

# A Two-Layered Game Approach Based Relay's Source Selection and Power Control for Wireless Cooperative Networks

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**Abstract.** Cooperative relay communication has become a promising technology to extend the network coverage and enhance the system performance. To avoid the interference among the relays assisted the same source and maximize the relay utility in multisource multirelay networks, we propose the two-layered game based distributed algorithm, which jointly considers power control and the relay's source selection. Power control and relay's source selection are formulated as a general non-cooperative game and an evolutionary game, respectively. By using the alternate iterations between the non-cooperative game and the evolutionary game, the proposed distributed algorithm can effectively suppress the interference and choose the optimal source. Simulation results are presented to analyze the performance of the proposed distributed algorithm.

**Keywords:** Power control · Relay's source selection  
Evolutionary game · Two-layered game · Distributed algorithm

## 1 Introduction

Cooperative relay communication has become an emerging transmitting strategy to extend the network coverage and enhance the system performance [1, 2]. The goal of cooperative transmission in wireless networks is to increase transmission diversity at less transmission power. The power control and relay's selection have attracted much research attention. Many papers have focused on cooperative communication for wireless networks over the past decade. In [3], the authors propose a relay-ordering based scheme, which can dynamically select relay and adjust power allocation based on the SNR and channel condition. The authors in [4] analyze the relay selection problem: when to cooperate and whom to cooperate with. By using a game approach, the authors of [5] study the distributed relay selection in randomized cooperation. Power control with a pricing is discussed in [6, 7]. In [8], the authors investigate distributed relay selection and power control for cooperative communication networks, which consists of one

source and multiple relays. Evolutionary game [9] is a useful tool to address the relay's selection problem in changeable environment. The authors in [10] propose an energy-aware dynamic cooperative partner selection for relay-assisted cellular networks, and the evolutionary game theory is first introduced to resolve the dynamic cooperative partner-selection problem with incomplete private information.

Different from the existing literatures that focus on the source's selection of relays, we concentrate on relay's selection of sources in multi-relay and multi-source networks. Because the relays occupy the frequency resource, they should select optimal source and determine their own transmit power, which can effectively suppress the interference and choose the optimal source. Therefore, for this multi-relay and multi-source networks, there are two main questions:

- (1) Among all source nodes, which is the optimal source node for relay nodes?
- (2) Once the optimal source node is selected, how the relay node determines the transmit power?

As an answer to these two questions, we present the two-layered game based distributed algorithm, which jointly considers the relays' source selection and power control. The proposed distributed algorithm can effectively suppress the interference and choose the optimal source.

## 2 System Model

We consider a cooperative relay networks, which consists of source node  $s \in \mathcal{S} \triangleq \{1, \dots, S\}$ , destination  $d \in \mathcal{D} \triangleq \{1, \dots, D\}$ , and relay node  $r \in \mathcal{R} \triangleq \{1, \dots, N\}$ . Furthermore, it is assumed that each relay node can only select one source to help with its feasible transmit power  $P_r$  and  $S$  is equal to  $D$ . Therefore, there are  $S$  source-to-destination pairs in the cooperative relay networks.

We can denote the path gain between node  $i$  and node  $j$  by  $G_{i,j}$ .  $\sigma^2$  represents the variance of additive white Gaussian noise (AWGN) at each node, which is assumed to be constant. As in [11], we employ the AF protocol in this paper. The SNR with relay's help at node  $d$  can be expressed as

$$\gamma_{s,r,d} = \frac{P_s P_r G_{s,r} G_{r,d}}{\sigma^2 (P_s G_{s,r} + P_r G_{r,d} + \sigma^2)}, \quad (1)$$

where  $P_s$  is the transmit power of source node  $s$ .

It is assumed that the maximal-ratio combining (MRC) detector is applied to node  $d$ . Then, we can get the combined rate as follows:

$$R_{s,r,d} = \log_2(1 + \gamma_{s,d} + \sum_{i \in L_s} \gamma_{s,i,d}), \quad (2)$$

where  $\gamma_{s,d} = \frac{P_s G_{s,d}}{\sigma^2}$  is the SNR of the direct link of source  $s$ , and  $L_s$  denotes the set of relay nodes that assist the source  $s$ .

We design a relay's utility function based on its contribution to source's rate. We adopt the similar Sharply method used in coalition game to guarantee fairness among relay nodes. Thus, relay node  $r$ 's utility function can be expressed as

$$u'_r = \alpha_s(R_{s,r,d} - R_{s,-r,d}), \quad (3)$$

where  $\alpha_s$  denotes relay node's gain per unit rate at the MRC output from source node  $s$ , and  $R_{s,-r,d}$  represents the source  $s$ 's transmission rate without relay node  $r$ 's help.

Substituting Eq. (2) into (3), we can get

$$u'_r = \alpha_s \log_2 \left( 1 + \frac{\gamma_{s,r,d}}{1 + \gamma_{s,d} + \sum_{i \in L_s, i \neq r} \gamma_{s,i,d}} \right). \quad (4)$$

It can be observed that relay node  $r$ 's utility is dependent on not only its own transmit power  $P_r$ , but also other relay nodes' selection and transmit power.

Each relay node's utility is a monotonically increasing function of its own transmit power. Therefore, each relay node has the incentive to transmit signal with its maximal transmit power, which results in the energy inefficiency. It is necessary to add a cost function with respect to transmit power, and then the relay node  $r$ 's payoff, or net utility function can be written as follows:

$$u_r = \alpha_s \log_2 \left( 1 + \frac{\gamma_{s,r,d}}{1 + \gamma_{s,d} + \sum_{i \in L_s, i \neq r} \gamma_{s,i,d}} \right) - c_r P_r, \quad (5)$$

where  $c_r$  is relay  $r$ 's cost per unit transmit power.

### 3 Problem Formulation

#### A. Relay's power control

The power control optimization problem can be formulated as a non-cooperative game, which is expressed as

$$\begin{aligned} \max \quad & u_r \quad \forall r \\ \text{s.t.} \quad & 0 \leq P_r \leq \bar{P}_r, \end{aligned} \quad (6)$$

where  $\bar{P}_r$  is the power upper bound for relay node  $r$ .

(1) Existence of the Equilibrium for the power control game:

By using the payoff function's concavity, we will proof the existence of Nash Equilibrium (NE) for power control game.

*Theorem 1:* A NE exists in game  $G = [\mathcal{R}, P(r), u_r]$ , if for all  $r \in \mathcal{R}$

- (1).  $P(r)$  is a non-empty, convex and compact subset of some Euclidean space.
- (2).  $u_r$  is continuous and quasi-concave in  $P_r$ .

*Proof:* For any  $r \in \mathcal{R}$ , each relay  $r$  has a strategy space of the transmit power for helping the selected source. For any  $P_r \in P(r) = [0, \bar{P}_r]$ , it is easy to proof that relay  $r$ 's power space  $P(r)$  is non-empty and compact. By utilizing the definition of the convex set, given any  $p1, p2 \in P(r)$  and any  $\epsilon \in [0, 1]$ , we have  $0 \leq \epsilon p1 \leq \epsilon \bar{P}_r$  and  $0 \leq (1 - \epsilon)p2 \leq (1 - \epsilon)\bar{P}_r$ . Based on the above two in equations, we can get  $0 \leq \epsilon p1 + (1 - \epsilon)p2 \leq \bar{P}_r$ . Thus, the power space  $P(r)$  is convex.

Then, we will show that the payoff function  $u_r$  is concave with respect to  $P_r$ . We can get the payoff function  $u_r$ 's second-order derivation

$$\frac{\partial^2 u_r}{\partial^2 P_r} = -\frac{\alpha_s}{\ln 2} \frac{P_s G_{s,r} G_{r,d} T + \sigma^2 T^2 \frac{\partial \gamma_{s,r,d}}{\partial P_r}}{(\Gamma'_{-r} + \gamma_{s,r,d}) \sigma^2 T^2 + P_s P_r G_{s,r} G_{r,d} T}, \tag{7}$$

where  $T$  is a positive value defined in Eq. (10). Then, we can derive the first-order derivation of  $\gamma_{s,r,d}$  as

$$\frac{\partial \gamma_{s,r,d}}{\partial P_r} = \frac{P_s G_{s,r} G_{r,d} (P_s G_{s,r} + \sigma^2)}{\sigma^2 T^2}, \tag{8}$$

which is a positive value.

Therefore, we can draw a conclusion that the payoff function of relay node  $r$  is concave. Theorem 1 follows.

(2) The optimal  $P_r^*$ :

Taking the derivative of Eq. (5), we can get

$$\frac{\partial u_r}{\partial P_r} = \frac{\alpha_s}{\ln 2} \frac{1}{1 + \gamma_{s,d} + \Gamma_{-r} + \gamma_{s,r,d}} \frac{\partial \gamma_{s,r,d}}{\partial P_r} - c_r, \tag{9}$$

where  $\Gamma_{-r} = \sum_{i \in L_s, i \neq r} \gamma_{s,i,d}$ .

Substituting Eq. (8) into (9), we can get

$$\frac{\partial u_r}{\partial P_r} = \frac{\alpha_s}{\ln 2} \frac{P_s G_{s,r} G_{r,d} (P_s G_{s,r} + \sigma^2)}{(\Gamma'_{-r} + \gamma_{s,r,d}) \sigma^2 T^2 + P_s P_r G_{s,r} G_{r,d} T} - c_r, \tag{10}$$

where  $T = P_s G_{s,r} + P_r G_{r,d} + \sigma^2$  and  $\Gamma'_{-r} = 1 + \gamma_{s,d} + \Gamma_{-r}$ .

Let Eq. (10) be zero, we can find that the equation can be rewritten as one quadratic function with respect to relay node  $r$ 's power  $P_r$ , which satisfies the following expression

$$AP_r^2 + BP_r + C = 0, \tag{11}$$

where

$$A = G_{r,d}^2 (P_s G_{s,r} + \sigma^2 \Gamma'_{-r}), \tag{12}$$

$$B = (P_s G_{s,r} + \sigma^2) (P_s G_{s,r} G_{r,d} + 2\sigma^2 G_{r,d} \Gamma'_{-r}), \tag{13}$$

and

$$C = \sigma^2 \Gamma'_{-r} (P_s G_{s,r} + \sigma^2)^2 - \frac{\alpha_s}{c_r \ln 2} P_s G_{s,r} G_{r,d} (P_s G_{s,r} + \sigma^2). \tag{14}$$

According to this function's properties, the necessary condition for the existence of one positive solution is  $C < 0$ . Then, we define relay's revenue-to-cost-ratio (RCR) as  $\rho_{s,r}$ , which should satisfy the following requirement

$$\rho_{s,r} > \rho_{s,r}^0, \quad (15)$$

where  $\rho_{s,r} = \frac{\alpha_s}{c_r}$  and  $\rho_{s,r}^0 = \frac{\ln 2 \sigma^2 \Gamma'_{-r}(P_s G_{s,r} + \sigma^2)}{P_s G_{s,r} G_{r,d}}$ . This means that if  $\rho_{s,r}$  is smaller than the threshold  $\rho_{s,r}^0$ , the relay node  $r$  will not help source node  $s$ .

Solving this quadratic function (11), we can get

$$\hat{P}_r = \frac{\sqrt{B^2 - 4AC} - B}{2A}. \quad (16)$$

Under relay's power constraint, the optimal power  $P_r^*$  is determined by

$$P_r^* = \begin{cases} 0, & \rho_{s,r} \leq \rho_{s,r}^0; \\ \hat{P}_r, & \hat{P}_r \leq \bar{P}_r; \\ \bar{P}_r, & \hat{P}_r > \bar{P}_r. \end{cases} \quad (17)$$

### B. Relay's source selection

Note that the payoff may be different for relay node  $r$  if it selects different source node  $s$  to help, even it transmits at the same power. Thus, to get a maximal payoff, the relays have the incentive to select the best source node.

Then, we can formulate this problem as an evolutionary game. Let  $n_s$  denote the number of relay nodes selecting source node  $s$ , and  $N = \sum_{s=1}^S n_s$ . The proportion of relay nodes selecting the strategy  $s$  can be denoted by  $x_s = n_s/N$ . The replicator dynamics of relay's selection game can be defined as

$$\frac{\partial x_s(t)}{\partial t} = \dot{x}_s(t) = \delta x_s(t)(u_s - \bar{u}), \quad (18)$$

where  $\delta$  controls the evolution speed, and

$$u_s = \frac{\sum_{i \in L_s} u_i}{n_s} \quad (19)$$

and then

$$\bar{u} = \sum_{s=1}^S x_s u_s. \quad (20)$$

With the evolution of all relay nodes, the evolutionary game will converge to the stable evolutionary strategy (ESS) which can be determined by solving such set of equations

$$\dot{x}_s = 0, \quad \forall s. \quad (21)$$

### C. Two-layered game's distributed algorithm

We can combine the non-cooperative game and evolutionary game into one two-layered game. By using the alternate iterations between the non-cooperative game of power control and the evolutionary game of source selection, the proposed distributed algorithm can effectively suppress the interference and choose the optimal source.

The distributed algorithm is described as follows:

- (1) Initial: For each relay, the transmit power and source are randomly chosen.
- (2) Power control game begins:
- (3) Each player adopts the optimal power according to Eq. (17) at time  $t$ .  
if  $\max_{r \in \mathcal{R}} \|P_r^*(t) - P_r^*(t+1)\| > \varepsilon$ , then  
Let  $t = t + 1$  and return to step (3).  
Else, go to step (4).
- (4) Power control game ends.
- (5) Relay's selection game begins:
- (6) The relays begin source selections according to Eqs. (19) and (20),  
if  $u_s < \bar{u}$ , then  
if  $\text{rand}() < \|\frac{\bar{u}-u_s}{\bar{u}}\|$ ,  
Give up the strategy  $s$  and choose the strategy  $k$ , where  $k = \arg \max_{i \in \mathcal{S}} u_i$ ,  
and return to step (6).  
Else if  $u_s = \bar{u} \quad \forall s$ , then go to step (7).
- (7) Source selection game ends.
- (8) Judge whether it comes the NE of the whole two-layered game.
- (9) If  $\max_{r \in \mathcal{R}} \|P_r^*(t) - P_r^*(t+1)\| > \varepsilon$ ,  
Return to step (2), and repeat this process.  
Else, go to step (10).
- (10) This two-layered game ends.

## 4 Simulation Results and Analysis

In this section, simulation results are presented to evaluate the proposed algorithm. In this simulation, all basic parameters are set as follows:  $N = 100$ ,  $S = D = 2$ ,  $\alpha_s = 20$ ,  $\bar{P}_s = 1 \quad \forall s$ ,  $G_{s,d} = 1 \quad \forall (s, d)$ ,  $\sigma^2 = 1$ ,  $G_{s,r} = 1 \quad \forall (s, r)$ ,  $G_{r,d} = 1 \quad \forall r, d$ , and  $c_r = 0.1 \quad \forall r$ .

### A. source's direct channel parameter $G_{s,d}$ impact

We consider two sources' direct links with  $G_{s,d} = 1$  and  $G_{s,d} = 4$ . From Fig. 1, it can be observed that each relay's payoff is same after certain iterations under different  $G_{s,d}$ . However, the number of relay nodes helping source 1 and 2 is different. Thus, source node's direct link condition has the effect on source selection, but has no impact on relay node's payoff. Furthermore, we can see that the worse the source node's direct link condition is, the larger the probability that the relay nodes select this source node.

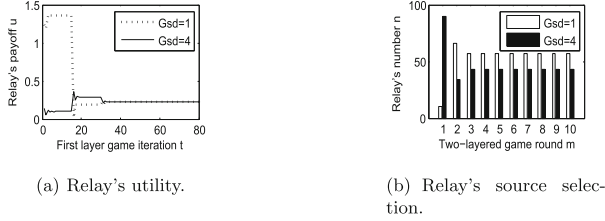


Fig. 1. Source's direct link's  $G_{s,d}$  impact

B. relay's channel parameter  $G_{s,r}$  impact

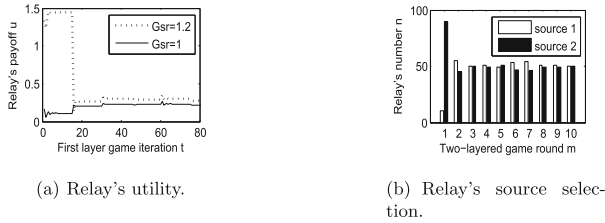


Fig. 2. Relay's channel condition's  $G_{s,r}$  impact

We set  $G_{s,r} = 1.2, r = 1, \dots, 50$  (group 1) and  $G_{s,r} = 1, r = 51, \dots, 100$  (group 2) to analyze its impact. From Fig. 2, we can see that the payoff of relay node from group 1 is higher than that of relay node from group 2. Furthermore, when the two-layered game reach the NE, there are about 50 relays helping source 1 and 2, respectively. Therefore, relay node's channel condition affects the relay node's payoff but not the source selection. Furthermore, we find that a better channel condition of relay node will result in a higher payoff.

### 5 Conclusions

In this paper, we propose a two-layered game approach based a distributed algorithm for relay's source selection and power control in multi-source and multi-relay networks. Simulation results demonstrate that the worse the source node's direct link condition is, the larger the probability that the relay nodes select this source node, and relay node with better channel conditions can get a higher payoff as compared with other relays.

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