forecasting [12] and cryptography [13]. In 1991, Pincus introduced the algorithm of approximate entropy (ApEn) [14]. Then in 2002, Bandt and Pompe developed the algorithm of permutation entropy (PE algorithm) [15], it is also the improvement of ApEn algorithm. Although these algorithms all can describe the complexity of continuous chaotic systems, but the Limpel-Ziv algorithm only estimates the time scale of chaotic sequences simply, and it needs coarse graining treatment for the non-pseudo-random sequences. When the ApEn algorithm is used to deal with the variations of different embedding dimensions, however, the problems of embedding dimensions and the resolution parameters will be involved during the process of calculation, and the calculated results are also affected by the subjective factors. At the same time, the calculated results of PE algorithm may be influenced by many factors too. These algorithms above are fast to the calculation of short sequences. While the length of the data increases to a certain amount, its calculated speed would slow down, and the practicality would be lower. Compared to the three algorithms, SE algorithm and C_0 algorithm are used to calculate the value of entropy based on Fourier transform (FFT). It not only has faster speed but also better reflect the structures of related sequence, and it can also measure the complexity of systems more effectively. Especially in the calculation of continuous stationary time series, the advantages of SE algorithm and C_0 algorithm are more obvious.

In this paper, we have analyzed the complexity characteristics of Lü chaotic system [16], Chua chaotic memristive system [17], Bao hyperchaotic system [18] and Chen hyperchaotic system [19] by using SE algorithm and C_0 algorithm. Then, theirs correctness were verified too. Through the dynamic contrastive analysis for two continuous chaotic systems and two continuous hyperchaotic systems, it shows that the superiority of SE complexity algorithm and C_0 complexity algorithm for calculating the continuous chaotic sequences. Meanwhile, we can also see that the chaotic systems have very rich dynamic characteristics. Finally, we compared and analyzed the maximum value and the average value of the four systems. The results shows that when we do the research of chaotic systems, the continuous chaotic systems and the continuous hyperchaotic systems are equivalent, there is no better or worse. All these above provided the theoretical source and the experimental basis for the application of chaotic theory.

2 SE Complexity Algorithm and C_0 Complexity Algorithm

2.1 SE Complexity Algorithm

At present, there are several algorithms for measuring the complexity of chaotic sequences. Among them, the SE [20–22] and C_0 [23–25] complexity algorithms have less parameters, faster calculation speed and higher accuracy. Spectral Entropy algorithm gets the corresponding Shannon entropy value based on the Fourier transform, the algorithm is described as follows:

(1) Remove the direct-current: using Eq. (1) to remove the DC part of pseudo-random sequence, which so that the spectrum can reflect the energy information of signal more accurately.

$$x(n) = x(n) - \bar{x} \tag{1}$$

where, $\bar{x} = (1/N)\sum_{n=0}^{N-1} x(n)$

(2) Do the discrete Fourier transform for Eq. (1)

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{N-1} x(n)W_N^{nk} \tag{2}$$

in which, k = 0, 1, 2 …, N − 1

(3) Calculate the relative power spectrum: calculate the front half of X(k), then we obtain the value of power spectrum in a certain frequency by using the Parseval theorem.

$$p(k) = \frac{1}{N}|\mathrm{X(k)}|^2 \tag{3}$$

where, k = 0, 1, 2 …, N/2 − 1, and the total power of sequence can be defined as:

$$p_{tot} = \frac{1}{N}\sum_{k=0}^{N/2-1} |X(k)|^2 \tag{4}$$

So, the probability of relative power spectrum can be expressed as:

$$P_k = \frac{p(k)}{p_{tot}} = \frac{\frac{1}{N}|X(k)|^2}{\frac{1}{N}\sum\limits_{k=0}^{N/2-1} |X(k)|^2} = \frac{|X(k)|^2}{\sum\limits_{k=0}^{N/2-1} |X(k)|^2} \tag{5}$$

where, $\sum\limits_{k=0}^{N/2-1} P_k = 1$

(4) Using Eqs. (3), (4) and (5), and the Shannon entropy, we can obtain the Spectral Entropy (SE) of signal:

$$se = -\sum_{k=0}^{N/2-1} P_k \ln P_k \tag{6}$$

If $P_k = 0$ in Eq. (6), we will define $P_k \ln P_k = 0$. And, the value of spectrum entropy converges to ln(N/2). In order to comparison and analysis, the spectral entropy can be normalized. Then, we obtain the normalized spectral entropy:

$$SE(N) = \frac{se}{\ln(N/2)} \tag{7}$$

Through the formulas above, we can obtain that the more unevenly the power spectrum distribution of sequence, the more simple the structure of sequence, the smaller the corresponding measured value.

2.2 C_0 Complexity Algorithm

The main idea of C_0 complexity algorithm is to divide the sequence into the regular part and the irregular part, the proportion of irregular part is what we need. The computational steps as follows:

(1) Do the discrete fourier transform for the time series

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{2\pi}{N}nk} = \sum_{n=0}^{N-1} x(n)W_N^{nk} \tag{8}$$

where, k = 0, 1, …, N − 1.

(2) Remove the regular part of Eq. (8), get the mean square value of $X(k)$:

$$G_N = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2 \tag{9}$$

The parameter r is added into Eq. (9), then retains the part which more than the r multiples of the mean square value, meanwhile set the remaining parts are zero, that is:

$$\tilde{X}(k) = \begin{cases} X(k), & |X(k)|^2 > rG_N \\ 0, & |X(k)|^2 < rG_N \end{cases} \tag{10}$$

(3) Do the Fourier inverse transform for Eq. (10)

$$\tilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}(k)e^{j\frac{2\pi}{N}nk} = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}(k)W_N^{-nk} \tag{11}$$

Where, n = 0, 1, …, N − 1

(4) With Eq. (11), the measure of C_0 complexity is defined as:

$$C_0(r, N) = \sum_{n=0}^{N-1} |x(n) - \tilde{x}(n)| \tag{12}$$

The C_0 complexity algorithm is calculated based on the fast Fourier transform algorithm, which deleted the regular part of the sequence, and retained the irregular part. The larger proportion of irregular part the sequence has, the higher value of complexity.

3 Dynamical Analysis of Continuous Chaotic Systems

3.1 Dynamic Analysis of Lü Chaotic System

Lü chaotic system is described as follows:

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cy - xz \\ \dot{z} = xy - bz \end{cases} \quad (13)$$

The mathematical model of Lü chaotic system is the simplest structure in the groups of Lorenz system. Setting the initial value is (1, 1, 1), and the time step is 0.001 s. We calculate the complexity characteristics by using SE algorithm and C_0 algorithm. When the parameters $a = 36$, $b = 3$, $c = 20$, the steady-state values of Lyapunov exponents are $LE_1 = 1.3657$, $LE_2 = 0$, $LE_3 = -20.3620$. In this case, we can calculate the corresponding Lyapunov dimension is 2.0671. The phase diagrams of chaotic attractor in Lü system as shown in Fig. 1. With the parameter b changing, the system is in chaotic state, periodic state, stable point state and so on. It shows that the algorithms of SE complexity and C_0 complexity can reflect the complexity of continuous chaotic systems accurately and effectually (Fig. 2).

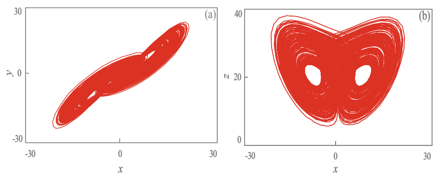

Fig. 1. Chaotic attractor of Lü system: (a) x-y plane (b) x-z plane

3.2 Dynamic Analysis of Chua Memristive Chaotic System

The Chua chaotic circuit is a classical circuit system, it is also a very hot circuit model of the scientific community in recent years. The Chua chaotic oscillation circuit is realized by using the parallel connection of a flue-controlled memristor and a negative conductance.

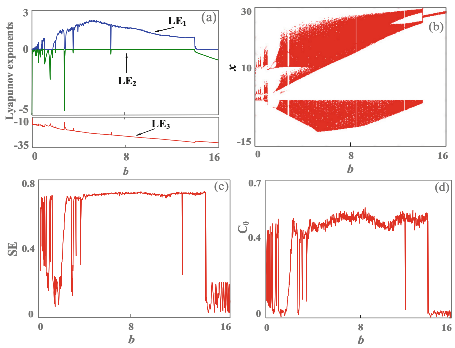

Fig. 2. Dynamic characteristics of Lü system: (a) Lyapunov exponents spectrum (b) bifurcation diagram (c) SE complexity (d) C_0 complexity

The equations of Chua circuit system is:

$$\begin{cases} \dot{x} = a[y - x + dx - W(w)x] \\ \dot{y} = x - y + z \\ \dot{z} = -by - cz \\ \dot{w} = x \end{cases} \tag{14}$$

In which,

$$q(w) = gw + hw^3,\ W(w) = dq(w)/dw = g + 3hw^2 \tag{15}$$

Setting the initial value of Eq. (14) is (0.1, − 0.1, 0.1, 0.1), the time step is 0.001 s. The complexity of x sequence is calculated by using SE algorithm and C_0 algorithm. When a = 10, b = 100/7, c = 0.1, d = 9/7, g = 1/7, h = 2/7, the steady-state values of Lyapunov exponents spectrum are LE_1 = 0.2935, LE_2 = 0, LE_3 = 0, LE_4 = −3.3719. Thus, the corresponding Lyapunov dimension is 3.0893. The Lyapunov exponents spectrum in Chua chaotic system is shown as Fig. 5(a). The path of the system into the chaotic state can be clearly seen from Fig. 5(b). All of these above show that the algorithms of SE complexity and C_0 complexity are right and effective dynamic analysis methods (Figs. 3 and 4).

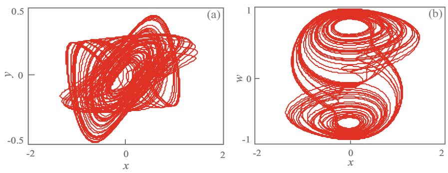

Fig. 3. Chaotic attractor of Chua memristive system: (a) x-y plane (b) x-w plane

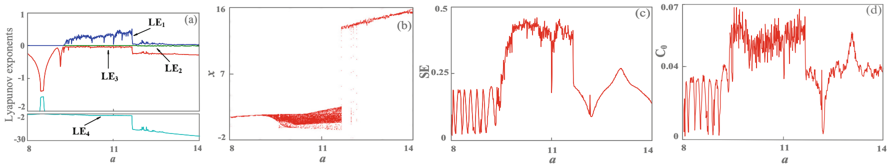

Fig. 4. Dynamic characteristics of Chua memristive system: (a) Lyapunov exponents spectrum (b) bifurcation diagram (c) SE complexity (d) C_0 complexity

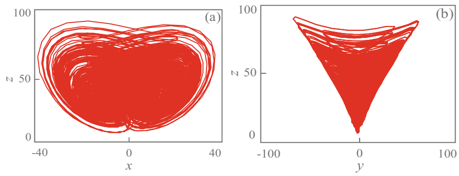

Fig. 5. Chaotic attractor of Bao hyperchaotic system: (a) *x*-*z* plane (b) *y*-*z* plane

3.3 Dynamic Analysis of Bao Hyperchaotic System

To generate a hyperchaotic signal from an autonomous dissipative system, the state equation of the system must satisfy the following two basic conditions: (1) The dimension of the state equation must be at least four, and the order of the state equation must be at least two. (2) The system has at least two positive Lyapunov exponents, and the sum of all the exponents is less than 0. So, the method of obtaining the hyperchaotic system is that adding a state feedback controller to the 3D continuous chaotic system. The 4D Bao hyperchaotic circuit by adding a state controller to the 3D Bao chaotic circuit.

According to the voltage-current characteristic relation of the circuit and the mathematically treated, we can get the mathematical model of Bao hyperchaotic system is:

$$\begin{cases} \dot{x} = a(x - y) \\ \dot{y} = xz - cy + w \\ \dot{z} = x^2 - bz \\ \dot{w} = d(x + y) \end{cases} \tag{16}$$

Setting the initial value of Eq. (16) is (10, 10, 10, 10), the time step is 0.01 s. When a = 20, b = 4, c = 32, d = 4, the steady-state values of Lyapunov exponents spectrum are $LE_1 = 2.0722$, $LE_2 = 0.0750$, $LE_3 = 0$, $LE_4 = -26.3259$, and the corresponding Lyapunov dimension is 3.0816. There are two positive Lyapunov exponents, so the system is a hyperchaotic system. With the parameter d varying, the system goes into the chaotic state from the hyperchaotic state. We can observed from the Fig. 6 that SE complexity and C_0 complexity also can precisely reflect the dynamical characteristics as the same to the conventional dynamic analysis methods.

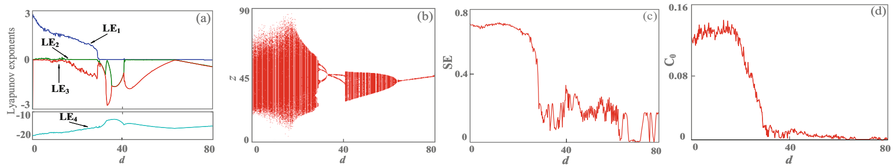

Fig. 6. Dynamic characteristics of Bao hyperchaotic system: (a) Lyapunov exponents spectrum (b) bifurcation diagram (c) SE complexity (d) C_0 complexity

3.4 Dynamic Analysis of Chen Hyperchaotic System

Chen system belongs to the groups of Lorenz system too. A feedback term is added to the equations of the classical 3D Chen system, and then we can get a 4D hyperchaotic Chen system. Chen hyperchaotic system can be described as follows:

$$\begin{cases} \dot{x} = a(y - x) + w \\ \dot{y} = dx - xz + cy \\ \dot{z} = xy - bz \\ \dot{w} = yz + ew \end{cases} \tag{17}$$

Let the initial value of Eq. (17) is (1, 0, 1, 0), the time step is 0.01 s. The complexity of x sequence is calculated by using SE algorithm and C_0 algorithm. When a = 35, b = 3, c = 12, d = 7, e = 0.083, the steady-state values of Lyapunov exponents spectrum are $LE_1 = 0.3745$, $LE_2 = 0.0405$, $LE_3 = 0$, $LE_4 = -26.3259$. We can calculate the corresponding Lyapunov dimension is 3.0158. The system has two positive Lyapunov exponents under the certain conditions, so it is a hyperchaotic system. When the parameter c varying, the state of system changing within periodic state, chaotic state and hyperchaotic state. The results of dynamic analysis in Fig. 10 are correspondence, it also shows that SE algorithm and C_0 algorithm are very correct and necessary for the dynamical analysis of the chaotic system (Figs. 7 and 8).

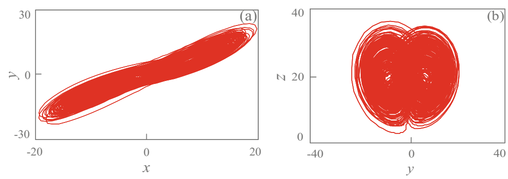

Fig. 7. Chaotic attractor of Chen hyperchaotic system: (a) x-y plane (b) y-z plane

3.5 Analysis of Complexity Characteristics for Continuous Chaotic Systems

Using the SE complexity algorithm and C_0 complexity algorithm, the complexity of four different continuous chaotic systems are compared and analyzed. The maximum value of complexity and the average value of complexity as shown in Table 1. We can

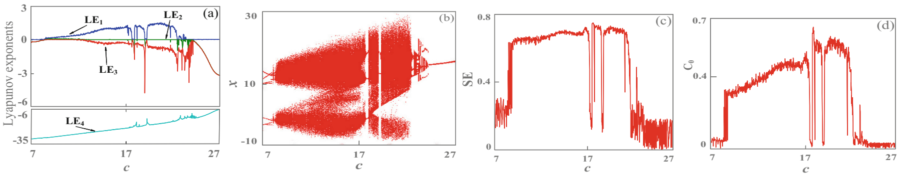

Fig. 8. Dynamic characteristics of Chen hyperchaotic system: (a) Lyapunov exponents spectrum (b) bifurcation diagram (c) SE complexity (d) C_0 complexity

Table 1. Analysis of complexity characteristics for continuous chaotic systems

	SE_{max}	$\overline{SE}$	C_{0max}	$\overline{C_0}$
Lü chaotic system	0.7318	0.5834	0.5598	0.3756
Chua chaotic system	0.4593	0.2422	0.0677	0.0382
Bao hyperchaotic system	0.7195	0.3432	0.1415	0.0426
Chen hyperchaotic system	0.7428	0.4948	0.6220	0.2838

obtain that the maximum value of complexity generated in Chen hyperchaotic system, the maximum average value of complexity generated in Lü chaotic system. And the minimum value of complexity generated in Chua memristive chaotic system, the minimum average value of complexity generated in Chua memristive chaotic system too. Through the comparative analysis to the Lü chaotic system and the Bao hyperchaotic system, we can see that the value of complexity in hyperchaotic system is not necessarily greater than that in chaotic system. Thus, the relationship of size of complexity in the different systems cannot be determined. The Table 2 shows the complexity of continuous chaotic systems in the chaotic state and the hyperchaotic state. Through the comparative analysis for the complexity of Chen hyperchaotic system in the chaotic state and in the hyperchaotic state, we can know that for a same system, the value of complexity in the hyperchaotic state is not necessarily greater than that in chaotic state. All the results show that the size of complexity for the different states in different chaotic systems can also not be determined. But, the varying tendencies of SE complexity and C0 complexity are basically consistent.

Table 2. Analysis of complexity characteristics in the chaotic state and the hyperchaotic state

System		SE_{max}	$\overline{SE}$	C_{0max}	$\overline{C_0}$
Lü chaotic system	Chaotic state	0.7318	0.6662	0.5598	0.4436
Chua chaotic system	Chaotic state	0.4593	0.4027	0.0677	0.0449
Bao hyperchaotic system	Chaotic state	0.6877	0.6160	0.1209	0.0740
	Hyperchaotic state	0.7195	0.7026	0.1415	0.1260
Chen hyperchaotic system	Chaotic state	0.7428	0.6839	0.6220	0.4714
	Hyperchaotic state	0.6623	0.6313	0.3810	0.3041

4 Conclusion

In this paper, we have done the analysis of complexity characteristics by using SE algorithm and C_0 algorithm for two chaotic systems and two hyperchaotic systems. From the results of comparison and analysis, we can obtain that the following conclusions: (1) SE complexity and C_0 complexity can reflect the complexity of continuous chaotic systems accurately and effectually. (2) For the different systems, the value of complexity in hyperchaotic system is not necessarily greater than that in chaotic system. For the same system, the value of complexity in hyperchaotic state is not necessarily greater than that in chaotic state. Generally, the maximum value of complexity generated in the chaotic state or the hyperchaotic state. The complexity characteristic of chaotic system is its inherent property, which is decided by the variation of parameters and the selection of initial value. Therefore, when we choose the chaotic system to study, the chaotic state and the hyperchaotic state all can be used as the experimental subjects, there is no better or worse. That is to say, when we choose the research object, the chaotic system and the hyperchaotic system are equivalent. (3) The varying tendency of SE complexity and C_0 complexity are basically consistent. Because they all reflect the complexity of sequence based on Fourier transform. However, the difference which between the specific value of two algorithms is larger, this is determined by the algorithm itself. (4) The value of complexity varying within a certain range, that is the complexity of continuous chaotic system has boundedness. This is one of the inherent characteristics with chaotic system. The research based on complexity characteristics of SE algorithm and C_0 algorithm, which provide a relevant theoretical basis and an experimental guidance for the applications of cryptography, secure communication and information security.

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