Influence of Inter-channel Error Distribution on Mismatch in Time-Interleaved Pipelined A/D Converter

Yongsheng Wang(✉), Chen Yin, and Xunzhi Zhou

Micro-electronic Department, Harbin Institute of Technology,
92 West Dazhi Street, Nangang District, Harbin, China
{yswang, yinchen}@hit.edn.cn

Abstract. Influence of channel error distribution on inter-channel mismatches of pipelined time-interleaved A/D converters (TIADCs) is discussed in this paper. TIADC systems can increase the maximum sample rate, but the mismatch between the channels significantly reduce the systems' performance. This paper analyzes the mismatch in frequency domain, and discusses the influence of amplitude distribution of the inter-channel errors on the mismatch. Finally comes to a conclusion that when the error of one channel is equal to the median of the two adjacent channel errors, them is match of the overall TIADC system is minimal. According to simulation results, it can instruct a way to reduce the mismatch of TIADCs.

Keywords: Time-Interleaved ADC · Offset mismatch · Gain mismatch
Time sampling mismatch · Channel error distribution

1 Introduction

ADC's accuracy and speed directly restrict the performance of the communication system, such as the software radio [1], the intelligent communication, the intelligent positioning and navigation [2] etc. In order to pursue both high resolution and high speed, the time-interleaved ADC (TIADC) is widely used. However, channel mismatches in this structure such as offset mismatch, gain mismatch and time sampling mismatch which caused by circuit structure asymmetry and process deviation significantly reduce the system performance [3]. While Inter-channel mismatch of TIADC has been analyzed by behavioral modeling [4], and the influence of offset error, gain error and time sampling deviation on mismatches has been researched in [5–7], they have only analyzed the effect of errors within each channel on mismatches. Spurs produced by channel mismatch in output signal spectrum have been investigated in [8], however, they have only discussed the relationship between the spurs amplitude and the error within each channel.

In this paper, the inter-channel mismatch is analyzed in frequency domain and the results of the analysis indicate that not only the errors within each channel determine the frequency and amplitude of the spurs in output spectrum, but also the amplitude distribution of the inter-channel errors affects the magnitude of the spurs. The

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remainder of the paper is organized as followers. Section 2 introduces a modeling of TIADC and analyzes the inter-channel mismatch in frequency domain. Section 3 analyzes the influence of inter-channel error samplitude distribution on mismatches. Section 4 gives the simulation results and Sect. 5 concludes the paper.

2 Modeling of TIADC Mismatches

2.1 Modeling of Multi-channel TIADC

Several channel ADCs with high resolution but relatively slower sample rateconvert the input signal parallel in TIADC [3]. Figure 1 shows the model of a typical TIADC system.

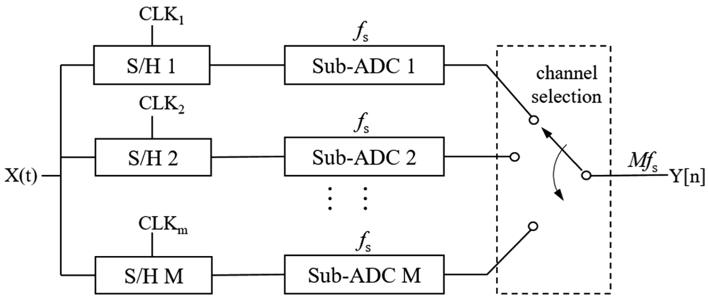

Fig. 1. Model of a typical TIADC system

When there are M channels and the time interval between adjacent channels is T_s, the sampling period of each sub-channel ADC is MT_s, and the sampling time of the *m-th* sub-channel ADC is $t_m = nMT_s + mT_s$, $n = 0, 1, 2\ldots$; $m = 0, 1, 2..M-1$. The discrete sampling sequence of each ideal sub-channel ADC can be obtained as follows [9],

$$\begin{cases} y_0 = [x(mMT_s)] \\ y_1 = [x(mMT_s + T_s)] \\ \quad\vdots \\ y_k = [x(mMT_s + kT_s)] \\ \quad\vdots \\ y_{M-1} = [x(mMT_s + (M-1)T_s)] \end{cases} \quad m = 0,\ 1,\ 2,\ \ldots;\ k = 0,\ 1,\ 2,\ \ldots,\ M-1 \tag{1}$$

2.2 Mismatches in Frequency Domain

Assume that the offset error, gain error and sampling time deviation in each channel are $os_0, os_1, os_2, \ldots os_{M-1}$, $g_0, g_1, g_2, \ldots g_{M-1}$ and $\Delta t_0, \Delta t_1, \Delta t_2, \ldots \Delta t_{M-1}$. Therefore, the discrete sampling sequence of each sub-channel ADC can be obtained as follows,

$$\begin{cases} y_0 = [g_0 x(mMT_s + \Delta t_0) + os_0] \\ y_1 = [g_1 x(mMT_s + T_s + \Delta t_1) + os_1] \\ \vdots \\ y_k = [g_k x(mMT_s + kT_s + \Delta t_k) + os_k] \\ \vdots \\ y_{M-1} = [g_{M-1} x(mMT_s + (M-1)T_s + \Delta t_{M-1}) + os_{M-1}] \end{cases} \quad m = 0, 1, 2, \ldots; k = 0, 1, 2, \ldots, M-1 \tag{2}$$

The above equation scan be seen as a sampling pulse sequence $p_g(t)$ sample the input signal $x(t + \Delta t_m)$, which deviate Δt_m with the ideal input signal $x(t)$. Where the sampling pulse sequence of the *k-th* channel is

$$p_g(t) = \sum_{m=0}^{M-1} g_m \sum_{k=-\infty}^{+\infty} \delta(t - kMT_s - mT_s) \tag{3}$$

Its Fourier transform is

$$P_g(j\omega) = 2\pi \sum_{m=-\infty}^{+\infty} C_{m,k} \delta\left(\omega - m\frac{\omega_s}{M}\right) \tag{4}$$

In the above equation,

$$\begin{aligned} C_{m,k} &= \frac{1}{MT_s} \int_{-MT_s/2}^{MT_s/2} p_k(t) e^{-j\frac{2\pi m}{MT_s}t} dt \\ &= \frac{1}{MT_s} \sum_{m=0}^{M-1} g_m \int_{-MT_s/2}^{MT_s/2} \sum_{n=-\infty}^{+\infty} \delta(t - nMT_s - kT_s) e^{-j\frac{2\pi m}{MT_s}t} dt \\ &= \frac{1}{MT_s} \sum_{m=0}^{M-1} g_m e^{-jkm\frac{2\pi}{M}} \end{aligned} \tag{5}$$

According to the convolution theorem, the sampling output signal containing the gain mismatch and the sampling time mismatch is

$$\begin{aligned} Y_{g,t}(j\omega) &= \frac{1}{2\pi} FT[x(t + \Delta t_m)] * P_g(j\omega) \\ &= \frac{1}{2\pi} \left[X(j\omega) e^{j\omega \Delta t_m}\right] * P_g(j\omega) \\ &= \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \left[\left(\frac{1}{M} \sum_{m=0}^{M-1} g_m e^{-jkm\frac{2\pi}{M}} e^{j\left(\omega - k\frac{\omega_s}{M}\right)\Delta t_m} \right) \cdot X\left(\omega - k\frac{\omega_s}{M}\right) \right] \end{aligned} \tag{6}$$

Since each channel samples the input signal as MT_s for the cycle, so the offset error can be expressed as

$$o(t) = \sum_{m=0}^{M-1} \sum_{n=-\infty}^{+\infty} os_m \cdot \delta(t - nMT - mT_s) \tag{7}$$

The Fourier transform of the above equation is

$$O(\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{+\infty} \left[\frac{1}{M} \sum_{m=0}^{M-1} os_m e^{-jkm\frac{2\pi}{M}} \cdot \delta\left(\omega - k\frac{\omega_s}{M}\right) \right] \tag{8}$$

According to the linear nature of Fourier transform, we can get the sampling signal spectrum that contains three kinds of mismatches as below

$$\begin{aligned} Y(j\omega) = {} & \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \left(\frac{1}{M} \sum_{m=0}^{M-1} g_m e^{-jkm\frac{2\pi}{M}} e^{j\left(\omega - k\frac{\omega_s}{M}\right)\Delta t_m} \right) X\left(j\left(\omega - k\frac{\omega_s}{M}\right)\right) \\ & + \frac{2\pi}{T_s} \sum_{k=-\infty}^{+\infty} \left(\frac{1}{M} \sum_{m=0}^{M-1} os_m e^{-jkm\frac{2\pi}{M}} \delta\left(\omega - k\frac{\omega_s}{M}\right) \right) \end{aligned} \tag{9}$$

Where we set

$$A(k) = \frac{1}{M} \sum_{m=0}^{M-1} g_m e^{-jkm\frac{2\pi}{M}} e^{j\left(\omega - k\frac{\omega_s}{M}\right)\Delta t_m} \tag{10}$$

$$B(k) = \frac{1}{M} \sum_{m=0}^{M-1} os_m e^{-jkm\frac{2\pi}{M}} \tag{11}$$

Equation (8) indicates that when there are M channels, the frequency of the spurs generated by the gain error and the sampling time deviation in the TIADC is located at $\omega = |\pm\omega_{in} + k\omega_s/M|$, $(k = 1, 2, \ldots M - 1)$ and the frequency of the spurs generated by the offset error is located at $\omega = k\omega_s/M$, $(k = 1, 2, \ldots M - 1)$. When there are three kinds of mismatches, the SINAD (Signal to noise and distortion ratio) is

$$\begin{aligned} SINAD &= 10\log_{10}\left(\frac{P_{signal}}{P_{noise} + P_{HD}}\right) \\ &= 10\log_{10}\left(\frac{A(0)^2}{\sum_{k=1}^{M-1} A(k)^2 + \sum_{k=0}^{M-1} B(k)^2}\right) \end{aligned} \tag{12}$$

3 Inter-channel Error Distribution

Equation (8) shows the amplitude of spurs is related to the coefficients $A(k)$ and $B(k)$. $A(k)$ and $B(k)$ can be regarded as the offset error, gain error and sampling time deviation of each sub-channel equidistant distribute in the unit circle $e^{-jk2\pi}$ and then accumulate. So the amplitude distribution of the errors between the channels will affect the coefficients $A(k)$ and $B(k)$. Next, a four-channel ADC will be used as an example to discuss the influence of the amplitude distribution of the offset error, the gain error and the sampling time deviation on the coefficients $A(k)$ and $B(k)$.

Equation (10) indicates that the magnitude of the offset error affects only the coefficient $B(k)$. When the number of channels $M = 4$, by Nyquist sampling theorem there should be $k\omega_s/M \leq \omega_s/2$ in Eq. (10), therefore $k \leq 2$.When $k = 0$, there should be $\omega = 0$ to make $\delta(\omega - k\omega_s/M) \neq 0$, it is expressed as a frequency independent of the DC component in the output spectrum, so needn't consider it. Then substitute $k = 1$ and $k = 2$ into Eq. (10), we can get

$$\begin{aligned} B(1) &= \frac{1}{4}\left(os_0 + os_1 e^{-j\frac{\pi}{2}} + os_2 e^{-j\pi} + os_3 e^{-j\frac{3}{2}\pi}\right) \\ &= \frac{1}{4}[os_0 - os_2 + j(os_1 - os_3)] \end{aligned} \tag{13}$$

$$\begin{aligned} B(2) &= \frac{1}{4}\left(os_0 + os_1 e^{-j\pi} + os_2 e^{-j2\pi} + os_3 e^{-j3\pi}\right) \\ &= \frac{1}{4}(os_0 - os_1 + os_2 - os_3) \end{aligned} \tag{14}$$

Since the mismatch between channels is caused by the relative error between the channels, the 0th channel is regarded as the standard channel, that is to make the 0th channel as a reference, therefore $os_0 = 0$. So the errors $os_k(k = 1, 2, 3)$ present in the remaining channels refer to the relative error between the *k-th* channel and the standard channel, rather than the absolute error relative to the ideal case.

Equation (11) indicates that the effect of spurious mismatches on SINAD is determined by the square sum of the spurious coefficient $B(k)$. Therefore, for the four-channel ADC there is

$$\begin{aligned} B(1)^2 + B(2)^2 &= \frac{1}{16}\left[(-os_2 + (os_1 - os_3))^2 + (os_2 - (os_1 + os_3))^2\right] \\ &= \frac{1}{8}\left(os_1^2 + os_2^2 + os_3^2 - 2os_1 os_2\right) \end{aligned} \tag{15}$$

Since the first channel and the third channel have a phase difference of π, and are symmetrically distributed at the center of the second channel, it is assumed that $|os_1| = |os_3|$, so the Eq. (14) becomes

$$B(1)^2 + B(2)^2 = \frac{1}{8}\left(2os_1^2 - 2os_1 os_2 + os_2^2\right) \tag{16}$$

In the same way, for gain error and sampling time deviation, there are

$$A(1)^2 + A(2)^2 = \frac{1}{8}\left(2g_1^2 - 2g_1 g_2 + g_2^2\right) \tag{17}$$

$$A(1)^2 + A(2)^2 = \frac{1}{8}\left(2e^{2j\omega_{in}\Delta t_1} - 2e^{j\omega_{in}(\Delta t_1 + \Delta t_2)} + e^{2j\omega_{in}\Delta t_2}\right) \tag{18}$$

So when $|os_1| = |os_3| = \frac{1}{2}|os_2|$, $|g_1| = |g_3| = \frac{1}{2}|g_2|$ and $\left|e^{j\omega_{in}\Delta t_1}\right| = \left|e^{j\omega_{in}\Delta t_3}\right| = \frac{1}{2}\left|e^{j\omega_{in}\Delta t_2}\right|$, the above Eqs. (15)–(17) respectively has a minimum $\frac{1}{16}|os_2|^2$, $\frac{1}{16}|g_2|^2$ and $\frac{1}{16}\left|e^{j\omega_{in}\Delta t_2}\right|^2$.

Therefore, it can be concluded that for the multi-channel ADC, when the absolute amplitude of the error in the *k-th* channel is the intermediate value of the absolute amplitude of the error in the $k-1th$ channel and the $k+1th$ channel, the spurs generated by the mismatch has the least effect on the SINAD of the whole ADC system, so that the ADC has the largest ENOB (Effective number of bits).

4 Simulation Results

In this section a four-channel 12bits ADC will be used as an example to simulate and verify the conclusion. The 0th channel of the ADC is regarded as a standard channel. That is to make the 0th channel as a reference with no mismatch error in it. The errors in the remaining channels refer to the relative error to the 0th channel, rather than the absolute error relative to the ideal case.

In the following simulation, 0.5% of the offset error, 10 dB gain error and 200ps sampling time deviation are added in the second channel, by changing the magnitude of the error in the first and third channels to verify the influence of the error amplitude distribution on the mismatch. The input signal frequency $\omega_{in} = 12.5488\,\text{MHz}$. According to the analysis in the Sect. 3, we assume that the errors in the 1st and 3rd channels are the same.

When the 1st and 3rd channel does not exist any error, the output signal spectrum is shown in Fig. 2. The amplitude of the spurs produced by offset mismatch at $\omega_s/4$ and $\omega_s/2$ are −52.64 dB and −67.69 dB respectively. The amplitude of the spurs produced by gain mismatch and time sampling mismatch at $\pm\omega_{in} + \omega_s/4$ and $\omega_s/2 - \omega_{in}$ are −48.72 dB, −48.23 dB and −49.04 dB respectively. The SINAD of the whole TIADCsystem is 42.7923 dB and the ENOB is 6.8153 bits.

When the error of the 1st and 3rd channel is the half of the error of the 2nd channel, that is 0.25% of the offset error, 5 dB gain error and 100ps sampling time deviation are added in the 1st and 3rd channels, the output signal spectrum is shown in Fig. 3. The amplitude of the spurs produced by offset mismatch at $\omega_s/4$ and $\omega_s/2$ are −52.64 dB and −83.11 dB respectively. The amplitude of the spurs produced by gain mismatch and time sampling mismatch at $\pm\omega_{in} + \omega_s/4$ and $\omega_s/2 - \omega_{in}$ are −48.72 dB,

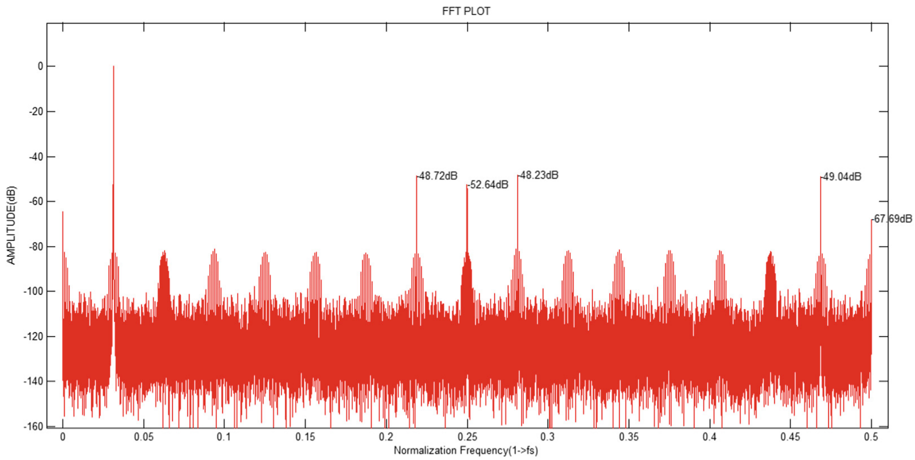

Fig. 2. Output signal spectrum without error in the 1st and 3rd channels

−48.24 dB and −80.08 dB respectively. The SINAD of the whole TIADC system is 44.2959 dB and the ENOB is 7.0651 bits.

When the error of the 1st and 3rd channel is the same as the error of the 2nd channel, that is 0.5% of the offset error, 10 dB gain error and 200ps sampling time deviation are added in the 1st and 3rd channels, the output signal spectrum is shown in Fig. 4. The amplitude of the spurs produced by offset mismatch at $\omega_s/4$ and $\omega_s/2$ are −52.64 dB and −67.71 dB respectively. The amplitude of the spurs produced by gain mismatch and time sampling mismatch at $\pm\omega_{in} + \omega_s/4$ and $\omega_s/2 - \omega_{in}$ are −48.73 dB, −48.24 dB and −49.06 dB respectively. The SINAD of the whole TIADC system is 42.7345 dB and the ENOB is 6.8057 bits.

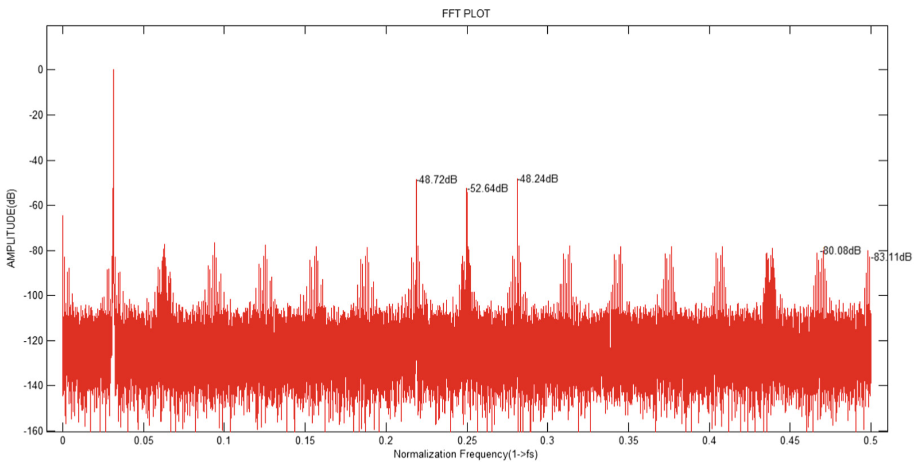

Fig. 3. Output signal spectrum under conditions that the 1st and 3rd channels' error is half of the 2nd channel

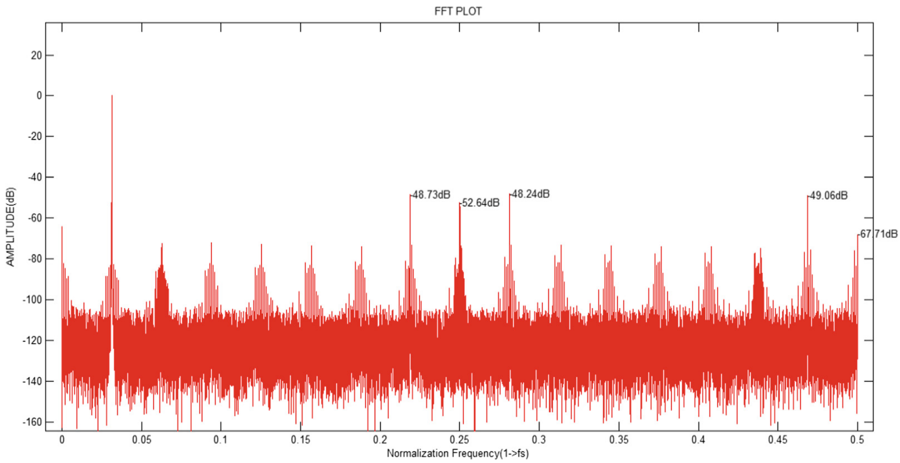

Fig. 4. Output signal spectrum under conditions that the 1st and 3rd channels' error is same as the 2nd channel

From simulation results, it is indicated that when the error of the 1st and 3rd channel is the half of the error of the 2nd channel, the amplitudes of the spurs which frequency at $\omega = \omega_s/2$ decreases almost 15 dB and that at $\omega = \omega_s/2 - \omega_{in}$ decreases almost 30 dB. The comparison of the results under three difference cases is shown in the Table 1.

Table 1. Comparison of results under three cases

	SINAD	ENOB	$\omega = \omega_s/2$	$\omega = \omega_s/2 - \omega_{in}$
No error	42.79 dB	6.8153	−67.69 dB	−49.04 dB
Half of the 2nd channel	44.30 dB	7.0651	−83.11 dB	−80.08 dB
Same as the 2nd channel	42.73 dB	6.8057	−67.71 dB	−49.06 dB

5 Conclusion

Three kinds of inter-channel mismatches in the frequency domain are analyzed in this paper. The influence of the amplitude distribution of the inter-channel errors on the mismatch is also studied. From results of the theoretical analysis and simulation results, it is indicated that not only the errors of each channel determine the frequency of the spurs which produced by mismatches in the output spectrum, but also the amplitude distribution of the inter-channel errors affects the magnitude of the spurs. Especially when the error of the 1st and 3rd channel is the half of the error of the 2nd channel, the amplitudes of part of the spurs decrease almost 15 dB and 30 dB respectively.

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