Data Association Based Passive Localization in Complex Multipath Scenario

Bing Zhao(✉) and Ganlin Hao

School of Information and Electronics, Beijing Institute of Technology, Beijing, China
{zhaobing,hglhust}@bit.edu.cn

Abstract. Complex scenarios are characterized by harsh multipath conditions. Recently, strong single reflections among multipath components (MPC) are proved to improve localization performance such as data-association (DA) and multipath components mitigation. We first propose a novel DA method, which figures out the relationship between the received signals and scatters based on an expectation maximization (EM) based Gaussian mixture model. Furthermore, sensors themselves often have uncertainties to be estimated, we propose a joint estimation method to obtain the final estimate. Simulation results show the effectiveness of the algorithm by considering sensors' uncertainties after demapping. As a result, the proposed algorithm can fit applications of large-scale wireless sensor networks (WSNs) in practice.

Keywords: Passive localization · Multipath components
Data association

1 Introduction

Wireless sensor networks (WSNs) [1] holds enough number of battery-powered sensors to transmit wireless signals and communicate with their neighbors. Sensors cooperatively estimate the state of one object by limited communication, ranging, and processing abilities. The idea of localization in WSNs has driven a myriad of applications like tracking, monitoring and control appliances [2].

In general, existing algorithms such as cooperative localization [3] and simultaneous localization and mapping (SLAM) [4] can work well in the desired line-of-sight (LOS) scenarios. However, in commercial shopping area, indoor, urban canyon or jungle scenario with scatters, these algorithms will experience severe performance declines, as each sensor may receive the same signals traveled from different paths in a time slot, i.e., multipath components (MPCs).

In [5], an iterative process is presented. Authors adopt time-of-arrival (TOA) measurements to estimate the ranging probability density function pdf. However, the static and i.i.d. assumptions of ranging pdf constrain its usage in practical scenarios. A TOA technique to utilize single reflections is presented in [6]. This research improves the performance but demands the whole map of layout and previous estimate to data-association (DA).

X. Gu et al. (Eds.): MLICOM 2017, Part II, LNICST 227, pp. 585–594, 2018.
https://doi.org/10.1007/978-3-319-73447-7_63

In this paper, we propose a expectation maximization (EM) method in Gaussian mixture model to realize DA without the information of entire layout. Here we focus on an expectation maximization (EM) process in Gaussian mixture model. Gaussian mixture model is typically used in WSNs localization like [7].

This paper is organized as follows. Section 2 introduces the signal model. Section 3 involves EM algorithm with Gaussian Mixture model. Section 4 elaborates the proposed algorithm to estimate object's location, followed by a comprehensive simulations in Sect. 5. Finally, concluding remarks are made in Sect. 6.

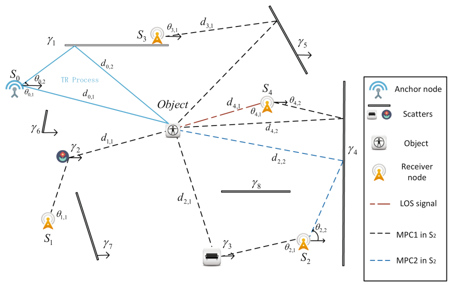

Fig. 1. A network with one anchor node $\boldsymbol{S}_0$, N(= 4) sensor nodes $\boldsymbol{S}_i$ and L (= 8) scatters with known number and tilt angles γ_k. $\boldsymbol{S}_0$ sends a TR signal. Then each node i may receive several measurements $(d_{i,j})$.

2 System Model

We set a two-dimensional localization problem in Cartesian coordinate and this work focuses on real multipath scenarios like Fig. 1.

Generally, sensor S_0 is chosen as the reference sensor. Ranging measurement $d_{0,1}$ with the most accurate pseudo-range measurement is chosen to be a reference, which is calculated by $d = \hat{\tau}_{1,\text{TX}} \times c$. Then the i-th sensor obtains its j-th TDOA measurement $\Delta\tilde{d}_{i,j}$ with zero mean Gauss white noise as

$$\Delta\tilde{d}_{i,j} = d_{i,j} - d_{0,1} = \mathbf{g}(\hat{\theta}_{i,j}, \gamma_k)^T(\mathbf{q} - \bar{\mathbf{p}}_i) - \mathbf{g}(\hat{\theta}_{0,1}, \gamma_{S_0})^T(\mathbf{q} - \mathbf{p}_0) + \tilde{n}_{i,j} \quad (1)$$

where object's ground-truth position $\mathbf{q} \triangleq [x_q \ y_q]^T$, the i-th sensor's original position $\bar{\mathbf{p}}_i \triangleq [\bar{x}_i \ \bar{y}_i]^T$, where $i = 1, \cdots, N$. In practice, the sensors may change around their original positions. So we assume position's uncertainty $\Delta\mathbf{p}_i$ with Gaussian distribution, which will discuss later. k denotes the index of scatter associated to the measurement $d_{i,j}$, and γ_k is the known orientation of the k-th scatter. We further denote

$$\mathbf{g}(\hat{\theta}_{i,j}, \gamma_k) = \frac{1}{\cos(\hat{\theta}_{i,j} - \gamma_k)}[\cos\gamma_k, \sin\gamma_k]. \quad (2)$$

which is decided by geometric Topology. $\hat{\theta}_{i,j}$ is the AOA measurement of the j-th signal path at sensor i. With measurement noise, the estimated AOA measurement is used to replace $\theta_{i,j}$ as $\hat{\theta}_{i,j} = \theta_{i,j} + \eta_{i,j}$, where $\eta_{i,j}$ is noise with uniform distribution, i.e., $\mathcal{U}[-\eta^0, \eta^0]$.

3 Data Association Algorithm

In [5], the single reflection MPCs can be distinguished from the received waveform. Let $\Delta\tilde{\mathbf{d}}_i$ be the ranging block containing M_i measurements obtained in sensor i. Generally, the ranging measurements comes from the scatters and LOS components, but the sensor i doesn't know the probabilities which measurement stemming from which scatter or object directly from LOS path. Assuming every range estimate has a certain weight $\alpha_{i,j,k}$, we obtain the Gaussian mixture model as

$$p(\Delta\tilde{d}_{i,j}, \hat{\theta}_{i,j}|\mathbf{q}, \Delta\mathbf{p}_i, \theta_{i,j}) = p(\hat{\theta}_{i,j}|\theta_{i,j}) \sum_{k=1}^{\mathrm{K_j}} \alpha_{i,j,k} \Phi(\Delta\tilde{d}_{i,j}|\mathbf{q}, \Delta\mathbf{p}_i, \theta_{i,j}) \quad (3)$$

$$\Phi(\Delta\tilde{d}_{i,j}|\mathbf{q}, \Delta\mathbf{p}_i, \theta_{i,j}) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(\Delta\tilde{d}_{i,j} - \mu_k)^2}{2\sigma^2}) \quad (4)$$

where $\alpha_{i,j,k} \geq 0, \sum_{k=1}^{\mathrm{K_j}} \alpha_{i,j,k} = 1$. $\mathbf{q} = (x_q, y_q)$.

As the first received signals in each sensor has the probability that coming from LOS path instead of the single reflection (NLOS) path from the scatter $k(k \in \mathrm{L})$. L is the scatters' number. So sensor i's first signal has $\mathrm{L}+1$ submodels in Gaussian mixture model as $\mathrm{K_j} = \mathrm{L} + 1 (j = 1)$ or $\mathrm{K_j} = \mathrm{L}(j! = 1)$, $(\mathrm{L}+1)$th submodel means the LOS estimate.

The key to obtain the mapping information lies on the latent variable $\rho_{i,j,k}$, which means one measurement coming from one certain submodel k.

$$\rho_{i,j,k} = \begin{cases} 1 & \text{the measurement j coming from the model k} \\ 0 & \text{else} \end{cases}$$

where $\rho_{i,j,k} \in \{0, 1\}$.

Having range estimate $\Delta\tilde{d}_{i,j}$ and latent variables $\rho_{i,j,k}$, we obtain the complete data like $(\Delta\tilde{d}_{i,j}, \rho_{i,j,1}, \cdots, \rho_{i,j,K_j})$. From the model assumptions, $\tilde{\theta}_i$ is independent of other variables in $(\Delta\tilde{d}_{i,j}, \rho_{i,j,1}, \cdots, \rho_{i,j,K_j})$. Besides, scatter's horizontal angle and TDOA ranging measurements among sensors are also independent.

Here we express data's log likelihood function in the following align

$$\begin{aligned} &\ln\ p(\Delta\hat{\mathbf{d}}, \hat{\boldsymbol{\theta}}, \boldsymbol{\rho}|\mathbf{x}) = \ln\ p(\{\{\{\Delta\hat{d}_{i,j}, \hat{\theta}_{i,j}, \rho_{i,j,k},\}_{k=1}^{\mathrm{K_j}}\}_{j=1}^{M_i}\}_{i=1}^{\mathrm{N}}|\mathbf{x}) \\ &= \sum_{i=1}^{\mathrm{N}} \sum_{j=1}^{M_i} \ln p(\hat{\theta}_{i,j}|\theta_{i,j}) + \ln p(\Delta\hat{\mathbf{d}}, \boldsymbol{\rho}|\mathbf{q}, \Delta\mathbf{p}, \boldsymbol{\theta}) \end{aligned} \quad (5)$$

where inaccurate sensors' positions $\bar{\mathbf{p}}$ are used to solve the mapping issue in subsection C, i.e. $\Delta\mathbf{p}$'s influence is negligible first.

For item $p(\Delta\hat{\mathbf{d}}, \boldsymbol{\rho}|\mathbf{q}, \Delta\mathbf{p}, \boldsymbol{\theta})$ in (5), we have a further mathematical expansion

$$p(\Delta\hat{\mathbf{d}}, \boldsymbol{\rho}|\mathbf{q}, \Delta\mathbf{p}, \boldsymbol{\theta}) = \prod_{i=1}^{N}\prod_{j=1}^{M_i} p(\Delta\hat{d}_{i,j}, \rho_{i,1}, \rho_{i,2}, \cdots, \rho_{i,M_i}|\mathbf{q}, \Delta\mathbf{p}, \boldsymbol{\theta})$$
$$= \prod_{i=1}^{N}\prod_{j=1}^{M_i}\prod_{k=1}^{K_j} [\alpha_{i,j,k}\Phi(\Delta\hat{d}_{i,j}|\mathbf{q}, \Delta\mathbf{p}_i, \theta_{i,j})]^{\rho_{i,j,k}} \tag{6}$$

Based on the TDOA and AOA method, each submodel is shown as

$$\Phi(\Delta\hat{d}_{i,j}|\mathbf{q}, \Delta\mathbf{p}_i, \theta_{i,j}) = \frac{1}{\sqrt{2\pi}\sigma_i}\exp(-\frac{1}{2\sigma_i^2}(\Delta\hat{d}_{i,j} + \mathbf{g}(\hat{\theta}_{i,j}, \gamma_k)^T\bar{\mathbf{p}}_i - \mathbf{g}(\theta_0, \gamma_k)^T\mathbf{p}_1$$
$$- (\mathbf{g}(\hat{\theta}_{i,j}, \gamma_k) - \mathbf{g}(\theta_0, \gamma_k))^T\mathbf{q})^2$$

Then the item in (6)'s log-likelihood function is

$$\ln p(\Delta\hat{\mathbf{d}}, \boldsymbol{\rho}|\mathbf{q}, \Delta\mathbf{p}, \boldsymbol{\theta}) = \sum_{i=1}^{N}\sum_{j=1}^{M_i}\sum_{k=1}^{K_j} \tag{7}$$
$$\rho_{i,j,k}\left[\ln\alpha_{i,j,k} + \ln(\frac{1}{\sqrt{2\pi}}) - \ln\sigma_i - \frac{1}{2\sigma_i^2}(\Delta\hat{d}_{i,j} - \mu_{i,j})^2\right]$$

where $\mu_{i,j} = \mathbf{g}(\hat{\theta}_{i,j}, \gamma_k)^T(\mathbf{q} - \mathbf{p}_i) - \mathbf{g}(\theta_0, \gamma_k)^T(\mathbf{q} - \mathbf{p}_0)$. We define n_k as the number of submodel k among all the measurements in sensors. So $n_k = \sum_{i=1}^{\mathrm{N}}\sum_{j=1}^{M_i}\rho_{i,j,k}, \sum_{k=1}^{K_j} n_k = \boldsymbol{N}$. So (7) can be reformulated as

3.1 E Step of the EM Algorithm

In order to obtain Q function in lth iteration, we have

$$\mathcal{Q}(\mathbf{x}, \mathbf{x}^l) = \mathrm{E}[\ln p(\Delta\hat{\mathbf{d}}, \hat{\boldsymbol{\theta}}, \boldsymbol{\rho}|\mathbf{x})|\boldsymbol{\rho}, \mathbf{q}^{(l)}, \Delta\mathbf{p}, \boldsymbol{\theta}] \tag{8}$$
$$=\mathrm{E}\Big\{\sum_{i=1}^{\mathrm{N}}\sum_{j=1}^{M_i}\ln p(\hat{\theta}_{i,j}|\theta_{i,j}) + \sum_{i=1}^{\mathrm{N}}\sum_{j=1}^{M_i}\sum_{k=1}^{K_j}\rho_{i,j,k}$$
$$\left[\ln\alpha_{i,j,k} + \ln(\frac{1}{\sqrt{2\pi}}) - \ln\sigma_i - \frac{1}{2\sigma_i^2}(\Delta\hat{d}_{i,j} - \mu_k)^2\right]$$

We define $\mathrm{E}(\rho_{i,j,k})$ as $\hat{\rho}_{i,j,k}$.

$$\hat{\rho}_{i,j,k}^{l+1} = \mathrm{E}(\rho_{i,j,k}) == \frac{\hat{\rho}_{i,j,k}^{l}\Phi(\Delta\hat{d}_{i,j}|\mathbf{q}^{l+1}, \Delta\mathbf{p}_i, \theta_{i,j})}{\sum_{k=1}^{K_j}\hat{\rho}_{i,j,k}^{l}\Phi(\Delta\hat{d}_{i,j}|\mathbf{q}^{l+1}, \Delta\mathbf{p}_i, \theta_{i,j})} \tag{9}$$

where $\hat{\rho}_{i,j,k}^{l+1}$ names the possible weight of model k to the observed data $\Delta\hat{d}_{i,j}$.

Using $\hat{\rho}_{i,j,k} = \mathrm{E}(\rho_{i,j,k})$

$$\mathcal{Q}(\mathbf{x}, \mathbf{x}^l) = \sum_{i=1}^{\mathrm{N}} \sum_{j=1}^{M_i} \ln p(\hat{\theta}_{i,j}|\theta_{i,j}) \sum_{i=1}^{\mathrm{N}} \sum_{j=1}^{M_i} \sum_{k=1}^{K_j} \hat{\rho}_{i,j,k} + \left[\ln \alpha_{i,j,k} + \ln(\frac{1}{\sqrt{2\pi}}) - \ln \sigma_i - \frac{1}{2\sigma_i^2} (\Delta \hat{d}_{i,j} - \mu_{i,j})^2 \right] \tag{10}$$

3.2 M Step of the EM Algorithm

After the E step, iterative M step for maximum $\mathcal{Q}$ is

$$\mathbf{q}^{l+1} = \arg\max_{\mathbf{q}} \mathcal{Q}(\mathbf{q}, \mathbf{q}^l)$$

After some manipulations, we can obtain

$$\mathbf{q}_x^{l+1} = \frac{\sum_{i=1}^{\mathrm{N}} \sum_{j=1}^{M_i} \sum_{k=1}^{K_j} \hat{\rho}_{i,j,k}^l (\Delta \hat{d}_{i,j} A_{i,j} - A_{i,j} B_{i,j} \mathbf{q}_y^{(l)} + A_{i,j} C_{i,j}) / \sigma_i^2}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} \sum_{k=1}^{K_j} \hat{\rho}_{i,j,k} A_{i,j}^2 / \sigma_i^2} \tag{11}$$

$$\mathbf{q}_y^{l+1} = \frac{\sum_{i=1}^{N} \sum_{j=1}^{M_i} \sum_{k=1}^{K_j} \hat{\rho}_{i,j,k}^l (\Delta \hat{d}_{i,j} B_{i,j} - A_{i,j} B_{i,j} \mathbf{q}_x^{(l)} + B_{i,j} C_{i,j}) / \sigma_i^2}{\sum_{i=1}^{N} \sum_{j=1}^{M_i} \sum_{k=1}^{K_j} \hat{\rho}_{i,j,k} B_{i,j}^2 / \sigma_i^2} \tag{12}$$

where $A_{i,j} = a_{i,j} - a_0, B_{i,j} = b_{i,j} - b_0, C_{i,j} = a_{i,j}\bar{x}_i + b_{i,j}\bar{y}_i - a_0 x_0 - b_0 y_0$, which $a_{i,j} = \frac{\cos\gamma_k}{\cos(\theta_{i,j} - \gamma_k)}$ $a_0 = \frac{\sin\gamma_k}{\cos(\theta_0 - \gamma_k)}$ $b_{i,j} = \frac{\sin\gamma_k}{\cos(\theta_{i,j} - \gamma_k)}$ $b_0 = \frac{\sin\gamma_k}{\cos(\theta_0 - \gamma_k)}$. This is a coarse position estimation without considering the AOAs' measurement errors and sensors' uncertainties, so we use it as the initial guess in the following section.

Furthermore, $\hat{\alpha}_{i,j,k}$ is obtained by $\hat{\mathbf{q}}$ and Laplace method under the constrain of $\sum_{k=1}^{\mathrm{K_j}} \hat{\alpha}_{i,j,k} = 1$.

$$\alpha_{i,j,k}^{l+1} = \arg\max_{\alpha_{\mathrm{i,j,k}}} \mathcal{Q}(\alpha_{\mathrm{i,j,k}}, {\alpha_{\mathrm{i,j,k}}}^{(l)}) = \hat{\rho}_{i,j,k}^{l+1} \tag{13}$$

where $k = 1, 2, \cdots, K_j$. Repeat this EM process $\mathrm{N_{iter}}$ times until log likelihood value are no longer changes obviously. The influence of $\boldsymbol{\Delta p}_i$ to mapping is discussed in simulations.

3.3 Demapping

After we obtain the coarse position of object, we use the updated Gaussian mixture model to realize the parameter evaluation, which means demapping. After calculation, if $\alpha_{i,j,k}$'s value is the biggest and exceed the empirical threshold in measurement $\Delta \hat{d}_{i,j}$, we choose the corresponding submodel to describe the likelihood distribution

$$p(\Delta \hat{d}_{i,j} | \mathbf{q}, \Delta \mathbf{p}_i, \theta_{i,j}) = p(\hat{\theta}_{i,j}|\theta_{i,j}) \sum_{k=1}^{K_j} [\alpha_{i,j,k} \Phi(\Delta \tilde{d}_{i,j} | \mathbf{q}, \Delta \mathbf{p}_i, \theta_{i,j})]^{\rho_{i,j,k}} \tag{14}$$

where $\rho_{i,j,k} = 1$ if and only if $\alpha_{i,j,k}$'s value is the biggest and exceed the threshold, otherwise, $\rho_{i,j,k} = 0$.

If $\rho_{i,j,k} = 1$ and $k \leq L$, the measurement is NLOS signal, else if the measurement is assumed LOS($\rho_{i,j,k} = 1$ and $k = \mathrm{L} + 1$).

4 Centralized Algorithm

As sensors' position may move as time passes by. Here we further consider sensor position's uncertainty $\Delta\mathbf{p}_i$, which turns to be the parameter of interest in $\mathbf{p}_i = \bar{\mathbf{p}}_i + \Delta\mathbf{p}_i$. To further improve the positioning performance by joint estimation, we will update the sensor uncertainty's influence in (1) as

$$\Delta\hat{d}_{i,j} = \mathbf{g}(\hat{\theta}_{i,j}, \gamma_k)^T(\mathbf{q} - \bar{\mathbf{p}}_i) - \mathbf{g}(\theta_0, \gamma_k)^T(\mathbf{q} - \mathbf{p}_0) + \hat{n}_i \tag{15}$$

Then we derive the likelihood function based on (14):

$$\tilde{p}(\Delta\hat{d}_{i,j}, \Delta\mathbf{p}_i|\mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j}) = p(\hat{\theta}_{i,j}|\theta_{i,j})\alpha''_{i,j,k_{i,j}}\tilde{\Phi}(\Delta\tilde{d}_{i,j}, \Delta\mathbf{p}_i|\mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j}). \tag{16}$$

Since the measurements are independent to each other,

$$\tilde{p}(\Delta\hat{\mathbf{d}}, \Delta\mathbf{p}_i|\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\theta}) = \prod_{i\in\mathcal{N}}\prod_{j\in M_i}\tilde{p}(\Delta\hat{d}_{i,j}, \hat{\theta}_{i,j}, \Delta\mathbf{p}_i|\mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j}). \tag{17}$$

Here we define sets called $\mathcal{N} = \{1, 2, ..., N\}$ and $\mathcal{M}_i = \{1, 2, ..., M_i\}$. Since $\Delta\mathbf{p}_i$ is independent of other random variables in the complete data, we fix other interested parameters to obtain the new $\tilde{\mathcal{Q}}$ function with the addition of $\Delta\mathbf{p}_i$ as

$$\begin{aligned}\tilde{\mathcal{Q}}(\mathbf{q}, \mathbf{q}') &= \mathrm{E}[\ln\tilde{p}(\Delta\hat{\mathbf{d}}, \Delta\mathbf{p}_i|\boldsymbol{q}, \boldsymbol{p}, \boldsymbol{\theta})|\Delta\hat{\mathbf{d}}, \boldsymbol{\alpha}'', \mathbf{q}'] \\ &= \sum_{i\in\mathrm{N}}\sum_{j\in M_i}\int p(\Delta\mathbf{p}_i|\Delta\hat{d}_{i,j}, \bar{\mathbf{p}}_i, \boldsymbol{q}') \times \ln\tilde{p}(\Delta\hat{d}_{i,j}, \Delta\mathbf{p}_i|\mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j})d\Delta\mathbf{p}_i\end{aligned} \tag{18}$$

in which

$$\ln\tilde{p}(\Delta\hat{d}_{i,j}, \Delta\mathbf{p}_i|\mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j}) = \ln p(\hat{\theta}_{i,j}|\theta_{i,j})\alpha''_{i,j,k} + \ln\tilde{\Phi}(\Delta\tilde{d}_{i,j}, \Delta\mathbf{p}_i|\mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j})$$

Substitute the align into (18). The first item doesn't contain the parameter of interest $\mathbf{q}$ to realize $\mathcal{Q}$ function minimization, which can be dropped. $\tilde{\mathcal{Q}}_i(\mathbf{q}, \mathbf{q}')$ can be reformulated as

$$\tilde{\mathcal{Q}}_i(\mathbf{q}, \mathbf{q}') = \int p(\Delta\mathbf{p}_i|\Delta\hat{d}_{i,j}, \bar{\mathbf{p}}_i, \boldsymbol{q}') \times \ln\tilde{\Phi}(\Delta\tilde{d}_{i,j}, \Delta\mathbf{p}_i|\mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j})d\Delta\mathbf{p}_i \tag{19}$$

By Bayes' rule, the posterior distribution of sensor i's position uncertainty in (19) is derived as

$$p(\Delta\mathbf{p}_i|\Delta\hat{d}_{i,j}, \bar{\mathbf{p}}_i, \boldsymbol{q}') \propto p(\Delta\mathbf{p}_i)\prod\nolimits_{j\in M_i}p(\Delta\hat{d}_{i,j}, \hat{\theta}_{i,j}|\mathbf{q}, \bar{\mathbf{p}}_i, \Delta\mathbf{p}_i, \theta_{i,j}) \tag{20}$$

Generally, the posterior distribution of sensor i's position uncertainty is intractable to be analyzed and calculated with low complexity, thus rendering the closed form of KullbackCLeibler divergence (KLD) as

$$D_{KL}(f \parallel p) = \int f(\Delta \mathbf{p}_i) \ln \frac{f(\Delta \mathbf{p}_i)}{p(\Delta \mathbf{p}_i)} d\Delta \mathbf{p}_i. \tag{21}$$

Single Reflections. Combining the updated model in (15), then the global $\Delta \mathbf{p}''$ with vector form can be expressed as

$$\Delta \mathbf{p}'' = \arg\min_{\Delta \boldsymbol{p}} \left\{ D_{KL}(\Delta \mathbf{p}; \boldsymbol{\alpha}'', \mathbf{q}', \hat{\boldsymbol{\theta}}) \right\} \tag{22}$$

As each sensor's uncertainty is i.i.d. with other sensors, the maximize the global $D_{KL}(\Delta \mathbf{p}; \boldsymbol{\alpha}'', \mathbf{q}', \hat{\boldsymbol{\theta}})$ is equivalent to obtain the extreme value in each $D_{KL}(\Delta \mathbf{p}_i; \boldsymbol{\alpha}''_i, \mathbf{q}', \hat{\boldsymbol{\theta}}_i)$.

The minimization of KLD can be derived by the partial derivatives of $D_{KL}(f \parallel p)$ with respect to $\Delta\bar{x}_i$, $\Delta\bar{y}_i$ and $\bar{\sigma}^2_{\Delta \boldsymbol{p}_i}$ and setting the results are zeros. After some manipulations, we have $\Delta \mathbf{p}'' = (\Delta x''_i, \Delta y''_i)$, where

$$\Delta x''_i = \frac{\frac{\Delta y'_i}{(1-\rho^2)\sigma_{\Delta x_i}\sigma_{\Delta y_i}} + \sum_{j=1}^{M_i} \frac{1}{\sigma_i^2}(E_{i,j} - a_{i,j}b_{i,j}\Delta y'_i)}{\frac{1}{(1-\rho^2)\sigma^2_{\Delta x_i}} - \frac{1}{\sigma_i^2}\sum_{j=1}^{M_i} a^2_{i,j}}, \tag{23}$$

$$\Delta y''_i = \frac{\frac{\Delta x'_i}{(1-\rho^2)\sigma_{\Delta x_i}\sigma_{\Delta y_i}} + \sum_{j=1}^{M_i} \frac{1}{\sigma_i^2}(F_{i,j} - a_{i,j}b_{i,j}\Delta x'_i)}{\frac{1}{(1-\rho^2)\sigma^2_{\Delta y_i}} - \frac{1}{\sigma_i^2}\sum_{j=1}^{M_i} b^2_{i,j}} \tag{24}$$

$$\bar{\sigma}_{\mathbf{p}_i} = \sqrt{\frac{2(1-\rho^2)}{(1-\rho^2)\sigma^2_{\Delta x_i}\sigma^2_{\Delta y_i}\sum_{j=1}^{M_i}(a^2_{i,j} + b^2_{i,j}) + \sigma_i^2(\sigma^2_{\Delta x_i} + \sigma^2_{\Delta y_i})}}\,\sigma_i\sigma_{\Delta x_i}\sigma_{\Delta y_i} \tag{25}$$

Then $x''_i = \bar{x}_i + \Delta x''_i$, $y''_i = \bar{y}_i + \Delta y''_i$.

Finally, we derive the closed form of $\mathcal{Q}$ function of the j-th measurement in sensor i as

$$\begin{aligned} \tilde{\mathcal{Q}}_{i,j}(\mathbf{q}, \mathbf{q}') &= \int f(\Delta \mathbf{p}_i | \Delta \hat{\mathbf{d}}_i, \bar{\mathbf{p}}_i, \mathbf{q}') \ln \tilde{\Phi}(\Delta \tilde{d}_{i,j}, \Delta \mathbf{p}_i | \mathbf{q}, \bar{\mathbf{p}}_i, \theta_{i,j}) d\Delta \mathbf{p}_i \\ &= -\frac{1}{2\sigma_i^2}\Big[(a_{i,j} - a_0)^2 q_x^2 + (b_{i,j} - b_0)^2 q_y^2 - 2(a_{i,j} - a_0) \\ &\quad \mathrm{H}_{i,j} q_x - 2(b_{i,j} - b_0)\mathrm{H}_{i,j} q_y + 2\mathrm{K}_{i,j} q_x q_y\Big] + \mathcal{C} \end{aligned} \tag{26}$$

where

$$\mathrm{H}_{i,j} = \Delta \tilde{d}_{i,j} + a_{i,j}x''_i + b_{i,j}y''_i - a_0 x_0 - b_0 y_0, \tag{27}$$

$$\mathrm{K}_{i,j} = a_{i,j}b_{i,j} + a_0 b_{i,j} + a_{i,j}b_0 + a_0 b_0. \tag{28}$$

For each sensor $i \in \mathrm{N_{LOS}}$ with M_i measurements, we obtain the global $\mathcal{Q}$ function as

$$\tilde{\mathcal{Q}}(\mathbf{q},\mathbf{q}') \propto -\sum_{i\in\mathrm{N_{LOS}}}\sum_{j=1}^{M_i}\frac{1}{2\sigma_i^2}\Big[(a_{i,j}-a_0)^2q_x^2+(b_{i,j}-b_0)^2q_y^2- 2(a_{i,j}-a_0)\mathrm{H}_{i,j}q_x-2(b_{i,j}-b_0)\mathrm{H}_{i,j}q_y+2\mathrm{K}_{i,j}q_xq_y\Big] \tag{29}$$

Finally, we can obtain the estimate of object like

$$q_x'' = \frac{\sum_{i\in\mathrm{N_{NLOS}}}\sum_{j=1}^{M_i}\frac{1}{\sigma_i^2}\Big[(a_{i,j}-a_0)\mathrm{H_{i,j}}+\mathrm{K_{i,j}}q_y'\Big]}{\sum_{i\in\mathrm{N_{NLOS}}}\sum_{j=1}^{M_i}\frac{1}{\sigma_i^2}(a_{i,j}-a_0)^2} \tag{30}$$

$$q_y'' = \frac{\sum_{i\in\mathrm{N_{NLOS}}}\sum_{j=1}^{M_i}\frac{1}{\sigma_i^2}\Big[(b_{i,j}-b_0)\mathrm{H_{i,j}}+\mathrm{K_{i,j}}q_x'\Big]}{\sum_{i\in\mathrm{N_{NLOS}}}\sum_{j=1}^{M_i}\frac{1}{\sigma_i^2}(b_{i,j}-b_0)^2} \tag{31}$$

5 Simulation Results

To evaluate the performance of the proposed algorithm in a centralized implementation, we realize the passive localization in a $100 \times 100\,\mathrm{m}^2$ plane with one anchor node S_0 and four receiver node $S_1, S_2, \cdots, S_4$ as shown in Fig. 1. The parameters related to the simulations are summarized in Table 1. Each nodes' positions are $\mathbf{p}_1 = [20\ 20]^T, \mathbf{p}_2 = [80\ 30]^T, \mathbf{p}_3 = [60\ 90]^T, \mathbf{p}_4 = [70\ 70]^T$. The corresponding scatter orientations is $\boldsymbol{\gamma} = [0°, 86°, 150°, 90°, 111°, 55°, 135°, 11°]$. Furthermore, the ground-truth AOAs are $\boldsymbol{\theta} = [45°; 135°; -135°\ 18.4°; 0°; -170°\ \ -15.9°]$. In this simulation scenario, sensor nodes (S_1–S_4) received number of measurements ($|\mathcal{M}_1|$–$|\mathcal{M}_4|$) as $[1, 2, 1, 2]^T$ respectively.

We consider a Monte Carlo experiment with 1000 independent trials in Fig. 2. An initial guess of the proposed algorithm is tested according to the proposed data association method in Sect. 3. However, without considering sensors' uncertainties, the value of each submodel's weights are fluctuated and improve the risk of mismatch in demapping process. Therefore, the positioning performance of the data association method with different level of sensor position uncertainty is generally worse than the ideal case without uncertainties. We also estimate the position based on [5] for the comparison purpose. The error of [5] is larger than our method as the assumption that all the TDOA ranging have the same noise pdf. Compared with these five CDFs, the quality of demapping is reliable with uncertainties and effective than [5] even with uncertainties.

More precisely, we optimize the positioning performance including sensors' uncertainties by aligns (30) and (31) in Fig. 3. After sufficient number of iterations, we can figure out the location errors are smaller than Fig. 2 as we iteratively update the object and sensors' positions simultaneously. For comparable reasons, we also estimate the method in As TOA based method in [5] is valuable to the i.i.d. assumption, the performance will be worse considering sensors' uncertainties in Fig. 2's description.

Table 1. Major parameters in data association based algorithms.

Parameter	Note	Value
L×W	Space dimensions	$100\,\text{m} \times 100\,\text{m}$
$\mathbf{p}_0$	Anchor node S_1	$[10\,\text{m}\ 70\,\text{m}]^T$
$\mathbf{q}$	Object node	$[40\,\text{m}\ 50\,\text{m}]^T$
$\alpha_{i,j,k}$	Submodels' weight	$1/K_j$
$\Delta\mathbf{p}_i$	Sensor i's uncertainty	$\Delta\mathbf{p}_i \sim \mathcal{N}(0, \sigma_{\Delta\mathbf{p}_i})$
$\sigma_{\Delta\hat{\mathbf{d}}}$	Ranging std. deviation for $\Delta\hat{\mathbf{d}}$	$1\,\text{m}$
$\eta_{i,j}$	Ranging std. deviation for AOA	$\eta_{i,j} \sim \text{Unif}[-3°, 3°]$
K_j	Submodels for first MPC	$K_1 = 6$
	Submodels for other MPCs	$K_j = 5, j \geq 1$

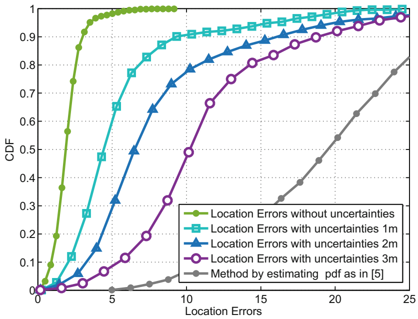

Fig. 2. Coarse location error based on different sensors' uncertainties.

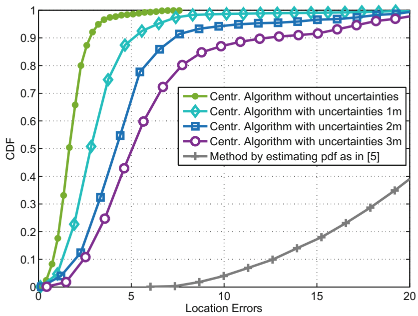

Fig. 3. Location errors based on proposed algorithm.

6 Conclusion

In this paper, we proposed a low complexity multipath aided algorithm to localization. For a further extension, we will study how to reduce the constrained known information to generalize the proposed algorithms and reduce the computational complexity.

References

1. Zhao, F., Guibas, L.J.: Wireless Sensor Networks: An Information Processing Approach. Morgan Kaufmann, Amsterdam (2004)
2. Gezici, S., Giannakis, G., Kobayashi, H., Molisch, A., Poor, H., Sahinoglu, Z.: Localization via ultra-wideband radios: a look at positioning aspects for future sensor networks. IEEE Sig. Process. Mag. **22**, 70–84 (2005)
3. Wymeersch, H., Lien, J., Win, M.Z.: Cooperative localization in wireless networks. In: Proceedings of IEEE, vol. 97, no. 2, pp. 427–450 (2009)
4. Grisetti, G., KuMmerle, R., Stachniss, C.: A tutorial on graph-based SLAM. IEEE Intell. Transp. Syst. Mag. **2**(4), 31–43 (2010)
5. Yin, F., Fritsche, C., Gustafsson, F., Zoubir, A.M.: TOA-based robust wireless geolocation and Cramr-Rao lower bound analysis in harsh LOS/NLOS environments. IEEE Trans. Sig. Process. **61**(9), 2243–2255 (2013)
6. Leitinger, E., Meissner, P., Rdisser, C.: Evaluation of position-related information in multipath components for indoor positioning. IEEE J. Sel. Areas Commun. **33**(11), 2313–2328 (2015)
7. Yin, F., Fritsche, C., Jin, D.: Cooperative localization in WSNs using Gaussian mixture modeling: distributed ECM algorithms. IEEE Trans. Sig. Process. **63**(6), 1448–1463 (2015)