Lorentzian Norm Based Super-Resolution Reconstruction of Brain MRI Image

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Abstract. Nowadays, SRR (super resolution image reconstruction) technology is a very effective method in improving spatial resolution of images and obtaining high-definition images. The SRR approach is an image late processing method that does not require any improvement in the hardware of the imaging system. In the SRR reconstruction model, it is the key point of the research to choose a proper cost function to achieve good reconstruction effect. In this paper, based on a lot of research, Lorenzian norm is employed as the error term, Tikhonov regularization is employed as the regularization term in the reconstruction model, and iteration method is employed in the process of SRR. In this way, the outliers and image edge preserving problems in SRR reconstruction process can be effectively solved and a good reconstruction effect can be achieved. A low resolution MRI brain image sequence with motion blur and several noises are used to test the SRR reconstruction algorithm in this paper and the reconstruction results of SRR reconstruction algorithm based on L2 norm are also be used for comparison and analysis. Results from experiments show that the SRR algorithm in this paper has better practicability and effectiveness.

Keywords: MRI image · Super resolution image reconstruction
Lorentzian norm · Regularization · Iteration

1 Introduction

In order to achieve a good visual effect, people always want to get clear images with high quality. In the current CCD imaging systems, for increasing image's spatial resolution by improving performance and structure of the imaging system often results in additional expenditure and noise. In addition, LR images can only be obtained in some image acquisition processes. In order to get HR images, super resolution image reconstruction (SRR) technology has become an effective method in image later processing process.

Super resolution image reconstruction technology refers to the process of image processing: from low resolution image sequences to reproduce one high resolution images. This technique can reconstruct one high resolution images by collecting

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additional information between sub pixels of low resolution image sequence taken from the same scene. By this method, the limitations of the intrinsic frequency of the imaging process can be overcome. Earlier work on super resolution image reconstruction was done by Tsai and Huang [1] in 1984, which is a SRR method of single-image in frequency-domain using DFT. Then, a serious of frequency domain super-resolution reconstruction method based on discrete-cosine-transform (DCT) [2], wavelet-transform [3] is presented. Because the frequency domain reconstruction effect is not ideal, in recent years, the research area has moved to the spatial domain. The typical spatial super-resolution image reconstruction model includes: IBP method [4], non-uniform- interpolation method [5], MAP method [6], ML method [7], POCS method [8], mixed-MAP/POCS method [9], and adaptive-filtering method [10], etc. Based on these algorithms, combining reconstruction and registration algorithm [11], multi-spectral and color image SRR algorithm [12], compressed sensing reconstruction algorithm [13], and Example-based SRR algorithm have also been proposed [14].

In this paper, the SRR algorithm based on Lorenzian norm [15] is introduced, and is applied to the reconstruction of low resolution MRI brain image sequence. Results from experiments show that the proposed algorithm is more efficient than that of L_2 norm based SRR algorithm in solving the outliers and edge preserving problems in super-resolution reconstruction process.

The structure of this paper includes: Sect. 2 introduces basic observation-model of HR image. Section 3 gives the SR image reconstruction algorithm based on Lorenzian norm and Tikhonov regularization. Section 4 gives the reconstruction results of two reconstruction methods based on a series of low resolution MRI brain images. Section 5 gives the conclusion of this paper.

2 Image Observation Model

The process to get low resolution image from high resolution image includes down sampling, blurring, warping, and noise addition. The image-observation-model [9] is

$$y_k = DBM_k x + n_k, \quad k = 1, 2, \ldots, P \tag{1}$$

Where x represents the initial HR image, the size of it is $L_1N_1 \times L_2N_2$, L_1 represents the horizontal direction down sampling factor, and L_2 represents the vertical direction down sampling factor. y_k is low-resolution-image sequence with the size of $N_1 \times N_2$. M_k represents motion matrix with moving, rotation, zoom motion and the size of it is $L_1N_1L_2N_2 \times L_1N_1L_2N_2$, B represents the blur matrix and the size of it is $L_1N_1L_2N_2 \times L_1N_1L_2N_2$, D represents sampling matrix, the size of it is $N_1N_2 \times L_1N_1L_2N_2$, n_k represents Gauss white noise, the size of it is $N_1N_2 \times 1$.

Formula (1) can also be expressed as

$$y_k = H_k x + n_k, \quad k = 1, 2, \ldots, P \tag{2}$$

where $H_K = DBM_K$ can be regarded as a composite degenerate operator.

3 SR Image Reconstruction Algorithm

The image reconstruction process of SR is estimating one HR image from LR image sequence taken from same scene using complementary information between sub-pixels. Super resolution image reconstruction is ill-posed-inverse-process, and reconstruction model is very sensitive. Small noise and error can lead to serious distortion of the reconstructed-image.

The key problem of reconstruct HR image from LR image-sequence is to get a proper cost function. From formulas (1) and (2) we get

$$\hat{x} = argmin\left[\sum\nolimits_k \rho(y_k - H_k x) + \alpha R(x)\right] \tag{3}$$

Where x is the initial HR image, $\hat{x}$ is the reconstructed SRR image, y_k is LR image sequence, *ρ(•)* is error estimation term (fidelity of the solution), *R(x)* is the regularization term (smoothness of the solution), $\boldsymbol{\alpha}$ is regularization-parameter (control the trade-off between $\boldsymbol{\rho}$ and *R(x)*).

3.1 Error Estimate Term

At present, the error-estimation-term used in SRR is the error estimate [6] based on the *Lp* norm. If the value of *P* is 1, it becomes the L_1 norm, and when the value of *P* is 2, it becomes the L_2 norm. A common problem with these methods is the over smoothing and edge ringing effects of reconstructed images.

The Lorenzian norm and its influence function are defined as

$$\rho_{LOR}(x) = \log\left[1 + \frac{1}{2}\left(\frac{x}{T}\right)^2\right] \tag{4}$$

$$\rho'_{LOR}(x) = \frac{2x}{2T^2 + x^2} \tag{5}$$

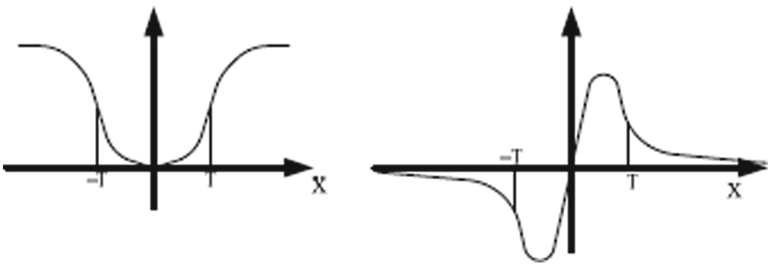
The Lorenzian norm is used as error estimation term, which concentrates the advantages of L_1 norm and L_2 norm, and can effectively suppress ringing and noise effect, especially salt & pepper noise.

$$\widehat{x} = argmin\left[\sum\nolimits_k \rho_{LOR}(y_k - H_k x) + \alpha R(x)\right] \tag{6}$$

where T is a soft threshold, when $x \leq T$, it is the L_2 norm, and when $x > T$, it is saturated. Therefore, only those values that are not greater than T are valid (Fig. 1).

3.2 Regularization Term

The regularization-term in the super-resolution-image-reconstruction model can restrict the solution space and improve the solution's stability by introducing a pri-knowledge.



(a) Lorentzian norm function (b) Lorentzian norm influence function

Fig. 1. Lorentzian norm and influence function.

The regularization method used in this paper is the effective and commonly used Tikhonov regularization.

$$\widehat{x} = argmin\left[\sum\nolimits_k \rho_{LOR}(y_k - H_k x) + \alpha\|Cx\|_2^2\right] \tag{7}$$

where C is a two-dimensional Laplacian operator. α is a regularization parameter that adjusts the ratio between the regularization and the error estimate term. If the value of alpha is too large, the solution will deviate from the real solution, and if the alpha value is too small, the solution will lead to instability. Therefore, the determination of an appropriate alpha value is a key factor in achieving better reconstruction effectiveness.

3.3 Iteration Reconstruction Method

In this paper, iterative reconstruction method is employed for the realization of SRR algorithm.

$$x_{n+1} = x_n + \lambda\left[\sum\nolimits_k \left(H_k^T y - (H_k^T H_k + \alpha(x_k) C^T C) x_k\right)\right] \tag{8}$$

Where λ means iteration step value.

4 Experiment

In this paper, the low-resolution MRI brain image sequence is used to test the SR image-reconstruction algorithm. Meanwhile, the SRR algorithm based on L_2 norm is also used for comparison of the reconstruction effect.

PSNR and RMSE are used for evaluating the performance of the reconstruction methods quantitatively.

$$\text{PSNR} = 10log_{10}\left(\frac{255^2}{MSE}\right) \tag{10}$$

$$\text{MSE} = \frac{1}{L_1N_1 \times L_2N_2}\sum\nolimits_{i=1}^{L_1N_1}\sum\nolimits_{j=1}^{L_2N_2}\left(\hat{x}(i,j) - x(i,j)\right)^2 \tag{11}$$

Where $L_1N_1L_2N_2$ is the size of the initial HR image, $\mathbf{x}$ represents initial HR image, and $\hat{x}$ represents the reconstructed SRR image.

In the experiments, two reconstruction methods are used for reconstruct SR image from LR image sequences and two types of noises is added. LR image sequence in the experiment is taken from the original HR MRI brain image (see Fig. 2) by global motion (assuming the motion parameters are known), 3×3 Gauss kernel blur, the factor of 2 down sampling of the horizontal direction and vertical direction, and the addition of two different types of noise (Gauss noise, salt & pepper noise), see Fig. 3. Based on the two reconstruction methods, the results of super resolution image reconstruction for different noise cases are shown in Figs. 4 and 5.

Fig. 2. Initial MRI brain image.

(a) noiseless (b) Gaussian noise (c) salt & pepper noise

Fig. 3. LR images with different type of noise.

The results from experiments show that the reconstruction effect of the super resolution image reconstruction method in this paper is better than that of based on the L_2 norm in the objective and subjective aspects. The reconstructed HR image with the algorithm in this paper has better visual and edge preservation effect. The reconstruction results show that this paper's algorithm can keep better robustness and adaptability in different kinds of noise, especially for the case of salt & pepper noise.

(a) noiseless (b) Gaussian noise (c) salt & pepper noise

Fig. 4. Reconstructed results for different type of noise by L_2 norm SRR.

(a) noiseless (b) Gaussian noise (c) salt & pepper noise

Fig. 5. Reconstructed results for different type of noise by Lorentzian norm SRR.

5 Conclusion

In order to solve the edge preserving and outliers problem in SR image reconstruction, the reconstruction algorithm based on Tikhonov regularization, Lorenzian error norm, and iterative method is employed. The results of the experiments show that the low resolution MRI brain images with different noise conditions are well reconstructed and the robustness and adaptability of this algorithm is better compared to L2 norm based reconstruction algorithm–not just remove the noise of different types, and also has better edge preserving effect. However, this method has the problem of large computation and slow processing speed. The work of next step is to improve the algorithm by considering adaptive regularization in the motion estimation model.

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