# Extension of 2FSK Signal Detection Utilizing Duffing Oscillator

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**Abstract.** Based on the sensitivity of the chaotic system to the initial value and the characteristics of the noise immunity, this paper presents a method to detect the FSK signal of closed carrier frequency under the low signal-to-noise ratio based on the Duffing oscillator, and then give the principle of FSK signal and its modulation. Furthermore, a method to solve the problem that frequency overlapping occurred between two closed frequencies FSK signal is proposed. Based on the theoretical analysis, the simulation model is established by using MATLAB and Simulink. The simulation results show that the model can solve the frequency overlapping of the FSK signal effectively; meanwhile, it has good detection precision and anti-noise performance.

**Keywords:** Duffing oscillator · Chaos system · Frequency overlapping
Weak signal detection · FSK

## 1 Introduction

Signal detection plays an important role in the communication system, in which the identification and extraction of signal characteristics is particularly important. When the transmission signal is weak, the effect of the traditional signal detection method is not ideal. Chaotic systems are widely used to detect the weak signal of the noise background. The effect of the noise in the detection is often ignored because of the characteristics of sensitive to certain signals and inert to noise. The sensitivity of the chaos theory to the initial value and the immunity to the noise are gradually applied to the weak signal detection and the result is much better [1].

In the conventional detection method, the linear detection method [2] mainly including three aspects, such as the detection in time domain, the detection in frequency domain and the detection in time-frequency domain. These methods mainly including correlation method, sampling integral and time domain averaging method, which are widely used in the detection of periodic signals, however, they have shortage apparently, for example, the lower detection efficiency and higher detection threshold [3]. As the rapidly development of nonlinear theory in recent years, in which the duffing oscillator in chaos theory transforms the presence or absence of weak signal into obvious state change of the system, that is to say, the chaotic state changes to large-scale periodic state, in that way can detect weak signal accurately [4].

X. Gu et al. (Eds.): MLICOM 2017, Part II, LNICST 227, pp. 43–52, 2018.
https://doi.org/10.1007/978-3-319-73447-7_6

The traditional detection method of FSK signal is to improve the signal to noise ratio of the input signal by filtering, then detect FSK signal by conventional method. Indeed, that way can reduce the noise in the weak FSK signal. Meanwhile, the useful signal in noise can also be affect in the process by filtering the noise, thereby the detection accuracy is affected [5]. On the contrary, chaos system detects the weak signal directly by utilizing the sensitivity of weak signal of the system without filter out the noise, in this way, it can be used in a lower signal to noise ratio [6, 7] situation. In the practical application, due to the characteristics of the communication channel, it will cause a higher bit error rate phenomenon at the receiving terminal as result of the delay of transmission signal.

In this paper, we present a new system aim at solve the phenomenon of frequency overlapping, in which is general in FSK signal detection. The proposed approach can identify the frequency overlapping phenomenon of FSK signal, then has good performance in distinguish the code overlapping, thereby reducing the bit error rate in the communication system, which is successfully shown in this paper through Simulink simulation.

## 2 Duffing Oscillator System Model

As the most classic oscillator in all kinds of chaotic systems, Duffing oscillator is often used by researchers to detect weak signal.

Now, the mature Duffing-Holmes equation as follow:

$$\ddot{x}(t) + k\dot{x}(t) - x(t) + x^3(t) = F\cos(\omega t) \tag{1}$$

Where $-x(t) + x^3(t)$ is non-linear restoring force; $k$ is damp ratio; $F\cos(\omega t)$ is main sinusoidal driving force; F is amplitude of driving force.

When $k$ is fixed, the system state enters the monoclinic orbit state, the periodic bifurcation state, the chaotic state and the large state periodic with the increase of $F$.

Let $t = \omega\tau$, then $x(t) = x(\omega\tau)$,

$$\dot{x}(t) = \frac{dx(t)}{dt} = \frac{dx(\omega\tau)}{d(\omega\tau)} = \frac{dx(\omega\tau)}{d(\tau)} \cdot \frac{d\tau}{d(\omega\tau)} = \frac{1}{\omega}\frac{dx(\omega\tau)}{d\tau} \tag{2}$$

$$\ddot{x}(t) = \frac{d\dot{x}(t)}{dt} = \frac{d\dot{x}(\omega\tau)}{d(\omega\tau)} = \frac{1}{\omega} \cdot \frac{dx(\omega\tau)}{d\tau} \cdot \frac{d\tau}{d(\omega\tau)} = \frac{1}{\omega^2}\frac{dx(\omega\tau)}{d\tau} \tag{3}$$

We can get:

$$\frac{1}{\omega^2}\ddot{x}(\omega\tau) + \frac{k}{\omega}\dot{x}(\omega\tau) - x(\omega\tau) + x^3(\omega\tau) = F\cos(\omega\tau) \tag{4}$$

Let $\dot{x}_\tau = \omega y$, then the equation became as follow:

$$\frac{1}{\omega}\dot{y} + ky - x + x^3 = F\cos(\omega\tau) \tag{5}$$

The Duffing equations is:

$$\begin{cases} \dot{x}_\tau = \omega y \\ \dot{y}_\tau = \omega(-ky + x - x^3 + F\cos(\omega\tau)) \end{cases} \tag{6}$$

Now, we make a research on the chaotic characteristics of Duffing system by building the Duffing system model through the Matlab/Simulink tool. The fourth-order Runge-Kutta method is used to simulate the Duffing oscillator, as shown in Fig. 1.

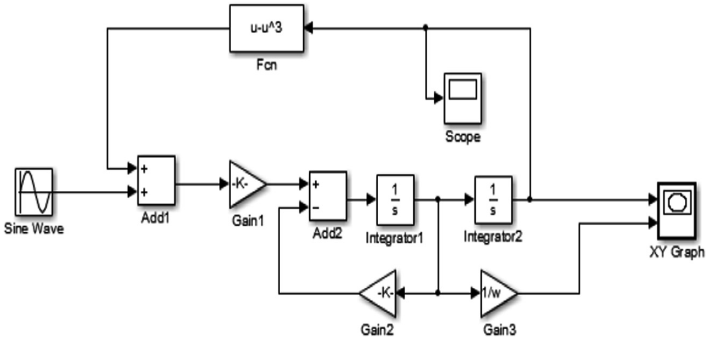
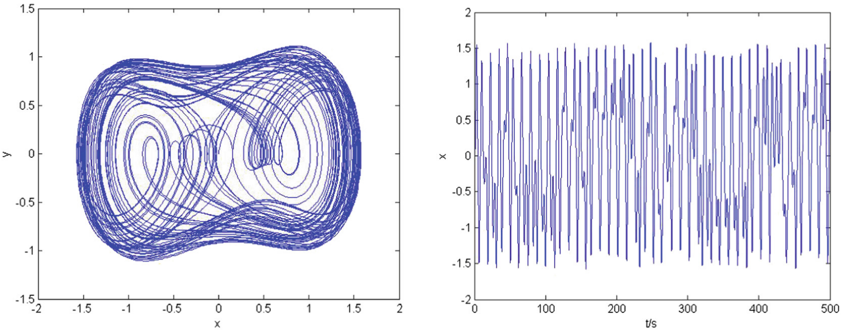


**Fig. 1.** Duffing oscillator Simulink model in Matlab

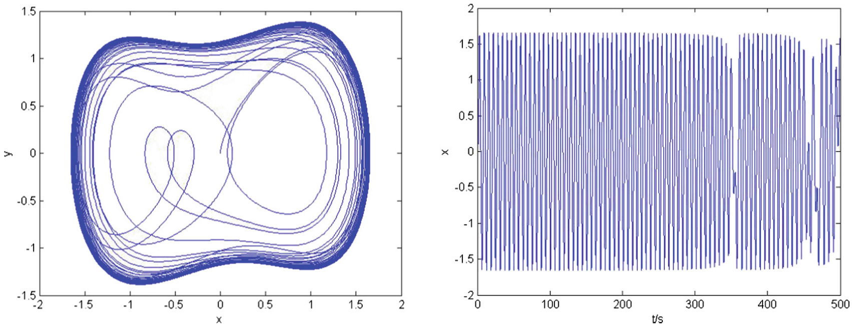
Where *Sine Wave* is the sinusoidal driving force, *Fcn* is the nonlinear restoring force, the integrator model *integrator* 1 and *integrator* 2 are used for closed-loop calculation of the first and second order, the multiplier *Gain* 1 adjust the coefficients of the integrator output in the Duffing oscillator, *Gain* 2 also represents the damp ratio, the multiplier *Gain* 3 adjusts the integrator coefficient when drawing the phase diagram. After the simulation runs, the oscilloscope *Scope* displays the time-domain waveform of the oscillator. The *XY Graph* shows the phase diagram of the oscillator, which is used to analyze and compare the properties of the vibrator in different environments and different parameters.

The value of the $k$ is set to 0.5, and the amplitude $F$ of the Duffing system cycle is gradually increased from a certain value. When running the Simulink module, we can find that the Duffing system will go through three system states, namely chaotic state, critical state (chaotic state transition to large-scale periodic state) and large-scale periodic state. After simulation experiments, the output state of the system can be seen equivalently from the $x$ output waveform of the system. The specific system output state phase diagram and the time domain waveform of $x$ are shown in Figs. 2, 3 and 4.



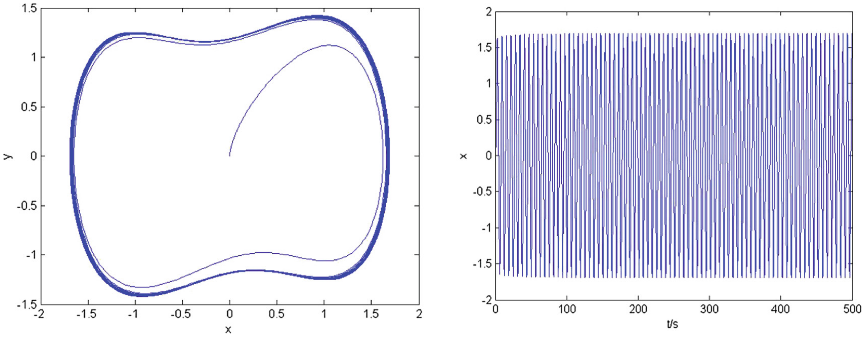
(a) Phase diagram (b) Time domain waveform

**Fig. 2.** Phase diagram and time domain waveform in chaos state $(F = 0.7)$



(a) Phase diagram (b) Time domain waveform

**Fig. 3.** Phase diagram and time domain waveform in critical state $(F = 0.8260)$



(a) Phase diagram (b) Time domain waveform

**Fig. 4.** Phase diagram and time domain waveform in large scale period state $(F = 0.83)$

In summary, the signal detection method utilizing Duffing oscillator is make the system stay at the critical state, when the weak signal input to the right side of the Duffing equation, the amplitude of driving force will increases to higher than the critical value, the oscillator will enter the large-scale periodic state; If there is no signal input, the amplitude of the driving force is still at the critical value, the oscillator state remains as it is, so that it can distinguish whether there is a weak signal whose frequency similar to the oscillator frequency or not, then detect the weak signal.

## 3 Spectrum Overlapping Signal Detection Through System

Frequency shift keying (FSK) is use the variation of carrier frequency to transfer digital information, the carrier frequency changed between the binary baseband signal frequency points, the expression of 2FSK signal is as follow:

$$e_{2FSK}(t) = [\sum_{n} a_n g(t - nT_s)] \cos \omega_1 t + [\sum_{n} \bar{a}_n g(t - nT_s)] \cos \omega_2 t \tag{7}$$

Where, $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, $\overline{a_n}$ is the reverse code of $a_n$, $g(t)$ is single rectangular pulse. The detection of the 2FSK signal utilizing Duffing oscillator is based on the sensitivity of the chaotic system to the specific frequency weak signal. The phase change of the system output can be used as a standard for the existence of the special signal that to be detected, then output the code. But in the actual transmission process, it can't be avoided to exist the error code under the influence of communication channel. In the communication system, there are two main factors of error code, one is the channel additive noise, the other one is inter-symbol interference. Using the chaotic system as the signal detection tool, the additive noise of the channel does not affect the system discrimination, it is only changed the trajectory of the chaotic state, the oscillator is still in the chaotic state; however, the inter-symbol interference became the core reason that affects whether the receiver can get the correct information directly.

The 2FSK signal with inter-symbol interference is detected by using an array of two Duffing oscillators. When the carrier frequency is the same as the frequency of the system, the corresponding Duffing oscillator will be changed from the initial chaotic state to the large-scale periodic state, and the Duffing oscillator with different frequencies will still be in the chaotic state. Thus, it is judged whether or not the received symbol signal is "1" or "0" based on the state of the system in each symbol period. Due to the other factors such as communication channel, the number $n$ code transmits delay, when the number $n + 1$ symbol arrives at the receiver, it contains information of the both frequency $f_1$ and $f_2$, so at this moment, the oscillator 1 and oscillator 2 will both be changed the chaotic state to large-scale periodic state. Further, the number $n$ code can be obtained according to the time at which the oscillator state changes, thereby solving the problem of inter-symbol interference. The flow chart of the principle as shown in Fig. 5.

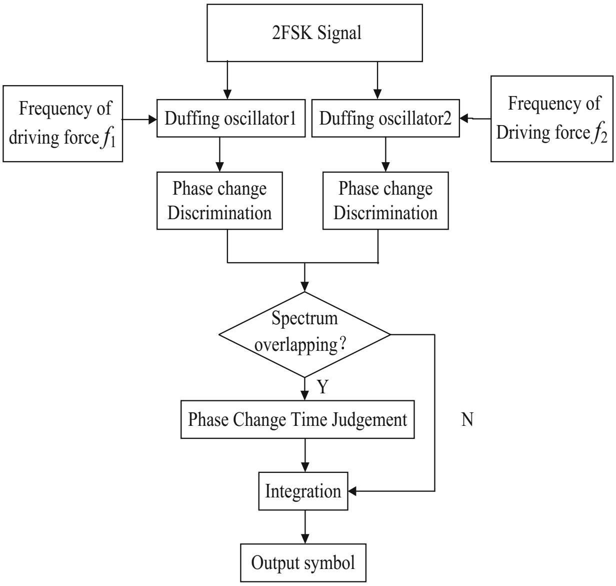


**Fig. 5.** The flow chart of the principle of 2FSK detection utilizing Duffing oscillator array

## 4 Simulation

According to the flow chart of the previous section, we can know that the step of detecting the spectrum overlapping 2FSK signal based on Duffing oscillator is:

Step 1: Set the 2FSK signal as shown in Fig. 6.

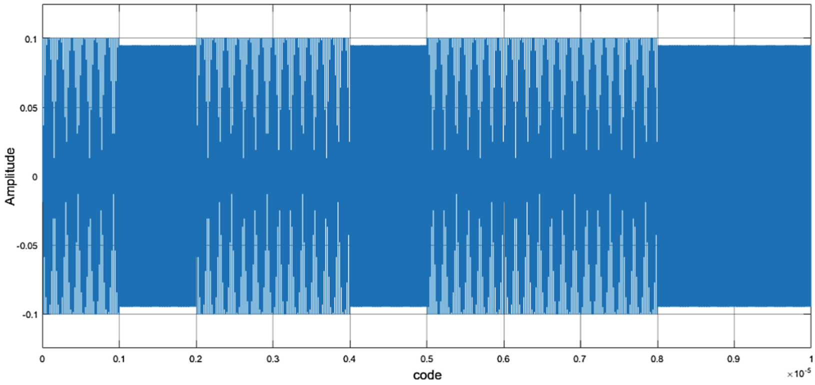
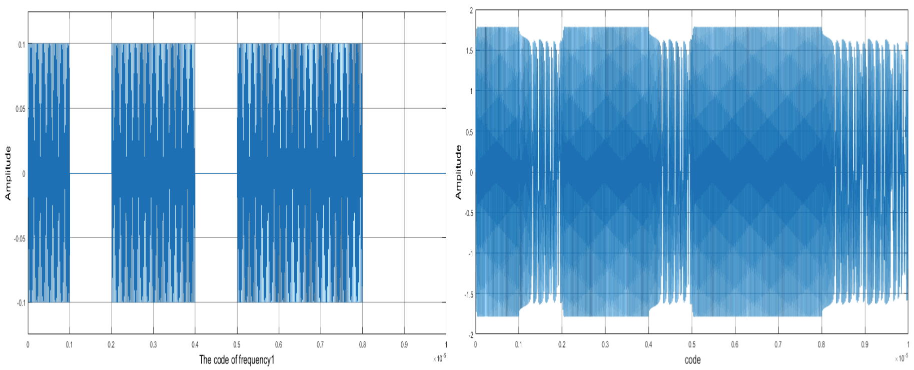


**Fig. 6.** The waveform of 2FSK signal

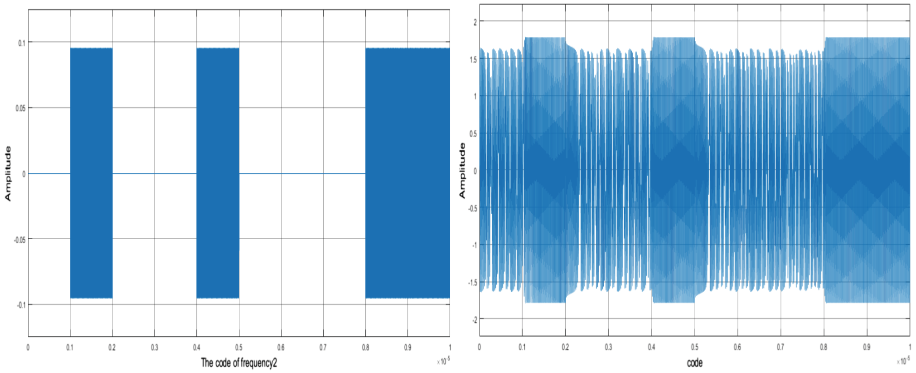
The binary symbol of 2FSK signal to be transmitted is [1,0,1,1,0,1,1,1,0,0], where $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, $f_2 = 10f_1$, $f_1$ represents signal “1” and $f_2$ represents “0”.

Step 2: Set the frequency of driving force in oscillator 1 and the oscillator 2 is $f_1$ and $f_2$, meanwhile, set its amplitude is the amplitude of critical state. Then input 2FSK signal to the duffing oscillator system. When the 2FSK signal input to the oscillator 1 and oscillator 2, the corresponding signal waveform and time domain waveform of output are shown in Figs. 7 and 8.



(a) (b)

**Fig. 7.** (a) The waveform of transmission code of $f_1$ and (b) the time domain waveform of duffing oscillator1



(a) (b)

**Fig. 8.** (a) The waveform of transmission code of $f_2$ and (b) the time domain waveform of duffing oscillator2

When the frequency of 2FSK signal is the same as the frequency of the oscillator, in this symbol period, the amplitude of the oscillator becomes larger than the critical state, the oscillator enters the large-scale periodic state, however, the oscillator whose frequency is different form the oscillator frequency is still in chaotic state.

Step 3: Due to the delay caused by the communication channel, the two adjacent signals in the transmission process are overlapped. There are two cases in the spectrum overlapping in 2FSK signal transmission. One is that the adjacent symbols of spectrum overlapping code are same, at this point the integration signal is not continuous, moreover the state of oscillator changed only one. The other one is that the adjacent symbols of spectrum overlapping code are not same, at this time, although the receive signal not continuous, but the two oscillators both changed.

[1]. For example, the 2FSK binary symbol is [1,0,1,1,0,1,1,1,0,0], when the third symbol "1" and the fourth symbol "1" generate inter-symbol interference, the received signal is discontinuous. As shown in Fig. 9.

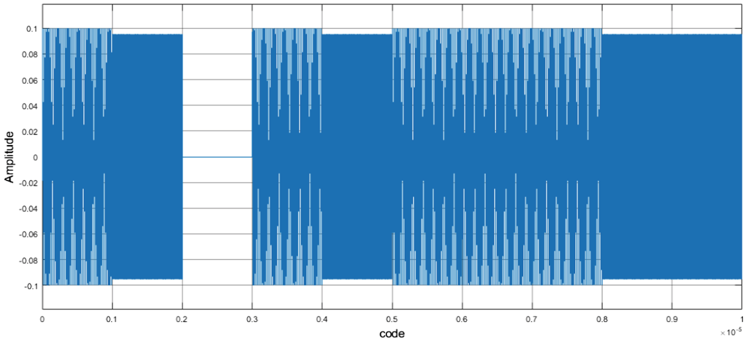
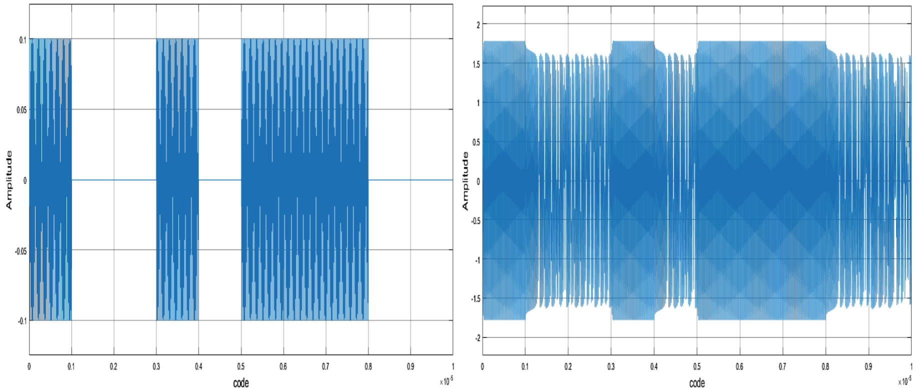


**Fig. 9.** The waveform of 2FSK signal with inter-symbol interference

The oscillator state of the third symbol period of oscillator 1 is still in chaotic state. The received signal and its time-domain waveforms of $f_1$ and $f_2$ are shown in Figs. 10 and 11. It can be seen that the system state of two oscillators are not changed during the third symbol period, while only one oscillator changed during the fourth symbol period. It can be concluded that the third symbol is the same as the fourth symbol, and the two symbols of spectrum overlapping occurs.

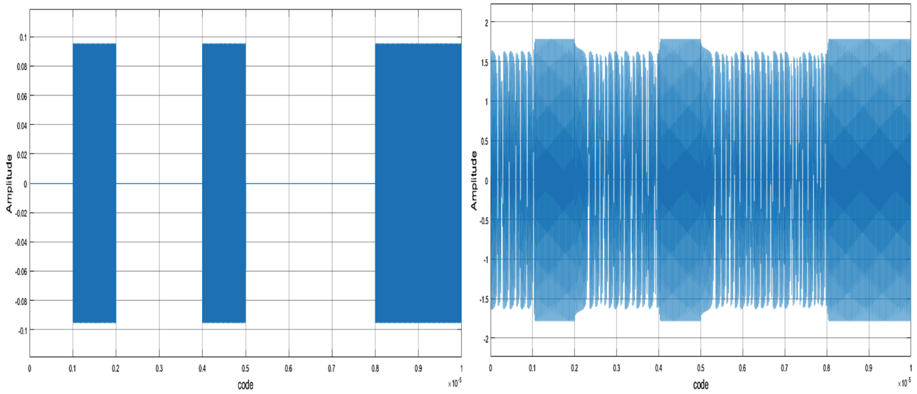
[2]. If the adjacent symbols of spectrum overlapping code are not same, when the second symbol "0" and the third symbol "1" generate inter-symbol interference, the received signal is not continuous. The output time domain waveforms of the oscillator 1 and the oscillator 2 are shown in Fig. 12.

It can be seen that the two oscillators are not changed during the second symbol period and both are changed during the third symbol period. It can be concluded that the second symbol is not the same as the third symbol, and the two symbols of spectrum overlapping occurs.



(a) (b)

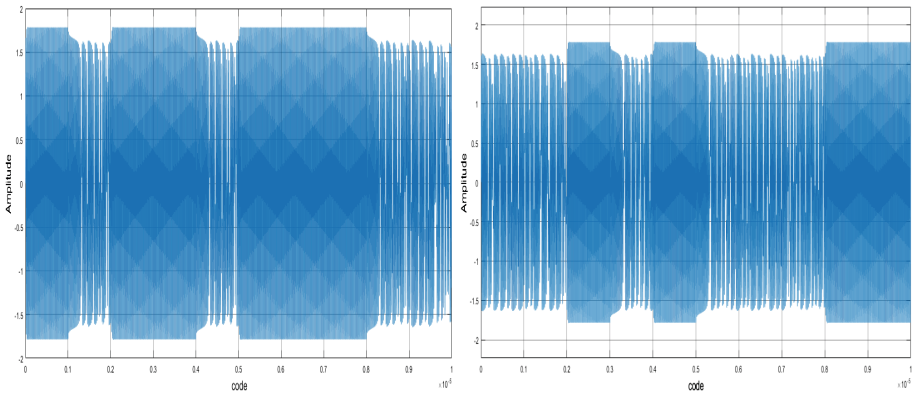
**Fig. 10.** (a) The waveform of received signal code of $f_1$ and (b) the time domain waveform of duffing oscillator1



(a) (b)

**Fig. 11.** (a) The waveform of received signal code of $f_2$ and (b) the time domain waveform of duffing oscillator2

Step 4: According to the time point at which the oscillator changed, the exact value of the two symbols can be discriminated, and the signal detection is completed.



(a) (b)

**Fig. 12.** The time domain waveform of duffing oscillator1 (a) and oscillator2 (b)

## 5 Conclusions

Compared with the traditional 2FSK signal detection method, the method proposed in this paper can solve the spectrum overlapping phenomenon of adjacent symbols in 2FSK caused by delay effectively. Through the research and simulation of this phenomenon, we improve the duffing oscillator accurate detection of the modulation signal. At the same time, it is proposed to determine the symbol when spectrum overlapping occurs according to the time point of the state change of the chaotic system, so as to determine the 2FSK signal further accurately.

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