Spectrum Pricing in Condition of Normally Distributed User Preference

Li Wang[1], Lele Cheng[1], Feng Li[1(✉)], Xin Liu[2], and Di Shen[1]

[1] Zhejiang University of Technology, Hangzhou 310023, Zhejiang, China
{liwang2002,fenglzj}@zjut.edu.cn, 478708892@qq.com, 814120631@qq.com
[2] Dalian University of Technology, Dalian 116024, Liaoning, China
liuxinstar1984@dlut.edu.cn

Abstract. During secondary user's dynamic access to authorized spectrum, a key issue is how to ascertain an appropriate spectrum price so as to maximize primary system's benefit and satisfy secondary user's diverse spectrum demands. In this paper, a scheme of pricing-based dynamic spectrum access is proposed. According to the diverse qualities of idle spectrum, the proposal applies Hotelling game model to describe the spectrum pricing problem. Firstly, establish a model of spectrum leasing, among which the idle spectrum with different qualities forms a spectrum pool. Then, divide the idle spectrum into equivalent width of leased channels, which will be uniformly sold in order. Secondary users can choose proper channels to purchase in the spectrum pool according to their spectrum usage preferences which are subject to normal distribution and affected by the spectrum quality and market estimation. This paper analyzes the effect of spectrum pricing according to the primary system's different tendencies to spectrum usage and economic income.

Keywords: Spectrum pricing · Cognitive radio · User preference
Spectrum quality

1 Introduction

With the rapid development of wireless communication technology and the establishment of next-generation 5g communication standard, high-quality idle spectrum is more scarce which has become one of the bottlenecks restricting the development of wireless communication technology [1]. Cognitive radio which is based on dynamic spectrum access has attracted more and more attention of academe and engineering recent years [2]. Various kinds of emerging network technology have begun to adopt dynamic spectrum detection and dynamic spectrum access to improve the efficiency of spectrum utilization. In the process of dynamic spectrum access, primary users owning licensed spectrum can lease the idle channels to secondary user to gain incomes. For primary users, how to identify an optimal channel pricing to maximize its own profit has become a significant issue. In this paper, we directly price the idle spectrum of authorized users according to the secondary user's diverse preferences. The spectrum

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pricing scheme has a prior estimate to the spectrum market. Compared with the spectrum auction, it doesn't need many overheads and improves the convenience of the spectrum access.

Spectrum trading provides an efficient way for secondary users to dynamically access licensed bands while the financial gains can encourage primary users to lease unused spectrum temporarily. Generally, the participants can perform the deal by auction-based method or pricing-based method. The spectrum auction mechanism can be divided into many kinds according to different application circumstances, such as trust-based auction which relaxes the credit limit appropriately in return for a higher economic efficiency to balance the honesty and the efficiency [3,4]. On the other hand, to lower the overhead and time cost for spectrum pricing, pricing-based spectrum trading has also been widespread concerned either [5,6].

In this paper, we investigate how to price the spectrum when heterogeneous spectrum and stochastic secondary user's preference are under consideration. A concept of spectrum pool is introduced to facilitate the following spectrum deal. A secondary spectrum customer will pick a high-quality channel for usage when its capital is ample or wide band is required to support essential service. We adopt Hotelling model which is proper to describe the product pricing issue in heterogeneous market. By analyzing the secondary user's preference parameter, an iterative algorithm for spectrum pricing is obtained by fixing the Nash equilibrium. Numerical results are further provided to evaluate how the pricing parameters affect the primary system's profits.

2 System Model

Suppose the idle spectrum leased by the primary system consists a spectrum sharing pool, where the spectrum can be divided into many uniform channels for selling. Besides, the qualities of these channels are not homogeneous. For high-quality channels, the secondary users suffers lower channel fading or adjacent channel interference. Thus, secondary users choose these channels according to their diverse preferences. The preference parameter is determined by the channel quality and channel price.

2.1 Utility Functions

In this paper, we consider the spectrum trading is performed without auction activities. During the course, primary systems have no prior knowledge of the secondary customer's spectrum preference. In spectrum trading, the utility function of a secondary user can be expressed as

$$U = \theta s - p, \tag{1}$$

where θ denotes the secondary user's preference, s denotes the channel quality and p denotes the channel price. In the spectrum sharing pool, it is assumed that two kinds of channels with diverse qualities can be chosen by secondary users as shown in Fig. 1. We use s_1 to denote the channel quality of high-quality channel and s_2 the low-quality channel. Then, we have $s_1 > s_2 > 0$. Here, different channel qualities means various transmission capacities. Furthermore, we suppose the secondary user's preference parameter θ is subject to normal distribution expressed as $g(\theta)$. θ locates in the region of $[\theta_L, \theta_H]$, and ρ is the corresponding probability distribution function denoted as $\rho = G(\theta)$. We adopt θ_0 to express the non-preference parameter of a cognitive user which means no demand difference existing between the high-quality channel and low-quality channel. Then, it can be calculated as $\theta_0 = \frac{p_1 - p_2}{s_1 - s_2}$, where p_1 and p_2 represent the two kinds of channels' prices. When a secondary user's spectrum preference θ_i is higher than θ_0, the user prefers to choose the high-quality channel. Otherwise, it would rather to choose the low-quality channel to lease.

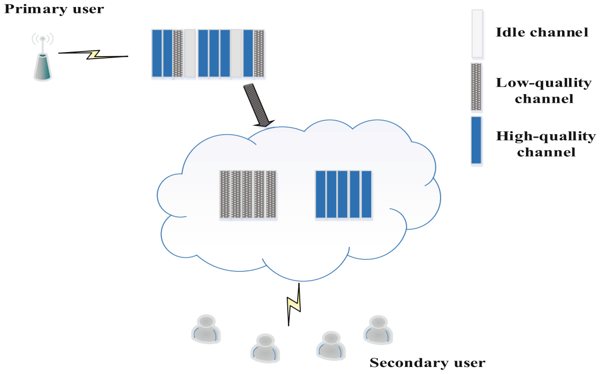

Fig. 1. Spectrum pool

2.2 Spectrum Pricing

Secondary user's preference parameter is considered to be non-uniform and obey normal distribution in practical application. Figure 2 shows the density curve of the standard normal distribution. The probability density can be given as

$$\varphi(x) = \frac{1}{\sqrt{2\pi} e^{-\frac{x^2}{2}}} \tag{2}$$

Then, the distribution function is

$$f(x) = \int \varphi(x) = \frac{1}{2\pi} \int_{-a}^{a} e^{-\frac{t^2}{2}}. \tag{3}$$

According to [7], $f(a)$ can be simplified as

$$f(a) = \sqrt{1 - e^{\frac{-a^2}{1.6058}}}. \tag{4}$$

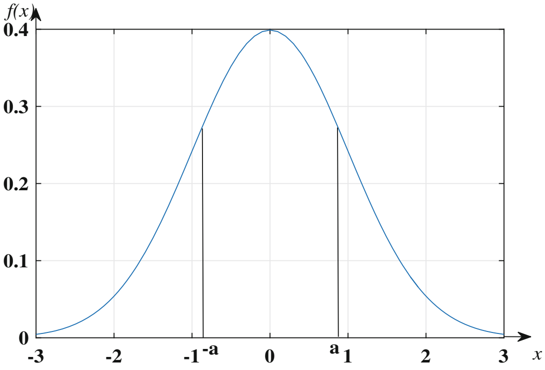

Fig. 2. Standard normal distribution

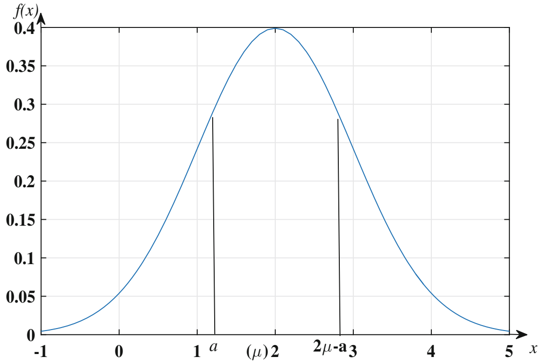

Fig. 3. General normal distribution

Thus, the probability can be approximately calculated in given region $[-a, a]$.

The conclusion can also be applied to the case of general normal distribution as shown in Fig. 3. When the distribution mean is μ, the probability calculated approximately in $[a, 2\mu - a]$ is obtained as

$$f(a) = \sqrt{1 - e^{-\frac{(u-a)^2}{1.6058}}}. \tag{5}$$

Furthermore, as shown in Fig. 4, the secondary customer whose preference parameters θ locates in $[\theta_L, \theta_0]$, will purchase low-quality channels. The user with preference parameters $\theta \in [\theta_0, \theta_H]$ chooses a high-quality channel.

Then, in order to obtain the specific solution, divide the red shadow part in Fig. 4 into two parts, where we have $\theta_0' = \theta_0 + \mu$ and $\theta_H - \theta_H' = \mu$.

According to the various regions of high-quality and low-quality channels in Fig. 4, we achieve the following equations

$$p_H = \frac{1}{2} \times (\sqrt{1 - e^{-\frac{(\mu-\theta_0)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\theta_H-\mu)^2}{1.6058}}}), \tag{6}$$

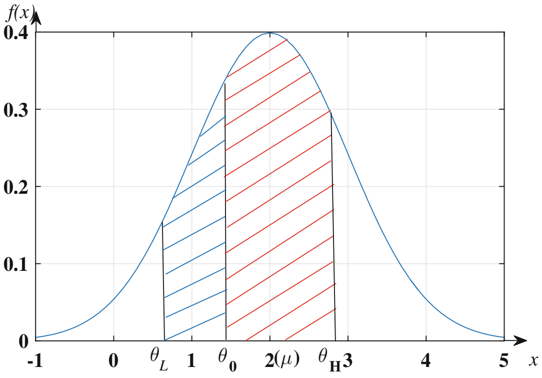

Fig. 4. Divide the channels into two kinds of qualities

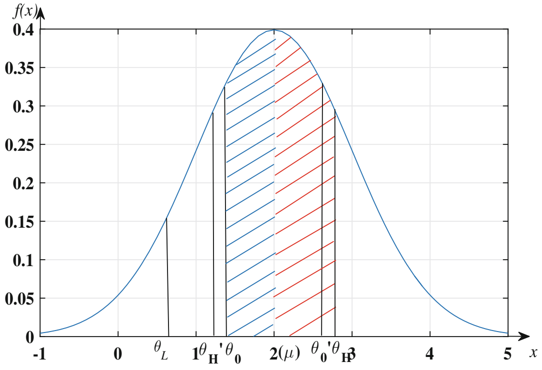

Fig. 5. Divide the high-quality channels into two parts

$$p_H = \frac{1}{2} \times (\sqrt{1 - e^{-\frac{(\mu-\theta_L)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\mu-\theta_0)^2}{1.6058}}}). \tag{7}$$

Assume the marginal cost of the primary user is related to the quality of the channel which can be expressed as $c_i = \alpha s_i (i = 1, 2)$, where α is the marginal cost factor. Formulating the problem by Berland game model, the profit functions of system H and L can be given as

$$\begin{aligned} \pi_H(p_1, p_2) &= N(p_1 - \alpha s_1) \times \frac{1}{2} \times \\ &= (\sqrt{1 - e^{-\frac{(\mu-\theta_0)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\theta_H-\mu)^2}{1.6058}}}), \end{aligned} \tag{8}$$

$$\begin{aligned} \pi_L(p_1, p_2) &= N(p_2 - \alpha s_2) \times \frac{1}{2} \times \\ &= (\sqrt{1 - e^{-\frac{(\mu-\theta_L)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\mu-\theta_0)^2}{1.6058}}}), \end{aligned} \tag{9}$$

where N is the number of secondary users. The non-preference parameter θ_0 is not fixed and changing in $[\theta_L, \theta_H]$. Besides, whether $\theta_0 > \mu$ will affect the deductions. The profits obtained above is under the situation when θ_0 locates at the left side of μ. Similarly, when θ_0 locates at the right side of μ, the corresponding profit functions can be deduced as

$$\begin{aligned}\pi_H(p_1, p_2) &= N(p_1 - \alpha s_1) \times \frac{1}{2} \times \\ &= (\sqrt{1 - e^{-\frac{(\theta_H - \mu)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\theta_0 - \mu)^2}{1.6058}}}),\end{aligned} \tag{10}$$

$$\begin{aligned}\pi_L(p_1, p_2) &= N(p_2 - \alpha s_2) \times \frac{1}{2} \times \\ &= (\sqrt{1 - e^{-\frac{(\mu - \theta_L)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\theta_0 - \mu)^2}{1.6058}}}).\end{aligned} \tag{11}$$

Based on the marginal utility function, we can achieve the optimal channel pricing as

$$\begin{aligned}p_1^{(t+1)} &= p_1 + \beta \times N \times \frac{1}{2} \times \\ &= (\sqrt{1 - e^{-\frac{(\mu - \theta_0)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\theta_H - \mu)^2}{1.6058}}}) + \\ &\beta \times N \times e^{-\frac{(\mu - \theta_0)^2}{1.6058}} \times (2\alpha - p_1^{(t)}) \times \frac{2\mu - 2p_1^{(t)} - 2p_2^{(t)}}{6.4232\sqrt{1 - e^{-\frac{(\mu - \theta_0)^2}{1.6058}}}},\end{aligned} \tag{12}$$

$$\begin{aligned}p_2^{(t+1)} &= p_2 + \beta \times N \times \frac{1}{2} \times \\ &= (\sqrt{1 - e^{-\frac{(\mu - \theta_L)^2}{1.6058}}} + \sqrt{1 - e^{-\frac{(\mu - \theta_0)^2}{1.6058}}}) + \\ &\beta \times N \times e^{-\frac{(\mu - \theta_0)^2}{1.6058}} \times \frac{1}{6.4232} \times (2p_2^{(t)} - 2p_1^{(t)} + 2\mu)\end{aligned} \tag{13}$$

Similarly, we can obtain the optimal channel pricing when $\theta_0 > \mu$.

3 Numerical Results

In this section, numerical results are provided to testify the effects of the proposed pricing method. In the dynamic access networks, we suppose the idle spectrum is controlled by the licensed users and we ignore the internet interference caused by adjacent cells. The secondary users who are eager to access the spectrum must participant in the spectrum trading and pay for the cost to the primary systems. As the proposed pricing solution is an iterative algorithm, we thus give the initial spectrum pricing for two kinds qualities of channels to be $s_1 = 2$, $s_2 = 1$, $N = 100$, $\alpha = 1$, $\mu = 2.2$, $\beta \in (0, 0.028)$. The cognitive user's preference locates at $[1, 3]$ which means $\theta_L = 1$, $\theta_H = 3$. Furthermore, since the

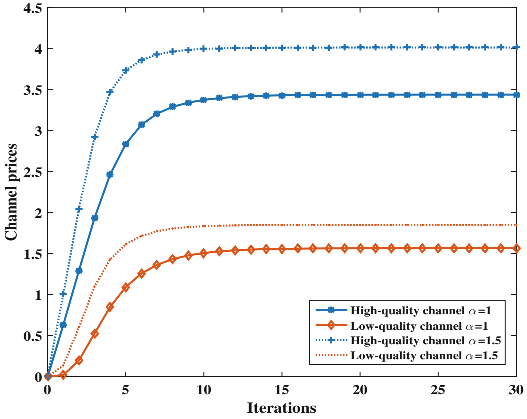

Fig. 6. Channel prices with different marginal factors

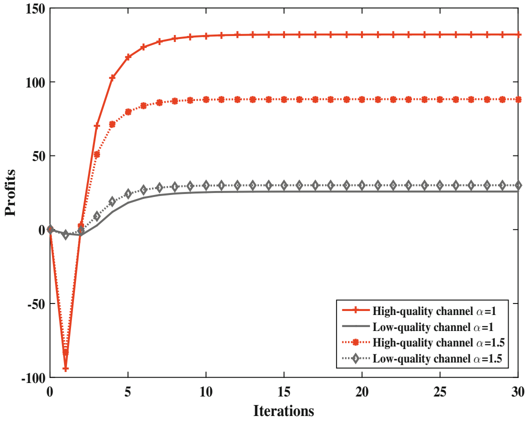

Fig. 7. System profits with different marginal factors

proposed pricing method is an iterative algorithm, we set the initial spectrum pricing for two kinds qualities of channels as $p_i^{(0)} = 0.01$. Then, we give the performances of the channel prices and system profits of the proposed method in the following tests.

In Fig. 6, we present the performances of the channel prices obtained in this paper with different marginal factors α. We can achieve from the figure that the optimal price of high-quality channel is much higher than that of low-quality channel. Furthermore, the channel pricing rises with increasing marginal factor α since higher marginal cost needs to be compensated for the primary system. We also can get the iterative algorithm converges very fast which will attain a stable value within 15 iterations. In the counterpart, Fig. 7 gives the performances of the system profits in optimal pricing with different marginal factors. We can obtain from Fig. 7 that the system profits decrease with increasing marginal factor α which means the close relationship between spectrum cost and system profit. Besides, it is obvious that the profit received on the high-quality channel overcomes that on the low-quality channel. It can be understandable that the primary system expects to reap more profits by its more excellent products.

4 Conclusion

In this paper, we investigate how to price the differential spectrum in respond to secondary users' stochastic selection preferences. The main contribution of this paper lies in that we introduce the Hotelling game model to formulate and address the differential spectrum pricing. In the paper, we assume the idle spectrum is collected and leased to potential secondary users centrally, then a centralized spectrum pricing by the primary system can be proceeded. Two kinds of spectrum is considered in the system model where it is foreseen the high-quality channels can incur more profits for the primary system. A preference factor is introduced to describe the secondary user's selection tendency on the spectrum leased.

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