Green Resource Allocation in Intelligent Software Defined NOMA Networks

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Abstract. Non-orthogonal multiple access (NOMA) with successive interference cancellation (SIC) is a promising technique for fifth generation wireless communications. In NOMA, multiple users can access the same frequency-time resource simultaneously and multi-user signals can be separated successfully with SIC. In this paper, with recent advances in software-defined networking (SDN), an architecture of SDN-NOMA network was proposed and the SDN controller has a global view of the network. We aim to investigate the resource allocation algorithms for the virtual resource blocks (VRB) assignment and power allocation for the downlink SDN-NOMA network. Different from the existing works, here, energy efficient dynamic power allocation in SDN-NOMA networks is investigated with the constraints of QoS requirement and power consumption. The simulation results confirm that the proposed scheme of SDN-NOMA system yields much better sum rate and energy efficiency performance than the conventional orthogonal frequency division multiple access scheme.

Keywords: Energy efficient · NOMA · SDN · Resource allocation

1 Introduction

With the explosive growth of smart mobile devices and the increasing demands for high spectral efficiency in recent years, orthogonal channel access in orthogonal frequency division multiple access (OFDMA) is becoming a limiting factor of spectrum efficiency since each subchannel can only be utilized by at most one user in each time slot [1]. Then, non-orthogonal multiple access (NOMA) has been envisioned as a promising technique to relieve the heavy burden of overloaded traffic in base station (BS) [2]. The improvement in spectral efficiency of NOMA network is significant by allowing multiple users to share the same subchannel in power domain [3]. The capacity region which achieved in NOMA

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is significantly outperforms the orthogonal multiple access schemes by power domain multiplexing at the transmitter and SIC at the receivers [4].

As a key technology in the 5G mobile communication, inter-user interference over each subchannel will be created when multiple users sharing the same subchannel [5]. As a multi-user detection technique, SIC can be applied at the end-user receivers to decode the received signals [6]. The outage performance of NOMA was evaluated in [7], while in [8], the authors investigated the system sum-rate of multiuser NOMA single-carrier systems as well as proposing a suboptimal power allocation and presenting a precoder design. A low-complexity suboptimal algorithm with power proportional factors determination for subchannel multiplexed users was investigated in [9]. By considering imperfect CSI, energy efficiency improvement for a downlink NOMA single-cell network is investigated and an iterative algorithm for user scheduling and power allocation is proposed in [10].

Software-defined networking (SDN) is proposed by Stanford University and has been regarded as a promising network platform which enables the adoption of new technologies and dynamic reconfiguration large scale complex networks [11]. By means of standardized interfaces (e.g., OpenFlow), independent devices of varies vendors can be fast control by the SDN controller [12]. The authors in [13] studied SDN information-centric cellular network virtualization with D2D communication and the subscribers from different mobile virtual network operators can share the virtualized contents. In order to reduce the emission of global greenhouse gas to protect our environment, the research of maximizing the system energy efficiency has been highly attractive [14]. However, most of the existing works considered resource allocation of energy efficiency using SDN only in OFDMA systems. To the best of the authors' knowledge, energy efficient resource allocation for SDN-NOMA networks has not been studied in previous works.

In this paper, we investigate the virtual resource blocks and power allocation respectively in a downlink SDN-NOMA network by considering energy efficiency, quality of service (QoS) requirements, power limits. Based on the novel energy efficient NOMA network optimization framework that we developed, we design a VRB assignment algorithm based on matching theory and a power allocation algorithm with multiple constraints.

2 SDN-NOMA System Model

We consider the downlink of a SDN-based resource sharing system of NOMA network. In the SDN framework, the control plane and the data plane are separated which eases resource management and network optimization. In the data plane, the distributed small cell base stations (SCBSs) which operated by the same or different network operators (service providers) provide data services to the users with different applications. All the transmitters are equipped with a single antenna. The users are uniformly distributed in the coverage of the base stations (BSs). As the high deployed density, the coverage areas of the heterogeneous BSs are overlapped seriously. For the control plane, the SDN controller has a global view of the network, including the traffic demands of users, available wireless resource and the channel status information. As the information of traffic demands arrived, the SDN controller design a resource allocation strategy between users and SCBSs with virtual resource blocks (VRBs).

In SDN-NOMA network, one user can receive signals from the BS through arbitrary VRB and one VRB can be allocated to multiple users at the same time. Since the user j on VRB n causes interference to the other users on the same VRB, each user j adopts SIC after receiving the superposed signals to demodulate the target message. The interference signals caused by user j whose channel gain is better than user u cannot be decoded and will be treated as noise this is because user u with higher channel gain can only decode the signals of user i with worse channel gain. Then, when the transmitted power of user u of SCBS k over VRB n is $p_{k,n}^{u}$, after SIC, the interference for user u caused by other users of the same SCBS k on the same VRB n is given by

$$\widetilde{I}_{k,n}^{u} = \sum_{s \in \left\{ U_k | g_{k,n}^{s} > g_{k,n}^{u} \right\}} a_{k,n}^{s} p_{k,n}^{s} g_{k,n}^{u} \tag{1}$$

where $g_{k,n}^{u}$ is the channel gain from small cell k of user u on VRB n, U_k is the set of users of SCBS k. $a_{k,n}^{u} = 1$ means that user u is allocated to the VRB n of SCBS k and ${\sigma_n}^2$ is the noise variance. Modeling this residual interference as additional AWGN, the received SINR of small cell user $u \in \mathcal{U} = \{1, 2, ..., U\}$ in SCBS k on the nth VRB is given by

$$\gamma_{k,n}^{u} = \frac{a_{k,n}^{u} p_{k,n}^{u} g_{k,n}^{u}}{I_{k,n}^{u} + \widetilde{I}_{k,n}^{u} + \sigma^2} \tag{2}$$

where $I_{k,n}^{u}$ is the interference caused by other SCBSs to user u in SCBS k on VRB n, which is given by

$$I_{k,n}^{u} = \sum_{l \neq k}^{K} b_{l.k} \sum_{s \in U_l} a_{l,n}^{s} p_{l,n}^{s} g_{l,k,n}^{s} \tag{3}$$

where $g_{l,k,n}^{s}$ is the channel gain from small cell l of user s to small cell k on VRB n. $b_{l.k} \in [0, 1]$ is the interference parameters between the SCBSs l and k. $b_{l.k} = 0$ denote the two BSs are operated by different operators and applying different licensed spectrum for direct transmission, otherwise, $b_{l.k} = 1$.

3 Resource Allocation for Energy Efficient Optimization

In this section, the VRB assignment is investigated in the NOMA network and the optimization problem for energy efficient is solved with the constraints of QoS requirements of users and power consumption of BSs.

3.1 Resource Blocks Matching

We assume that all the users can transmit on the VRB n of SCBS k arbitrarily in a SDN-NOMA system. Considering the complexity of decoding and the fairness of users, each VRB can only be allocated to at most D_n users and each user can only occupy at most one VRB of one SCBS. The dynamic matching between the users and the VRBs of SCBSs is considered as a two-sided matching process between the set of $\mathcal{U}$ users and the set of $\mathcal{N}$ VRBs of SCBSs. User u is matched with VRB n of SCBSs k if $a_{k,n,u} = 1$. Based on the channel state information, we assume user u prefers channel n_1 of SCBS k_1 over n_2 of SCBS k_2 if and only if $g_{k_1,n_1,u} > g_{k_2,n_2,u}$. Then, the preference lists of the users can be denoted by

$$\Pr ef\_U = [\Pr ef\_U(1), ..., \Pr ef\_U(u), ... \Pr ef\_U(U)]^T \tag{4}$$

where $\Pr ef\_U(u)$ is the preference list of user u which is in the descending order of channel gains of VRBs of SCBSs. We propose a suboptimal matching algorithm for VRB allocation as follows.

Algorithm 1. Suboptimal Matching Algorithm for VRB Allocation

```
Initialize the matched list $S_{k,n}$ and $S_u$ to denote the number of users matched with VRB $n$ ($\forall n \in \{1, 2, ..., N\}$) of SCBS $k$ ($\forall n \in \{1, 2, ..., K\}$) and the number of VRBs of SCBSs matched with user $u$ ($\forall n \in \{1, 2, ..., U\}$), respectively;
Initialize preference lists $\Pr ef\_U(u)$ for all the users according to channel state information;
Initialize the set of not matched users $S_{U\_F}(u)$ to denote users who have not been matched with a VRB of a SCBS;
while $S_{U\_F}(u) \neq \phi$ do
  for $u = 1$ to $U$ do
    if $S_u < 1$ then
      User $u$ sends a matching request to its most preferred VRB $\widehat{n}$ of SCBS $\widehat{k}$ according to $\Pr ef\_U(u)$;
      if $S_{\widehat{k},\widehat{n}} < D_n$ then
        Set $a_{\widehat{k},\widehat{n},u} = 1$, $S_u = S_u + 1$ and $S_{\widehat{k},\widehat{n}} = S_{\widehat{k},\widehat{n}} + 1$;
      else if $S_{\widehat{k},\widehat{n}} = D_n$ then
        Find the minimum channel gain of users $g_{\widehat{k},\widehat{n},\widehat{u}}$ on channel $\widehat{n}$ of SCBS $\widehat{k}$ and compare it with $g_{\widehat{k},\widehat{n},u}$;
        if $g_{\widehat{k},\widehat{n},\widehat{u}} < g_{\widehat{k},\widehat{n},u}$ then
          Set $a_{\widehat{k},\widehat{n},u} = 1$, $a_{\widehat{k},\widehat{n},\widehat{u}} = 0$, $S_u = S_u + 1$, and $S_{\widehat{u}} = S_{\widehat{u}} - 1$;
        else
          Remove VRB $\widehat{n}$ of SCBS $\widehat{k}$ from the $\Pr ef\_U(u)$ and find the next $(\widehat{k}, \widehat{n})$ of user $u$ according to $\Pr ef\_U(u)$.
        end if
      end if
    end if
  end for
end while
```

3.2 Total Capacity and Power Consumption

We denote the total bandwidth of VRBs with each SCBS as $\mathcal{B}$. Using the Shannon's capacity formula, we can write the capacity of user $u \in \mathcal{U} = \{1, 2, ..., U\}$ in SCBS k on the nth VRB as

$$r_{k,n}^{u} = \frac{B}{N} \log_2(1 + \gamma_{k,n}^{u}). \tag{5}$$

The capacity of user u can be written as

$$r^{u} = \sum_{k=1}^{K} \sum_{n=1}^{N} r_{k,n}^{u}, \forall u \in \mathcal{U}. \tag{6}$$

The total capacity of all the users of the BS is

$$R_{tot} = \sum_{u=1}^{U} r^{u}. \tag{7}$$

In order to specify the QoS of users, we let R^u be the QoS requirement in terms of minimum capacity of user u which is thus given as

$$C1 : r^{u} \geq R^{u}, \forall u \in \mathcal{U}. \tag{8}$$

Denote by p_k and P_{tot} the transmit power of SCBS k and the total power consumption of the BSs respectively, which can be written as

$$p_k = \sum_{u \in U_k} \sum_{n=1}^{N} p_{k,n}^{u}, \forall k \in \mathcal{K}. \tag{9}$$

and

$$P_{tot} = \sum_{k=1}^{K} (p_k + p_C^k) \tag{10}$$

where p_C^k accounts for the circuit power consumption of SCBS k. The power constraints of SCBS k is denoted by P_k, which can be given by

$$C2 : p_k \leq P_k, \forall k \in \mathcal{K}. \tag{11}$$

Let EE denote the energy efficient which is the ratio of the total data capacity to the corresponding total power consumption. It is given as

$$EE = \frac{R_{tot}}{P_{tot}}. \tag{12}$$

3.3 Optimization Problem Formulation

In this subsection, when considering all constraints, the utility function is expressed as

$$\begin{aligned} &\max EE = \tfrac{R_{tot}}{P_{tot}} \\ &s.t. C1, C2 \\ &C3 : \sum_{k=1}^{K} \sum_{n=1}^{N} a_{k,n}^{u} \leq 1, \forall u \in \mathcal{U} \\ &C4 : \sum_{u=1}^{U} a_{k,n}^{u} \leq D_n, \forall k \in \mathcal{K}, u \in \mathcal{U} \end{aligned} \tag{13}$$

where the constraint $C1$ ensures the QoS of users; $C2$ is the maximum transmit power of SCBS k; $C3$ denote that user u is allocated at most one VRB of SCBSs; and $C4$ ensures one VRB of each SCBS can be allocated to at most D_n users.

4 Solution of the Optimization Problem

In this section, we introduce a transformation of objective function (14) which is a non-convex function. We focus on the equivalent objective function with the constrains above.

4.1 Equivalent Objective Function

We define the optimal energy efficient EE^{opt} as

$$EE^{opt} = \frac{R_{tot}(p^*)}{P_{tot}(p^*)} = \max \frac{R_{tot}(p)}{P_{tot}(p)} \tag{14}$$

where p^* denotes the optimal power allocation that yields EE^{opt}. We introduce Theorem 1 as follows.

Theorem 1 (Ghoussoub-Preiss). *The optimal energy efficient EE^{opt} can be reached if and only if*

$$\begin{aligned} &\max R_{tot}(p) - EE^{opt} P_{tot}(p) = R_{tot}(p^*) - EE^{opt} P_{tot}(p^*) = 0 \\ &for R_{tot}(p) \geq 0, P_{tot}(p) \geq 0 \end{aligned} \tag{15}$$

□

Proof: The proof of the theorem is omitted due to space limitations. A similar detailed proof can be found in [15]. Then, in the rest of this paper, we can only focus on the function $R_{tot}(p) - EE^{opt} P_{tot}(p)$ which is a non-convex mixed integer programming problem.

4.2 Iterative Algorithm for Power Allocation

According to Theorem 1, the (non-convex) optimization problem (15) can be rewritten in the more tractable form

$$\begin{aligned} &\max_{p} R_{tot}(p) - \eta_{EE} P_{tot}(p) \\ &s.t. C1, C2 \end{aligned} \tag{16}$$

where the definition of η_{EE} is as shown in Algorithm 2. It is the ratio of the data capacity to the corresponding total power consumption in each iteration of the main loop. The above proposed approach based on KKT condition for solving the EE optimization problem in (17) can be summarized in Algorithm 2.

Algorithm 2. Iterative Power Allocation Algorithm

```
Initialize the $a_{k,n}^{u}$ using suboptimal Algorithm 1;
Initialize $p_{k,n}^{u}$ using equal power allocation;
Initialize the maximum number of iterations $L_{\max}$ and the maximum tolerance $\delta$;
Set current maximum value of energy efficiency $\eta_{EE} = \frac{R_{tot}}{P_{tot}}$ and iteration index $l = 0$;
repeat
    Obtain the allocation policies of power $\hat{p}_{k,n}^{u}$ in the current iteration according to (34);
    Calculate the value of $\hat{R}_{tot}$ and $\hat{P}_{tot}$ by solving (7) and (10);
    if $\hat{R}_{tot} - \eta_{EE}\hat{P}_{tot} \prec \delta$ then
        Convergence=1
        obtain $p_{k,n}^{*u} = \hat{p}_{k,n}^{u}$ and $EE^{opt} = \frac{\hat{R}_{tot}}{\hat{P}_{tot}}$.
    else
        Convergence=0
        set $\eta_{EE} = \frac{\hat{R}_{tot}}{\hat{P}_{tot}}$ and $l = l + 1$.
    end if
    Update Lagrangian multipliers of $\lambda$, $\beta$ by solving (35);
until Convergence or certain stopping criteria is met
```

Let $\omega_{k,n}^{u} = a_{k,n}^{u} p_{k,n}^{u}, \forall u \in \mathcal{U}, n \in \mathcal{N}, k \in \mathcal{K}$; then we can rewrite the SINR of user u in SCBS k on VRB n as

$$\gamma_{k,n}^{u} = 1 + \frac{\omega_{k,n}^{u} g_{k,n}^{u}}{\sum\limits_{l \neq k}^{K} b_{l,k} \sum\limits_{s \in U_l} \omega_{l,n}^{s} g_{l,k,n}^{s} + \sum\limits_{s \in \left\{ U_k | g_{k,n}^{s} > g_{k,n}^{u} \right\}} \omega_{k,n}^{s} g_{k,n}^{u} + \sigma^2} \tag{17}$$

To satisfy the series of constraints, the Lagrange function of the problem (17) can be expressed as

$$
\begin{aligned}
&F(\lambda,\beta,p) = \max L(\lambda,\beta,p)\\
&= R_{tot}(p) - \eta_{EE} P_{tot}(p) + \sum_{u=1}^{U} \lambda_u \left(\sum_{k=1}^{K} \sum_{n=1}^{N} r_{k,n}^{u} - R^u \right) + \sum_{k=1}^{K} \beta_k \left(P_k - \sum_{u \in U_k} \sum_{n=1}^{N} p_{k,n}^{u} \right)\\
&= \sum_{u=1}^{U} \left[\left(\tfrac{B}{N} + \lambda_u \right) \sum_{k=1}^{K} \sum_{n=1}^{N} \log_2 (1 + \frac{\omega_{k,n}^{u} g_{k,n}^{u}}{\sum\limits_{l \neq k}^{K} b_{l,k} \sum\limits_{s \in U_l} \omega_{l,n}^{s} g_{l,k,n}^{s} + \sum\limits_{s \in \left\{ U_k | g_{k,n}^{s} > g_{k,n}^{u} \right\}} \omega_{k,n}^{s} g_{k,n}^{u} + \sigma^2}) \right] \cdot\\
&- \sum_{k=1}^{K} \left[(\eta_{EE} + \beta_k) \sum_{u=1}^{U} \sum_{n=1}^{N} \omega_{k,n}^{u} \right] - \left(\sum_{u=1}^{U} \lambda_u R^u - \eta_{EE} \sum_{k=1}^{K} P_C + \sum_{k=1}^{K} \beta_k P_k \right)
\end{aligned}
\tag{18}
$$

where λ, β are the Lagrange multiplier vectors for the constraints in (17). Taking the first order derivation of $F(\lambda,\beta)$ with respect to $\omega_{k,n}^{u}$, we can get the optimal power allocation as

$$
p_{k,n}^{u*} = \frac{\omega_{k,n}^{u*}}{a_{k,n}^{u}} = \frac{\frac{B}{N} + \lambda_u}{\ln 2 (\eta_{EE} + \beta_k)} - \frac{\sum\limits_{l \neq k}^{K} b_{l,k} \sum\limits_{s \in U_l} \omega_{l,n}^{s} g_{l,k,n}^{s} + \sum\limits_{s \in \left\{ U_k | g_{k,n}^{s} > g_{k,n}^{u} \right\}} \omega_{k,n}^{s} g_{k,n}^{u} + \sigma^2}{g_{k,n}^{u}} \cdot \tag{19}
$$

Based on the subgradient method [16], the master dual problem in (17) can be solved by

$$
\begin{aligned}
\lambda_u^{l+1} &= \left[\lambda_u^{l} - \varepsilon_\lambda^{l} \left(\sum_{k=1}^{K} \sum_{n=1}^{N} r_{k,n}^{u} - R^u \right) \right]^{+}, \forall u \in \mathcal{U}\\
\beta_k^{l+1} &= \left[\beta_k^{l} - \varepsilon_\beta^{l} \left(P_k - \sum_{u \in U_k} \sum_{n=1}^{N} p_{k,n}^{u} \right) \right]^{+}, \forall k \in \mathcal{K}
\end{aligned}
\tag{20}
$$

5 Simulation Results and Discussions

In this section, simulation results are given to evaluate the performance of the proposed algorithms. For the simulation, the number of SCBS is $K = 5$. The maximum transmit power and circuit power consumption of each SCBS is set as 3 *Watt* and 0.5 *Watt* respectively. The maximum of users can be allocated to each VRB n of SCBS k is $D_n = 2$. The QoS requirement of each user is $R_u = 3$ bps/Hz. The number of VRB is depend on the number of users and they are nearly full matched in the SDN-NOMA system.

In Fig. 1, the performance of EE is evaluated versus the number of users with different D_n which is maximum number of matched users of each VRB. It is shown that, with the increase of users, the value of EE decreases. And for the same value of user number, the larger value of D_n leads to larger value of energy efficient. This is because a larger D_n leads to more selection of users and bigger bandwidth of each VRB. And it is shown that the energy efficient in NOMA is better than the average EE in OFDMA.

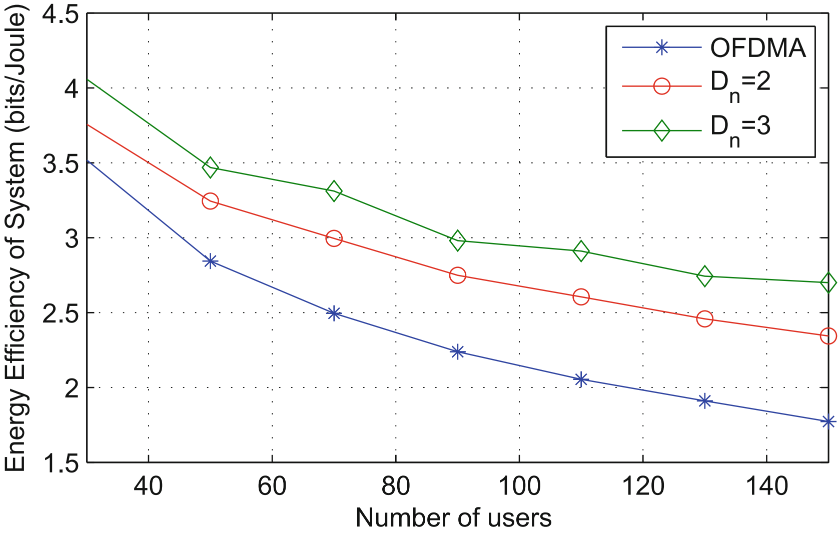

Fig. 1. Energy efficient performance versus user number with different D_n.

6 Conclusions

We investigated the dynamic resource allocation in downlink SDN-NOMA networks. We developed a framework in NOMA network by means of SDN technology. We considered the energy efficient of the network as optimization function. We proposed a suboptimal VRB assignment algorithm based on the two-side matching method. By considering minimum QoS requirement and maximum power constraint, we formulated the power allocation as a mixed integer programming problem as the considered problem was transformed into an equivalent problem with a tractable iterative solution. The mathematical analysis and simulation results demonstrated that the effectiveness of the proposed algorithms.

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