

# Joint Mode Selection and Beamformer Optimization for Full-Duplex Cellular Systems

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**Abstract.** We investigate a novel mode selection scheme for full-duplex (FD) cellular system where the base station (BS) and the user equipments (UEs) are equipped with multiple-input multiple-output (MIMO) antennas. We consider that FD is utilized at the BS, i.e. it enables simultaneous transmission and reception at the same frequency band, while UEs work in the conventional half-duplex (HD) way. Since FD system can not always outperform HD system due to residual self interference (RSI) at the base station, the mode selection is mainly determined by system performance. To address this issue, a joint mode selection and beamformer optimization problem with power constraints is formulated to achieve the maximal weighted sum rate (WSR). On account of the non-convex of original problem, a heuristic algorithm based on decoupling is proposed, which decomposes the original problem into two sub-problems. One is mode selection sub-problem and the other one is mean square error (MSE) minimization sub-problem. By means of simulation, the proposed algorithm shows the ability to choose the mode with greater performance in achievable rate.

**Keywords:** Full-duplex · Cellular system · Self-interference  
Mode selection · WSR

## 1 Introduction

System rate improvement has become one of the most significant characteristics in 5G cellular communication systems since the great demand for data transmission. Most modern communication systems apply bi-directional communication, which in traditional way needs two different channels for inverse directions, typically using time division duplex (TDD) or frequency division duplex (FDD) technique, to provide isolation between transmission and reception. As we all known, these communication systems utilize half-duplex (HD) technology. Full-duplex (FD) technology can inconceivably increase system throughput and spectral efficiency by enabling fully

utilizing of both time and frequency resource, which has attracted considerable interest both in industry and academia.

The major obstruction of applying full-duplex technology in real communications is the self-interference (SI) which is generated by simultaneous transmission and reception on the same end. A large amount of research has considered the problem of SI in FD communications by studying various system architectures and self-interference cancellation (SIC) techniques to mitigate SI signal. Generally, SIC techniques can be divided into three types: antenna, analog and digital cancellation. Analog cancellation tries to remove SI before the deteriorated signals are digitized, whereas digital cancellation eliminates the SI after the signals are digitized. Generally one approach alone usually still leaves a majority amount of SI. Recent researches in SIC have made a great progress in reducing SI into a low level. New antenna designs, together with analog and digital cancellation, are applied to eliminate most SI. Digital cancellation in transceiver design was considered in [1] along with taking actual implementation facts of FD systems into account. Moreover, certain experimental results were displayed in [2–4] which can reach 50–80 dB of SI cancellation. A combination of analog and digital cancellation method was proposed in [5], which showed 85 dB cancellation effect over a 20 MHz WLAN channel. Besides the hardware based research, most theoretical works were stemming from array processing technique, that is known as beamforming [6, 7].

However, due to channel estimation errors, the SI cannot be eliminated fully. Hence, the residual self-interference (RSI) remains at a high level, which needs to be cancelled via signal processing at baseband [8]. In other words, RSI cancellation results will dominate that whether the FD system outperforms HD system or not. Therefore, in ideal FD systems, a mode selection method is needed to between FD mode and traditional HD mode. To address this issue, a joint mode selection and beamformer optimization problem is formulated. To find the maximal objective which is the weighted sum data rate, a heuristic algorithm based on decoupling is proposed, which decomposes the original problem into two sub-problems. One is mode selection sub-problem and the other one is weighted mean square error (WMSE) minimization sub-problem.

The remainder of the paper is organized as follows: a multiuser FD MIMO system model and the joint optimization problem are presented in Sect. 2. The specific analysis of the proposed heuristic algorithm is illustrated in Sect. 3. Section 4 provides numerical simulations of overall system performance. Section 5 concludes our work.

## 2 System Model and Problem Formulation

Consider a multiuser FD MIMO system as depicted in Fig. 1, in which the base station (BS) communicates with  $K$  uplink (UL) users and  $J$  downlink (DL) users simultaneously. Both the BS and users are equipped with multiple antennas. For simplicity, we assume the same antennas at transmit and receive ends.  $\mathbf{H}_k^{UL}$  represents channel from the  $k$ -th UL user to BS and  $\mathbf{H}_j^{DL}$  represents the channel from BS to the  $j$ -th DL user. We assume FD only applied at BS, users work at HD mode because of the hardware

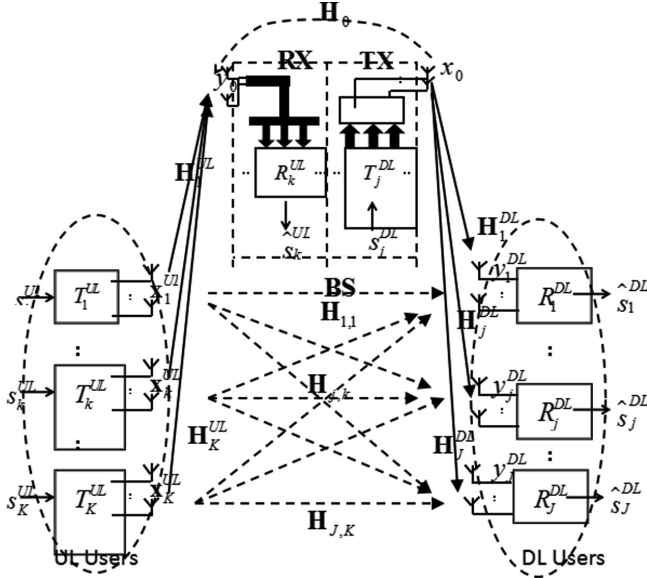


Fig. 1. FD cellular system model

complexity. Thus SI exists only at BS, and SI channel is denoted as  $\mathbf{H}_0$ .  $\mathbf{H}_{j,k}$  denotes the interference channel between UL and DL users.

The signal transmitted by the  $k$ -th UL user and the  $j$ -th DL user are denoted as  $\mathbf{s}_k^{UL}$  and  $\mathbf{s}_j^{DL}$ , respectively, which are assumed with independent identical distribution and unit power. Denoting the transmit filters for transmit signals as  $\mathbf{T}$  and the receive filters for the received data as  $\mathbf{R}$ . The signal received by the BS and the  $j$ -th DL user are denoted as

$$\mathbf{y}_0 = m \left[ \sum_{k=1}^K \mathbf{H}_k^{UL} \mathbf{T}_k^{UL} \mathbf{s}_k^{UL} + \sqrt{\eta} \mathbf{H}_0 \sum_{j=1}^J \mathbf{T}_j^{DL} \mathbf{s}_j^{DL} + \mathbf{n}_0 \right] + (1-m) \left[ \sum_{k=1}^K \mathbf{H}_k^{UL} \mathbf{T}_k^{UL} \mathbf{s}_k^{UL} + \mathbf{n}_0 \right] \quad (1)$$

$$\mathbf{y}_j^{DL} = \mathbf{H}_j^{DL} \sum_{j=1}^J \mathbf{T}_j^{DL} \mathbf{s}_j^{DL} + \sum_{k=1}^K \mathbf{H}_{j,k} \mathbf{T}_k^{UL} \mathbf{s}_k^{UL} + \mathbf{n}_j^{DL} \quad (2)$$

where  $m \in \{0, 1\}$  is the mode selection parameter. When  $m = 1$ , FD mode is selected, otherwise HD mode is selected.  $\eta$  is the RSI factor. Meanwhile,  $\mathbf{n}_0$  and  $\mathbf{n}_j^{DL}$  are additive white Gaussian noise at receivers.

As is shown in (1), the first term contains the inter-user interference; the second term is SI. The second term in (2) is the inter-channel interference. Since FD mode cannot keep outperforming HD mode since RSI exists at the BS, the mode selection parameter  $m$  is obviously determined by the performance of the system. Thus, in order to achieve the great performance of this FD MIMO cellular system, we should propose

an optimized design of transmit and receive filters to deal with those interference and to maximize system rate.

Since the value of  $m$  could only be 0 or 1, the covariance matrices of noise plus interference are written as

$$\mathbf{C}_k^{UL} = m \left[ \sum_{i=1, k \neq i}^K \mathbf{H}_i^{UL} \mathbf{T}_i^{UL} (\mathbf{T}_i^{UL})^H (\mathbf{H}_i^{UL})^H + \sum_{j=1}^J \eta \mathbf{H}_0 \mathbf{T}_j^{DL} \mathbf{T}_j^{DL} \mathbf{H}_0^H + \mathbf{I} \right] \\ + (1 - m) \left[ \sum_{i=1, k \neq i}^K \mathbf{H}_i^{UL} \mathbf{T}_i^{UL} (\mathbf{T}_i^{UL})^H (\mathbf{H}_i^{UL})^H + \mathbf{I} \right] \quad (3)$$

$$\mathbf{C}_j^{DL} = \sum_{k=1}^K \mathbf{H}_{j,k} \mathbf{T}_k^{UL} (\mathbf{T}_k^{UL})^H (\mathbf{H}_{j,k})^H + \sum_{i=1, i \neq j}^J \mathbf{H}_i^{DL} \mathbf{T}_i^{DL} \mathbf{T}_i^{DL} \mathbf{H}_i^{DL} + \mathbf{I} \quad (4)$$

where  $\mathbf{I}$  is the identity matrix.

The received signals  $\mathbf{R}_k^{UL}$  and  $\mathbf{R}_j^{DL}$  are filtered by receiving beamformers. Thus, the estimated signal at the BS is given as  $\hat{\mathbf{s}}_k^{UL} = (\mathbf{R}_k^{UL})^H \mathbf{y}_0$ . According to [9], the lower bound of the achievable rate of uplink and downlink users under Gaussian signaling are expressed as

$$I_k^{UL} = \log_2 |\mathbf{I} + \mathbf{H}_k \mathbf{T}_k^{UL} (\mathbf{T}_k^{UL})^H (\mathbf{H}_k^{UL})^H (\mathbf{C}_k^{UL})^{-1}| \quad (5)$$

$$I_j^{DL} = \log_2 |\mathbf{I} + \mathbf{H}_j^{DL} \mathbf{T}_j^{DL} (\mathbf{T}_j^{DL})^H (\mathbf{H}_j^{DL})^H (\mathbf{C}_j^{DL})^{-1}| \quad (6)$$

We try to maximize weighted system sum rate. In general, the rate of uplink and downlink can not be simply added together because they are of different importance. Hence, two weight factors  $\mu_k^{UL}$  and  $\mu_j^{DL}$  are introduced here to get system sum rate given by

$$I = \sum_{k=1}^K \mu_k^{UL} I_k^{UL} + \sum_{j=1}^J \mu_j^{DL} I_j^{DL} \quad (7)$$

Therefore, the joint problem with mode selection and beamformer optimization can be formulated as

$$\max_{m, \mathbf{T}, \mathbf{R}} \quad \sum_{k=1}^K \mu_k^{UL} I_k^{UL} + \sum_{j=1}^J \mu_j^{DL} I_j^{DL} \\ s.t. \quad tr\{\mathbf{T}_k^{UL} (\mathbf{T}_k^{UL})^H\} \leq P_k \\ \sum_j tr\{\mathbf{T}_j^{DL} (\mathbf{T}_j^{DL})^H\} \leq P_T \quad (8)$$

where  $P_k$  is power constraint for UL user, and  $P_T$  denotes the power constraint for BS.

### 3 Algorithm of Optimization

According to [9], based on the relationship between weighted sum rate (WSR) and weighted minimum mean squared error (WMMSE) problems for FD cellular system, the MMSE receive beamformer applied at BS is expressed as

$$\mathbf{R}_k^{UL} = (\mathbf{T}_k^{UL})^H (\mathbf{H}_k^{UL})^H [\mathbf{H}_k^{UL} \mathbf{T}_k^{UL} (\mathbf{T}_k^{UL})^H (\mathbf{H}_k^{UL})^H + \mathbf{C}_k^{UL}]^{-1} \quad (9)$$

Substitute (9) into  $\hat{\mathbf{s}}_k^{UL} = (\mathbf{R}_k^{UL})^H \mathbf{y}_0$  and  $\mathbf{E}_k^{UL} = \mathbb{E}[(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H]$ , the MSE matrix is written as

$$\mathbf{E}_k^{UL} = [\mathbf{I} - (\mathbf{T}_k^{UL})^H (\mathbf{H}_k^{UL})^H (\mathbf{C}_k^{UL})^{-1} \mathbf{H}_k^{UL} \mathbf{T}_k^{UL}]^{-1} \quad (10)$$

It is obviously that the relationship between achievable rate and MMSE can be denoted as

$$I_k^{UL} = \log_2 |(\mathbf{E}_k^{UL})^{-1}| \quad (11)$$

Similarly, the MMSE beamformer, the MSE matrix and the achievable rate of  $j$ -th DL user are as follows,

$$\mathbf{R}_j^{DL} = (\mathbf{T}_j^{DL})^H (\mathbf{H}_j^{DL})^H [\mathbf{H}_j^{DL} \mathbf{T}_j^{DL} (\mathbf{T}_j^{DL})^H (\mathbf{H}_j^{DL})^H + \mathbf{C}_j^{DL}]^{-1} \quad (12)$$

$$\mathbf{E}_j^{DL} = [\mathbf{I} - (\mathbf{T}_j^{DL})^H (\mathbf{H}_j^{DL})^H (\mathbf{C}_j^{DL})^{-1} \mathbf{H}_j^{DL} \mathbf{T}_j^{DL}]^{-1} \quad (13)$$

$$I_j^{DL} = \log_2 |(\mathbf{E}_j^{DL})^{-1}| \quad (14)$$

Now we can formulate the WMMSE problem as

$$\begin{aligned} \min_{\mathbf{m}, \mathbf{T}, \mathbf{R}} \quad & \sum_{k=1}^K \text{tr}\{\mathbf{W}_k^{UL} \mathbf{E}_k^{UL}\} + \sum_{j=1}^J \text{tr}\{\mathbf{W}_j^{DL} \mathbf{E}_j^{DL}\} \\ \text{s.t.} \quad & \text{tr}\{\mathbf{T}_k^{UL} (\mathbf{T}_k^{UL})^H\} \leq P_k \\ & \sum_j \text{tr}\{\mathbf{T}_j (\mathbf{T}_j^{DL})^H\} \leq P_T \end{aligned} \quad (15)$$

where  $\mathbf{W}_k^{UL}$  and  $\mathbf{W}_j^{DL}$  are weight matrix. The WSR and WMMSE problems are equivalent as long as carefully choose weights, which are denoted as

$$\mathbf{W}_k^{UL} = \mu_k^{UL} (\mathbf{E}_k^{UL})^{-1} / \ln 2 \quad (16)$$

$$\mathbf{W}_j^{DL} = \mu_j^{DL} (\mathbf{E}_j^{DL})^{-1} / \ln 2 \quad (17)$$

After elaborately choose MSE weights as listed in (16) and (17), the KKT conditions of the WSR and WMMSE problems can be satisfied simultaneously. Therefore the original problem (8) can be settled by solving WMMSE problem (15). Here, we propose a heuristic algorithm by decoupling problem (15) into two sub-problems: beamformer optimization and mode selection.

### 3.1 Beamformer Optimization

The optimal transmit beamformer  $\mathbf{T}_k^{UL}$  of each UL user can be calculated by utilizing the Lagrange method. Then the optimal  $\mathbf{T}_k^{UL}$  is expressed as

$$\mathbf{T}_k^{UL} = (\mathbf{X}_k^{UL} + \lambda_k \mathbf{I})^{-1} (\mathbf{H}_k^{UL})^H (\mathbf{R}_k^{UL})^H \mathbf{W}_k^{UL} \tag{18}$$

where  $\mathbf{X}_k^{UL} = (\mathbf{H}_k^{UL})^H (\mathbf{R}_k^{UL})^H \mathbf{W}_k \mathbf{R}_k^{UL} \mathbf{H}_k^{UL}$  and  $\lambda_k$  is the Lagrange multiplier, which can be obtained by taking the singular value decomposition of  $\mathbf{X}_k = \mathbf{U}_k \mathbf{\Lambda}_k (\mathbf{U}_k)^H$ .

Rewrite the power constraint in problem (15) as

$$\begin{aligned} \text{tr}\{\mathbf{T}_k^{UL} (\mathbf{T}_k^{UL})^H\} &= \sum_{k=1}^K \frac{g_{ki}}{(\lambda_k + \Delta_{ki})^2} \\ &= P_k \end{aligned} \tag{19}$$

where  $g_{ki}$  is the  $i$ -th diagonal element of  $(\mathbf{U}_k)^H (\mathbf{H}_k^{UL})^H (\mathbf{R}_k^{UL})^H \mathbf{W}_k (\mathbf{W}_k)^H \mathbf{R}_k^{UL} \mathbf{H}_k^{UL} \mathbf{U}_k$  and  $\Delta_{ki}$  denotes the  $i$ -th diagonal element of matrix  $\mathbf{\Lambda}_k$ . After that  $\lambda_k$  is calculated using bisection method.

Similarly, we can obtain close form solutions of the optimal transmit filters  $\mathbf{T}_j^{DL}$  under the sum-power constraint as follows

$$\mathbf{T}_j^{DL} = \alpha \bar{\mathbf{T}}_j^{DL} \tag{20}$$

where

$$\alpha = \sqrt{\frac{P_T}{\sum_j \text{tr}\{\bar{\mathbf{T}}_j^{DL} (\bar{\mathbf{T}}_j^{DL})^H\}}} \tag{21}$$

and  $\bar{\mathbf{T}}_j^{DL}$  is computed as

$$\bar{\mathbf{T}}_j^{DL} = (\mathbf{X}_j + \frac{\sum_k \text{tr}\{(\mathbf{R}_j^{DL})^H \mathbf{W}_j^{DL} \mathbf{R}_j^{DL}\}}{P_T})^{-1} (\mathbf{H}_j^{DL})^H (\mathbf{R}_j^{DL})^H \mathbf{W}_j^{DL} \tag{22}$$

where  $\mathbf{X}_j = (\mathbf{H}_j^{DL})^H (\mathbf{R}_j^{DL})^H \mathbf{W}_j^{DL} \mathbf{R}_j^{DL} \mathbf{H}_j^{DL}$ .

### 3.2 Mode Selection

Since both in the original WSR problem (8) and equivalent MMSE problem (13), the mode selection parameter  $m$  is irrelevant to any of the other variables,  $m$  is only determined by the rate performance of different mode. Then, we can compare the system sum rate performance of FD and HD modes and choose  $m$  to get the better performance.

### 3.3 Convergence

Therefore, the optimal solution of original WSR problem (8) can be achieved by solving the equivalent WMMSE problem (13), which can be decoupled into two sub-problems and using the iterative alternating algorithm listed in Table 1.

**Table 1.** WSR maximum algorithm.

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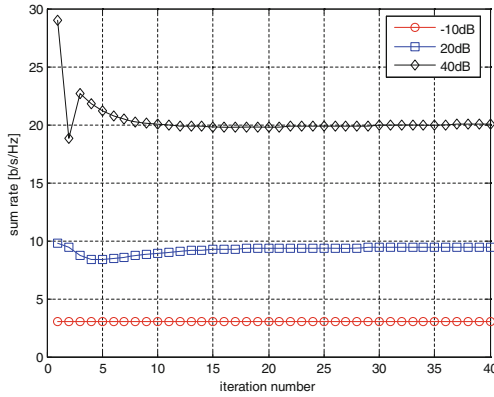
1) Set mode selection $m=1$
2) set the iteration number $N$ and start from $n=0$ , initialize the transmit filters $\mathbf{T}_k^{UL}$ and $\mathbf{T}_j^{DL}$ .
3) $n=n+1$ .
4) Calculate the receive filters $\mathbf{R}_i^{UL}$ and $\mathbf{R}_j^{DL}$ using (9) and (12).
5) Calculate the weight matrices $\mathbf{W}_k^{UL}$ and $\mathbf{W}_j^{DL}$ using (16) and (17).
6) Update the transmit filters $\mathbf{T}_k^{UL}$ and $\mathbf{T}_j^{DL}$ using (18) and (20).
7) Repeat step 3) ~ 6) until convergence, or $N$ is reached.
8) Get the achievable rate of FD system using (7).
9) Set mode selection $m=0$ .
10) Repeat step 2) ~ 7) until convergence, or $N$ is reached.
11) Get the achievable rate of HD system using (7).
12) Compare the achievable rate of FD scheme with HD scheme, set the mode as the one with higher rate.

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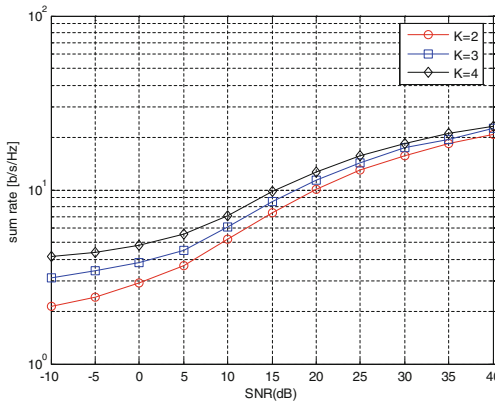
## 4 Numerical Simulations

This section is devoted to examine system sum rate performance of the proposed optimization scheme through simulation. For simplicity, we assume the same transmit power constraint.

Figure 2 elaborates the convergence behavior of our algorithm. We can find that the WSR problem converges in a few iterations under different power constraints. We illustrate the achievable sum rate with different numbers of users in Fig. 3. The sum rate of the system increases with the user number, because of the user diversity gain. Figure 4 compares the achievable sum rate under different INR (self-interference noise ratio). It has been seen from the figure that the sum rate of HD mode is constant to INR and the performance of FD mode decreases with INR. When INR is large enough, HD



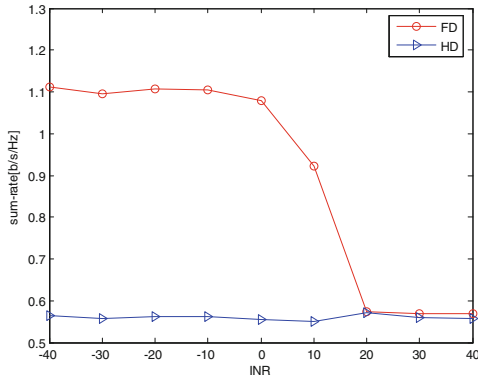
**Fig. 2.** Convergence behavior of the proposed algorithm



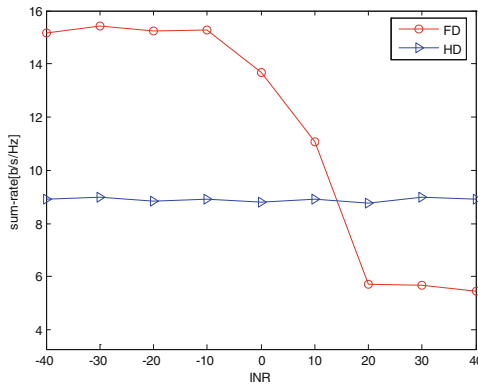
**Fig. 3.** Achievable rate comparison for different users

system outperform FD system, in other words the advantage of FD system disappears. We can also find in Fig. 4 that the sum rate of FD mode drops below that of HD mode around INR = 0 dB when SNR = 20 dB. Figure 5 gives the performance of the proposed heuristic algorithm. At low-to-mid range of INR, FD mode holds a better performance over HD mode, so the system remains working in FD mode. But when INR increase, sum rate of HD scheme outperforms that of FD scheme, the system will switch to the HD mode. In all, the proposed optimization can provide the freedom to switch between the FD and HD mode then to insure the best performance of the system.

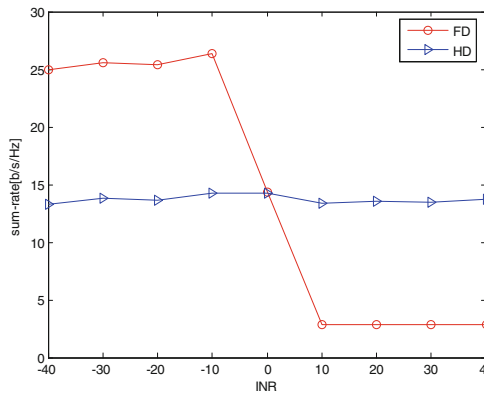




(a) SNR=-20dB

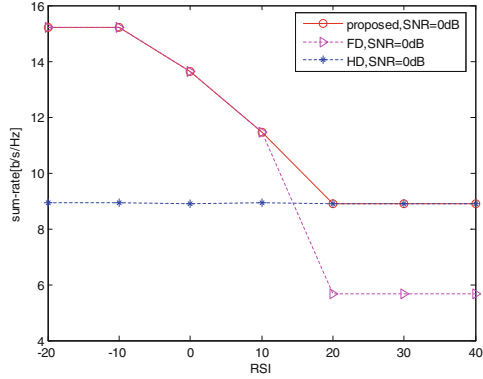


(b) SNR=0dB

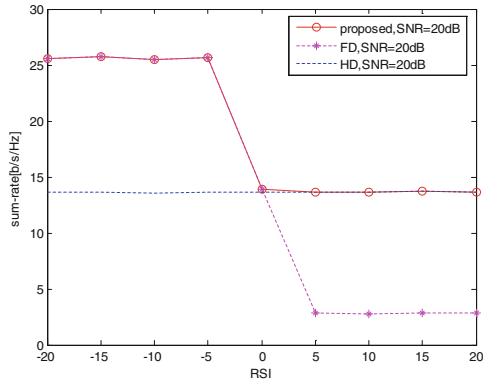


(c) SNR=20dB

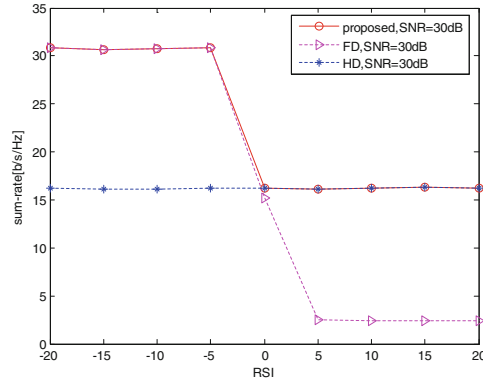
**Fig. 4.** Sum-rate comparison for different duplex schemes under different RSI



(a) SNR=0dB



(b) SNR=20dB



(c) SNR=30dB

Fig. 5. Sum-rate comparison of proposed algorithm under different RSI

## 5 Conclusion

In this work, we have considered a full-duplex cellular communication system with the BS transmitting and receiving simultaneously to (from) multiple DL (UL) MSs. The main challenge of such system lies in SI of the BS and will cause the awkward situation that FD mode cannot outperform HD mode. We formulate a joint mode selection and beamformer optimization problem, which has the maximal weighted sum rate as objective and power constraints. Since the non-convex characteristic of the problem, global optimal solution is hard to achieve. Thus a heuristic algorithm based on decoupling is proposed, which decomposes the original problem into two sub-problems. One is mode selection sub-problem and the other one is minimization of weighted sum mean square error sub-problem. Numerical results demonstrate that the proposed algorithm can always choose the mode with greater performance in achievable rate.

**Acknowledgments.** This work is sponsored by National Natural Science Foundation of China (Grant No. 61601409 and Grant No. 61471322).

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