On Sampling of Bandlimited Graph Signals

Mo Han, Jun Shi(✉), Yiqiu Deng, and Weibin Song

Communication Research Center, Harbin Institute of Technology,
Harbin 150001, China
junshi@hit.edu.cn

Abstract. The signal processing on graphs has been widely used in various fields, including machine learning, classification and network signal processing, in which the sampling of bandlimited graph signals plays an important role. In this paper, we discuss the sampling of bandlimited graph signals based on the theory of function spaces, which is consistent with the pattern of the Shannon sampling theorem. First, we derive an interpolation operator by constructing bandlimited space of graph signals, and the corresponding sampling operator is also obtained. Based on the relationship between the interpolation and sampling operators, a sampling theorem for bandlimited graph signals is proposed, and its physical meaning in the graph frequency domain is also given. Furthermore, the implementation of the proposed theorem via matrix calculation is discussed.

Keywords: Sampling · Signal processing on graphs · Graph signals

1 Introduction

With the rapid development of information technology, the demand for large-scale data processing is growing, such as signals from social, biological, and sensor networks. Different from traditional timeseries or images, these structured signals are interconnected. The underlying connectivities between data points naturally reside on the structure of graphs, which leads to the emerging field of signal processing on graphs. In recent years, the graph signal processing has been widely used in various application domains such as machine learning, classification and network signal processing [1,2].

The sampling theory plays a fundamental role in digital signal processing. The traditional Shannon sampling theorem bridges the continuous and discrete domains. Unlike traditional sampling, the sampling for graph signals is more challenging because the paradigm of leveraging frequency folding phenomenon cannot be defined for graph signal due to its irregular structure. Therefore, the sampling for graph signals has drawn lots of attention. Unfortunately, existing works on sampling of graph signals [4–7] do not reveal the clear physical meaning in the graph frequency domain, and the implementation of graph signal sampling and reconstruction is still not discussed in the literature. Towards this end,

X. Gu et al. (Eds.): MLICOM 2017, Part II, LNICST 227, pp. 577–584, 2018.
https://doi.org/10.1007/978-3-319-73447-7_62

we propose a new derivation of the sampling for bandlimited graph signals based on the theory of function spaces. We first derive an interpolation operator by constructing bandlimited space of graph signals, and then obtain its corresponding sampling operator. Based on the relationship between these two operators, a sampling theorem for bandlimited graph signals is proposed. The physical meaning of the sampling and reconstruction process in the graph frequency domain is also given. Furthermore, the implementation of the proposed theorem via matrix calculation is presented. Finally, a numerical example of the derived results is given.

The rest of the paper is organized as follows. Some facts of signal processing on graphs are introduced in Sect. 2. Section 3 discusses the sampling for bandlimited signals defined on graph. Finally, a conclusion is made in Sect. 4.

2 Preliminaries

In this chapter, some basic concepts of discrete signal processing on graphs [1–3] are given, which are generalized from the traditional discrete signal processing.

Graph Signal. Discrete signal processing on graphs is focused on the signal with irregular and complex internal structure, which can be represented by a graph $G = (\mathcal{V}, \mathrm{A})$, where $\mathcal{V} = [v_0, v_1, \cdots, v_{N-1}]$ denotes the set of nodes and $\mathrm{A} \in \mathbb{C}^{N\times N}$, the weighted adjacency matrix, means the *graph shift*. Given a graph representation $G = (\mathcal{V}, \mathrm{A})$, a *graph signal* is defined as the map on the graph nodes that assigns the signal coefficient $f_n \in \mathbb{C}$ to the node v_n. The edge weight $\mathrm{A}_{m,n}$ between v_m and v_n can express the correlation and similarity between the signals defined on those two nodes. When the order of the nodes is determined, the graph signal can be represented by a vector

$$\mathrm{f} = [f_0 \ f_1 \ \cdots \ f_{N-1}]^T \in \mathbb{C}^N. \tag{1}$$

For simplicity, assume A can be completely decomposed as follows (unless A should be decomposed on a set of Jordan eigenvectors)

$$\mathrm{A} = \mathrm{V}\Lambda\mathrm{V}^{-1} \tag{2}$$

where the columns of matrix V is the eigenvectors of A, and Λ is the diagonal matrix of corresponding eigenvalues $\lambda_0, \cdots, \lambda_{N-1}$ with $\lambda_0 >, \cdots, > \lambda_{N-1}$.

Graph Fourier Transform. Generally, a Fourier transform can achieve the expansion of a signal on a set of basis functions which are invariant to filtering. The eigenvectors (or the Jordan eigenvectors) of the graph shift A just satisfy the requirement [1,3], so the *graph Fourier transform* and the *inverse graph Fourier transform* can be respectively defined as

$$\hat{\mathrm{f}} = \mathrm{V}^{-1}\mathrm{f} \tag{3}$$

$$\mathrm{f} = \mathrm{V}\hat{\mathrm{f}}. \tag{4}$$

Eigenvalues $\lambda_0 >, \cdots, > \lambda_{N-1}$ denote the lowest to the highest frequencies of graph signals, with a descending order of the eigenvalues [3]. Eigenvectors of different frequencies correspond to different graph frequency components.

3 A Sampling Theorem of Bandlimited Graph Signals

For finite-dimentional discrete signal, sampling and interpolation mean the decrease and increase of the dimension of input signal. Thus the sampling and interpolation of a graph signal $\mathrm{f} \in \mathbb{C}^N$ can be respectively expressed as

$$\mathrm{g} = \Psi \mathrm{f} \in \mathbb{C}^M \tag{5}$$

$$\tilde{\mathrm{f}} = \Phi \mathrm{g} = \Phi\Psi\mathrm{f} = \mathrm{P}\mathrm{f} \in \mathbb{C}^N, \tag{6}$$

where $M<N$, g is the sampled graph signal, and matrix $\Psi \in \mathbb{C}^{M\times N}$ and $\Phi \in \mathbb{C}^{N\times M}$ denote the sampling and interpolation operators respectively, and

$$\mathrm{P} = \Phi\Psi \in \mathbb{C}^{N\times N} \tag{7}$$

with

$$\Psi^* = (\psi_0, \cdots, \psi_{M-1}) \in \mathbb{C}^{N\times M} \tag{8}$$

$$\Phi = (\phi_0, \cdots, \phi_{M-1}) \in \mathbb{C}^{N\times M} \tag{9}$$

where $\psi_i \in \mathbb{C}^N$ and $\phi_i \in \mathbb{C}^N$. If vectors $\psi_0, \cdots, \psi_{M-1}$ and $\phi_0, \cdots, \phi_{M-1}$ constitute two sets of basis of signal space $\mathrm{S_s} = \mathrm{span}\{\psi_0, \cdots, \psi_{M-1}\}$ and $\mathrm{S_i} = \mathrm{span}\{\phi_0, \cdots, \phi_{M-1}\}$, the sampling (5) and interpolation (6) can be regarded as the expansion and combination of signal f in the two spaces, where $\mathrm{S_s}$ and $\mathrm{S_i}$ represent the sampling and interpolation spaces respectively.

3.1 Sampling and Interpolation in Bandlimited Graph Signal Space

Similar to the Shannon theorem, for the possibility of perfect recovery, we consider bandlimited graph signals, i.e., the input signal f is in bandlimited space.

A graph signal f is called *bandlimited* when there exists a $K \in \{0, \cdots, N-1\}$ such that its graph Fourier transform $\hat{\mathrm{f}}$ satisfies

$$\hat{f}_i = 0 \quad \text{for all} \quad i \geq K. \tag{10}$$

The smallest K is the *bandwidth* of f. All the graph signals in $\mathbb{C}^N$ with bandwidth of at most K can form a closed bandlimited subspace, represented by BL_K.

Perfect recovery equals to achieve $\tilde{\mathrm{f}} = \mathrm{f}$. Thus given $\mathrm{f} \in \mathrm{BL}_K$, $\tilde{\mathrm{f}} \in \mathrm{BL}_K$ must be satisfied. From (6) we can know $\tilde{\mathrm{f}} \in \mathrm{S_i}$, so the problem has been transformed into the construction of the bandlimited interpolation space which should satisfy:

$$\mathrm{S_i} = \mathrm{span}\{\phi_0, \cdots, \phi_{M-1}\} = \mathrm{BL}_K. \tag{11}$$

The graph Fourier transform of interpolation operator Φ can be written as

$$\begin{aligned}\hat{\Phi} &= \mathrm{V}^{-1}\Phi = (\mathrm{V}^{-1}\phi_0, \cdots, \mathrm{V}^{-1}\phi_{M-1}) \\ &= (\hat{\phi}_0, \cdots, \hat{\phi}_{M-1})\end{aligned} \tag{12}$$

where $\hat{\phi}_i$ is the graph Fourier transform of vector ϕ_i. For vector $\mathrm{u} \in \mathrm{S_i}$ with expansion coefficients $a_0, \cdots, a_{M-1}$ on basis $\phi_0, \cdots, \phi_{M-1}$, $\hat{\mathrm{u}}$ is as follows

$$\begin{aligned}\hat{\mathrm{u}} &= \mathrm{V}^{-1}\mathrm{u} = \mathrm{V}^{-1}(\phi_0, \cdots, \phi_{M-1})(a_0, \cdots, a_{M-1})^T \\ &= (\hat{\phi}_0, \cdots, \hat{\phi}_{M-1})(a_0, \cdots, a_{M-1})^T \\ &= a_0\hat{\phi}_0 + \cdots + a_{M-1}\hat{\phi}_{M-1}\end{aligned} \tag{13}$$

where $\hat{\phi}_0, \cdots, \hat{\phi}_{M-1}$ form a new set of basis in graph frequency domain, and $\hat{\mathrm{u}} \in \mathrm{span}\{\hat{\phi}_0, \cdots, \hat{\phi}_{M-1}\}$. Thus if $\hat{\phi}_0, \cdots, \hat{\phi}_{M-1}$ satisfy (10), (11) holds true.

If $\hat{\phi}_0, \cdots, \hat{\phi}_{M-1}$ satisfy (10), then we have

$$\begin{aligned}\hat{\Phi} &= \hat{\Phi}_{BL} = \mathrm{V}^{-1}\Phi = \mathrm{V}^{-1}(\phi_0, \cdots, \phi_{M-1}) = (\hat{\phi}_0, \cdots, \hat{\phi}_{M-1}) \\ &= \left(\begin{pmatrix} \hat{\phi}_0(1) \\ \vdots \\ \hat{\phi}_0(K) \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} \hat{\phi}_{M-1}(1) \\ \vdots \\ \hat{\phi}_{M-1}(K) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \right) = \begin{pmatrix} \hat{\phi}_{01} & \cdots & \hat{\phi}_{(M-1)1} \\ \vdots & \ddots & \vdots \\ \hat{\phi}_{0K} & \cdots & \hat{\phi}_{(M-1)K} \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \\ &= \underbrace{\begin{pmatrix} \mathrm{Q} \\ 0 \cdots 0 \\ \vdots \ddots \vdots \\ 0 \cdots 0 \end{pmatrix}}_{N\times M} = \underbrace{\begin{pmatrix} \mathrm{I}_{K\times K} \\ 0 \cdots 0 \\ \vdots \ddots \vdots \\ 0 \cdots 0 \end{pmatrix}}_{N\times K} \mathrm{Q}\end{aligned} \tag{14}$$

where I is the unit matrix, and the coefficient matrix $\mathrm{Q} \in \mathbb{C}^{K\times M}$ includes all the nonzero frequency contents of bandlimited vectors $\hat{\phi}_0, \cdots, \hat{\phi}_{M-1}$:

$$\mathrm{Q} = \left(\begin{pmatrix} \hat{\phi}_0(1) \\ \vdots \\ \hat{\phi}_0(K) \end{pmatrix}, \cdots, \begin{pmatrix} \hat{\phi}_{M-1}(1) \\ \vdots \\ \hat{\phi}_{M-1}(K) \end{pmatrix} \right) = \begin{pmatrix} \hat{\phi}_{01} & \cdots & \hat{\phi}_{(M-1)1} \\ \vdots & \ddots & \vdots \\ \hat{\phi}_{0K} & \cdots & \hat{\phi}_{(M-1)K} \end{pmatrix}. \tag{15}$$

And $\mathrm{V}^{-1}\mathrm{V} = \mathrm{I}_{N\times N}$ is true, so we have

$$\mathrm{V}^{-1} \cdot \mathrm{V}_{(K)} = \begin{pmatrix} \mathrm{I}_{K\times K} \\ 0 \cdots 0 \\ \vdots \ddots \vdots \\ 0 \cdots 0 \end{pmatrix} \tag{16}$$

where $\mathrm{V}_{(K)} \in \mathbb{C}^{N\times K}$ denotes the first K columns of $\mathrm{V} \in \mathbb{C}^{\mathrm{N}\times\mathrm{N}}$, and satisfies

$$\mathrm{V}_{(K)} \cdot \mathrm{Q} = \mathrm{V}\left(\begin{pmatrix} \hat{\phi}_0(1) \\ \vdots \\ \hat{\phi}_0(K) \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} \hat{\phi}_{M-1}(1) \\ \vdots \\ \hat{\phi}_{M-1}(K) \\ 0 \\ \vdots \\ 0 \end{pmatrix}\right) = \mathrm{V}\hat{\Phi}_{BL} = \Phi. \tag{17}$$

Then combining (16) and (17), (14) can be expressed as

$$\hat{\Phi}_{BL} = \begin{pmatrix} \mathrm{Q} \\ 0 \cdots 0 \\ \vdots \ddots \vdots \\ 0 \cdots 0 \end{pmatrix} = \begin{pmatrix} \mathrm{I}_{K\times K} \\ 0 \cdots 0 \\ \vdots \ddots \vdots \\ 0 \cdots 0 \end{pmatrix} \mathrm{Q} = \mathrm{V}^{-1} \cdot \mathrm{V}_{(K)} \cdot \mathrm{Q} = \mathrm{V}^{-1}\Phi \tag{18}$$

so that we can get

$$\Phi = \mathrm{V}_{(K)}\mathrm{Q}. \tag{19}$$

By (19), the interpolation operator Φ can be constructed uniquely using a given coefficient matrix Q, and simultaneously the interpolation space $\mathrm{S_i}$ satisfies (11).

The interpolation operator Φ can be built through (19), so the next step of perfect recovery is to find the corresponding sampling operator Ψ.

Under the sampling theory for finite-dimensional vectors discussed in [8], two requirements must be satisfied for perfect recovery: (1) input signal $\mathrm{f} \in \mathrm{S_i} = \mathrm{span}\{\phi_0, ..., \phi_{M-1}\} = \mathrm{BL}_K$; (2) $\mathrm{P} = \Phi\Psi$ is a projection operator, satisfying

$$\mathrm{P}^2 = \mathrm{P}. \tag{20}$$

The first requirement can be guaranteed by (19) when given a graph signal $\mathrm{f} \in \mathrm{BL}_K$, and the second one (20) just implies the relation between Φ and Ψ.

From (7) and (19) we obtain

$$\mathrm{P} = \Phi\Psi = \mathrm{V}_{(K)}\mathrm{Q}\Psi \tag{21}$$

so (20) can be written as

$$\begin{aligned} \mathrm{P}^2 &= \Phi\Psi \cdot \Phi\Psi = \mathrm{V}_{(K)}\mathrm{Q}\Psi \cdot \mathrm{V}_{(K)}\mathrm{Q}\Psi \\ &= \mathrm{V}_{(K)} \cdot \left(\mathrm{Q}\Psi\mathrm{V}_{(K)}\right) \cdot \mathrm{Q}\Psi = \mathrm{V}_{(K)} \cdot \mathrm{W}_1 \cdot \mathrm{Q}\Psi \\ &= \mathrm{V}_{(K)}\mathrm{Q} \cdot \left(\Psi\mathrm{V}_{(K)}\mathrm{Q}\right) \cdot \Psi = \mathrm{V}_{(K)}\mathrm{Q} \cdot \mathrm{W}_2 \cdot \Psi \\ &= \mathrm{P} = \mathrm{V}_{(K)}\mathrm{Q}\Psi \end{aligned} \tag{22}$$

where $\mathrm{W}_1 = \mathrm{Q}\Psi\mathrm{V}_{(K)} \in \mathbb{C}^{K\times K}$ and $\mathrm{W}_2 = \Psi\mathrm{V}_{(K)}\mathrm{Q} \in \mathbb{C}^{M\times M}$. To achieve $\tilde{\mathrm{f}} = \mathrm{f} \in \mathrm{BL}_K$, sampled signal $\mathrm{g} \in \mathbb{C}^M$ must include at least K graph frequencies to avoid the truncation error. Thus the dimension M should satisfy $M \geq K$. Then for $Q \in \mathbb{C}^{K\times M}$, $\operatorname{rank}\left(W_2 = \Psi V_{(K)} Q\right) \leq \operatorname{rank}(Q) \leq K \leq M$; and $W_2 = I_{M\times M}$ only when $M = K$. So to make (22) true, $W_1 = I_{K\times K}$ should be satisfied, i.e.,

$$Q\Psi V_{(K)} = I_{K\times K}. \tag{23}$$

The relation between Φ and Ψ is given by (23). From (19) and (23), Φ and Ψ can be uniquely obtained. However, there are 3 unknowns in this problem: Φ, Ψ and Q, so one of them must be built first to fix the rest. For the feasibility and simplicity of sampling, we construct the interpolation operator Ψ first and conclude the following sampling theorem for graph signals.

Theorem 1. *For the sampling operator $\Psi \in \mathbb{C}^{M\times N}$ and interpolation operator $\Phi \in \mathbb{C}^{N\times M}$ of a bandlimited graph signal $f \in BL_K \in \mathbb{C}^N$, if $M \geq K$ and $\operatorname{rank}(\Psi) \geq K$ are true, then the perfect recovery of f can be achieved, where M is the total sample number, with*

$$\Phi = V_{(K)} Q \quad \text{and} \quad Q\Psi V_{(K)} = I_{K\times K}. \tag{24}$$

The restriction of $M \geq K$ and $\operatorname{rank}(\Psi) \geq K$ in Theorem 1 provides the instruction for building sampling operator $\Psi \in \mathbb{C}^{M\times N}$, and the results are varied. When this restriction is not satisfied or $f \notin BL_K$, perfect recovery is impossible due to the truncation error in the graph frequency domain.

By Theorem 1, the implementation steps for the sampling and interpolation of bandlimited graph signals are as follows:

(i) Select the total sample number M, satisfying $M \geq K$;
(ii) Build operator Ψ with $\operatorname{rank}(\Psi) \geq K$ and sample the input signal: $g = \Psi f$;
(iii) Calculate Q using $Q\Psi V_{(K)} = I_{K\times K}$;
(iv) Obtain Φ by $\Phi = V_{(K)} Q$ and recover the signal: $\tilde{f} = \Phi\Psi f$.

3.2 Numerical Example

The perfect recovery of graph signals can be obtained via the given steps, and the choices of sampling operator Ψ are varied. Next, we take one of them as an example to demonstrate the validity of the proposed theorem.

We consider a 5-node graph with adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{25}$$

and the corresponding inverse graph Fourier transform matrix is

$$V = \begin{bmatrix} 0.45 & 0.29 & 0.71 & 0.41 & 0.22 \\ 0.45 & 0.29 & 0 & -0.82 & 0.22 \\ 0.45 & 0.29 & 0.71 & 0.41 & 0.22 \\ 0.45 & -0.87 & 0 & 0 & 0.22 \\ 0.45 & 0 & 0 & 0 & -0.90 \end{bmatrix}. \tag{26}$$

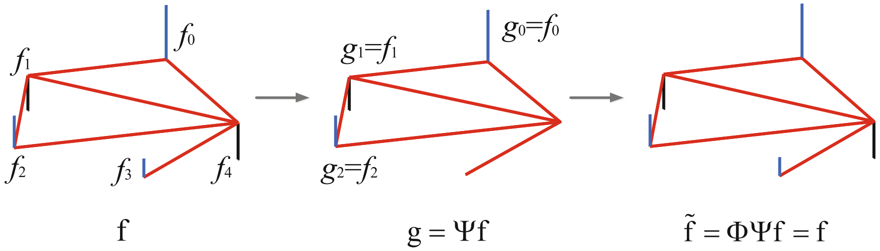

Fig. 1. Sampling and interpolation of graph signal $\mathrm{f} = [f_0\ f_1\ f_2\ f_3\ f_4]^T$.

We input a graph signal f with bandwidth $K = 3$ as

$$\mathrm{f} = \left[3.5643\ -2.2893\ 2.1501\ 1.2213\ -2.4374\right]^T. \tag{27}$$

Without loss of generality, we let the number of samples be $M = K = 3$, and let $\mathrm{rank}\,(\Psi) = K$. One possible sampling operator Ψ with simple form is

$$\Psi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{28}$$

and we can obtain the following interpolation operator Φ by matrix calculation

$$\Phi = \begin{bmatrix} 1.0000 & -0.0000 & -0.0000 \\ 0.0000 & 1.0000 & 0.0000 \\ -0.0000 & 0.0000 & 1.0000 \\ 0.5411 & 0.8172 & 0.5411 \\ -0.9086 & -1.2033 & -0.9086 \end{bmatrix}. \tag{29}$$

Then we can get following sampled signal g and recovered signal $\tilde{\mathrm{f}}$

$$\mathrm{g} = \left[3.5643\ -2.2893\ 2.1501\right]^T \tag{30}$$

$$\tilde{\mathrm{f}} = \left[3.5643\ -2.2893\ 2.1501\ 1.2213\ -2.4374\right]^T \tag{31}$$

where (31) implies the perfect recovery achieved and can be expressed as Fig. 1.

As mentioned above, the choices of the sampling operator Ψ are not unique. The following options of Ψ can also lead to perfect recovery

$$\Psi_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \Psi_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \quad \Psi_3 = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 1 & 0 & 10 \end{bmatrix} \tag{32}$$

as long as the restriction in Theorem 1 can be guaranteed.

4 Conclusion

In this paper, a new derivation for the sampling theorem of bandlimited graph signals is proposed based on the theory of function space. After introducing necessary preliminaries of signal processing on graphs, an interpolation operator is derived by constructing bandlimited space of graph signals, and the corresponding sampling operator is also obtained. On the basis of the relationship between the interpolation and sampling operators, a sampling theorem for bandlimited graph signals is obtained. Our proposed result states that perfect recovery is possible for bandlimited graph signals, and the theorem can be achieved easily in practice via matrix calculation, with the implementation given in the paper.

Acknowledgments. This work was supported in part by the National Natural Science Foundation of China under Grants 61501144 and 61671179, in part by the Fundamental Research Funds for the Central Universities under Grant 01111305, and in part by the National Basic Research Program of China under Grant 2013CB329003.

References

1. Sandryhaila, A., Moura, J.M.F.: Discrete signal processing on graphs. IEEE Trans. Sig. Process. **61**, 1644–1656 (2013)
2. Sandryhaila, A., Moura, J.M.F.: Big data analysis with signal processing on graphs: representation and processing of massive data sets with irregular structure. IEEE Sig. Process. Mag. **31**, 80–90 (2014)
3. Sandryhaila, A., Moura, J.M.F.: Discrete signal processing on graphs: frequency analysis. IEEE Trans. Sig. Process. **63**, 6510–6523 (2012)
4. Pesenson, I.Z.: Sampling in Paley-Wiener spaces on combinatorial graphs. Trans. Am. Math. Soc. **360**, 5603–5627 (2008)
5. Anis, A., Gadde, A., Ortega, A.: Towards a sampling theorem for signals on arbitrary graphs. In: IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 3864–3868 (2014)
6. Wang, X., Liu, P., Gu, Y.: Local-set-based graph signal reconstruction. IEEE Trans. Sig. Process. **63**, 2432–2444 (2015)
7. Chen, S., Varma, R., Sandryhaila, A., et al.: Discrete signal processing on graphs: sampling theory. IEEE Trans. Sig. Process. **63**, 6510–6523 (2015)
8. Vetterli, M., et al.: Foundations of Signal Processing. Cambridge University Press, Cambridge (2014)