

High-Resolution Sparse Representation of Micro-Doppler Signal in Sparse Fractional Domain

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Abstract. In order to effectively improve radar detection ability of moving target under the conditions of strong clutter and complex motion characteristics, the principle framework of Short-Time sparse Time-Frequency Distribution (ST-TFD) is established combining the advantages of TFD and sparse representation. Then, Short-Time Sparse FRactional Ambiguity Function (ST-SFRAF) method is proposed and applied to radar micro-Doppler (m-D) detection and extraction. It is verified by real radar data that the proposed methods can achieve high-resolution and low complexity TFD of time-varying signal in time-sparse domain, and has the advantages of good time-frequency resolution, anti-clutter, and so on. It can be expected that the proposed methods can provide a novel solution for time-varying signal analysis and radar moving target detection.

Keywords: Sparse representation · Micro-Doppler signal
Sparse time-frequency distribution (STFD)
Short-time sparse fractional ambiguity function (ST-SFRAF)

1 Introduction

Doppler signature extraction and analysis of moving target are quite important for radar target detection and recognition [1]. The traditional Fourier spectrum cannot exhibit time-varying Doppler signature with low spectrum resolution. Recently, the micro-Doppler (m-D) theory has attracted extensive attention worldwide for accurate description of a target's motion [2]. The m-D features reflect the unique dynamic and structural characteristics, which are useful for target recognition and classification. The motion of a marine target is rather complex especially for marine target and maneuvering target [3, 4]. Therefore, how to effectively detect m-D signal is the key step for the following target detection and estimation.

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Time-frequency distributions (TFDs) provide an image of frequency contents as a function of time, which reveals how a signal changes over time. However, the classic TFDs, such as short-time Fourier transform (STFT) or Wigner-Ville distribution (WVD), suffer from the poor time-frequency resolution or cross-terms. Moreover, it is difficult to separate the complex background (clutter) from the weak m-D signal in time domain or frequency domain [5].

In the last decade, sparsity has been proved as a promising tool for a high-resolution solution. Sparse transform is proposed to increase resolution in different transform domains [6], such as the sparse FFT [7, 8] and sparse FRFT (SFRFT) [9], et al. M-D signal can be approximated as sum of frequency-modulated (FM) signals and it can be considered to be sparse in the TF plane [10]. In this paper, the merits of TFD and sparse representation are combined together and a novel method, i.e., short-time sparse fractional ambiguity function (ST-SFRAF) is proposed for high-resolution representation of time-varying m-D signal in the sparse time-frequency domain.

2 Radar M-D Signal Model

Suppose there is a target moving towards radar, and only radial velocity component is considered. Then, the radar line-of-sight (RLOS) distance $r_s(t_m)$ can be modeled as a polynomial function of slow time, i.e.,

$$r_s(t_m) = \sum_i a_i t_m^{i-1} = r_0 - vt_m - \frac{1}{2!} v' t_m^2 - \frac{1}{3!} v'' t_m^3 - \dots, t_m \in [-T_n/2, T_n/2] \quad (1)$$

where t_m is the slow-time, v is target’s velocity and T_n is coherent integration time.

Suppose radar transmits linear frequency modulated (LFM) signal, and after demodulation and pulse compression, the radar returns of a target can be expressed as

$$s_{PC}(t_m) = A_r \text{sinc} \left[B \left(t - \frac{2r_s(t_m)}{c} \right) \right] \exp \left(-j \frac{4\pi r_s(t_m)}{\lambda} \right) \quad (2)$$

where A_r is amplitude, B is the bandwidth of transmitted signal, c represents speed of light, and λ is the wavelength.

Therefore, for the m-D signal, which mostly has the form of nonuniformly translational and rotational motions, can be expressed as a FM signal with fluctuated amplitudes, i.e.,

$$x(t_m) = \sum_{i=1}^I a_i(t_m) e^{j\varphi_i(t_m)} \quad (3)$$

where $\varphi(t_m) = 2\pi f_d(t_m) = \sum_k 2\pi a_k t_m^{k-1}$.

In real engineering applications, the m-D signal can be approximated as a LFM signal according to the observation time and integration time. The LFM signal can represents the moving parameters of a target, such as initial velocity and acceleration. For the m-D signal model of (3), its time-frequency distribution (TFD) has the following form.

$$\rho_x(t_m, f) = \sum_{i=1}^I a_i^2(t_m) \delta[f - \dot{\varphi}_i(t_m)/2\pi] \tag{4}$$

where $\dot{\varphi}_i(t_m)$ is the estimation of m-D frequency.

3 Principle of STFD and ST-SFRAF

From the above analysis, the m-D signal can be regarded as sum of multiple instantaneous frequency components. In the time-frequency domain, the m-D signal can exhibit an obvious peak at the location of $f = \dot{\varphi}_i(t)$ via sparse representation. And the time-frequency domain $\rho_x(t, f)$ is the sparse representation domain of m-D signal. For example, for an LFM signal, $a_1(t) = 1$, $\dot{\varphi}_1(t) = 2\pi(f_o + at)$.

Any signal can be represented in terms of basis or atoms g [10],

$$\mathbf{x} = \sum_m \alpha_m g_m \tag{5}$$

where m is the number of atoms, and the coefficient α_m denotes the similarity of the signal and the atoms.

It can be found by comparing (9) and (10) that the TFD is a special case of the sparse representation, i.e.,

$$\rho_{\mathbf{x}}(t, f) = \sum_m \alpha_m(t) h(t) g_m(t, f) \tag{6}$$

where $h(t)$ is a window function and $g_m(t, f)$ is the atoms combined with frequency modulated signal.

For the condition without noise, the sparse representation of (11) can be regarded as the optimization problem and solved by l_1 -norm minimization,

$$\min \|\rho_{\mathbf{x}}(t, f)\|_1, \text{ s.t. } o\{\rho_{\mathbf{x}}(t, f)\} = b \tag{7}$$

where o is the sparse operator with $K \times N$ dimension. The above equation can be relaxed by the following constraint, i.e.,

$$\min \|\rho_{\mathbf{x}}(t, f)\|_1, \text{ s.t. } \|o\{\rho_{\mathbf{x}}(t, f)\} - b\|_2 \leq \varepsilon. \tag{8}$$

When $\varepsilon = 0$, (12) and (13) have the same form. Then the framework of STFD can be defined by the calculation from (11) to (13).

When ρ_x is the Fourier transform (FT) and b is the signal component in the FT domain, the STFD is named as short-time sparse FT (ST-SFT), i.e.,

$$\min \|\mathcal{F}(t, f)\|_1, \text{ s.t. } \|o\{\mathcal{F}(t, f)\} - f\|_2 \leq \varepsilon \tag{9}$$

When ρ_x is the fractional ambiguity function (FRAF) and b is the signal component in the FRAF domain, the STFD is named as short-time sparse FRAF (ST-SFRAF),

$$\min \|\mathcal{R}^\alpha(t, f)\|_1, \text{ s.t. } \|o\{\mathcal{R}^\alpha(t, f)\} - f(\alpha, u)\|_2 \leq \varepsilon. \tag{10}$$

where $\mathcal{R}^\alpha(\cdot)$ is the FRAF operator, $\alpha \in (0, \pi]$ denotes transform angle, $K_\alpha(t, u)$ is the transform kernel.

$$\mathcal{R}^\alpha(\tau, u) = \int_{-\infty}^{\infty} R_x(t, \tau) K_\alpha(t, u) dt \tag{11}$$

$$K_\alpha(t, u) = \begin{cases} A_\alpha \exp\{j[\frac{1}{2}t^2 \cot \alpha - ut \csc \alpha + \frac{1}{2}u^2 \cot \alpha]\}, & \alpha \neq n\pi \\ \delta[u - (-1)^n t], & \alpha = n\pi \end{cases} \tag{12}$$

where τ is the time delay, $A_\alpha = \sqrt{(1 - j \cot \alpha)/2\pi}$, $R_x(t, \tau)$ is the instantaneous auto-correlation function of $x(t)$,

$$R_x(t, \tau) = x(t + \tau/2)x^*(t - \tau/2) \tag{13}$$

Suppose there is a m-D signal modeled as a QFM signal, i.e.,

$$\begin{aligned} x(t) &= \sigma_0 \exp\left[j4\pi \frac{r_s(t)}{\lambda}\right] = \sigma_0 \exp\left(\sum_{i=0}^3 j2\pi a_i t^i\right) \\ &= \sigma_0 \exp[j2\pi(a_0 + a_1 t + a_2 t^2 + a_3 t^3)] \end{aligned} \tag{14}$$

where a_i ($i = 1, 2, 3$) represents the coefficient of the polynomial of QFM signal.

If the time delay τ is fixed, the relation between $\mathcal{R}^\alpha(\tau, u)$ and $\mathcal{R}^\alpha(t, f)$ is derived as follows,

$$\begin{cases} a_2 = \frac{u}{4\pi\tau} \csc \alpha \\ a_3 = -\frac{1}{12\pi\tau} \cot \alpha \end{cases} \tag{15}$$

And $f(t) = 2\pi(a_1 + 2a_2 t + 3a_3 t^2) = 2\pi(v_0 + \mu t + \frac{1}{2}gt^2)$, where v_0 is the initial velocity, μ is the chirp rate, and g denotes the jerk motion.

Therefore, the proposed STFD and ST-SFRAF are the generalized forms of the classical TFD and FRAF, which indicates promising applications.

4 Sparse Representation of Radar M-D Signal via ST-SFRAF

Flowchart of the ST-SFRAF-based m-D signal extraction method is shown in Fig. 1, which mainly consists of four steps, i.e.,

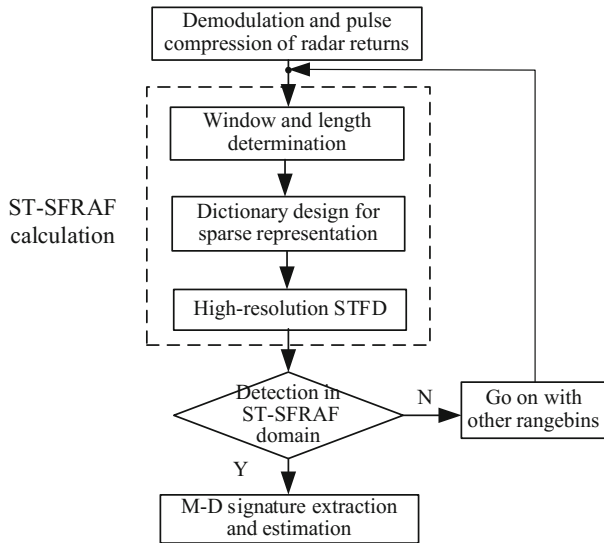


Fig. 1. Observation model of radar and typical micromotion target.

- Perform demodulation and pulse compression of radar returns, which achieves high-resolution in range direction;
- ST-SFRAF, which is the most important procedure, consists of three parts, i.e., window and length determination, dictionary design for sparse representation, and high-resolution STFD;
- Signal detection in ST-SFRAF domain by comparing an adaptive detection threshold;
- M-D signature extraction and estimation.

It should be noted that the choice of dictionary can be determined according to the prior information of target to satisfy the sparsity condition, such as the types of observed target and different sea states. For the micromotion target whose main motion components are the rotation or high mobility, its m-D signal exhibits periodical frequency modulated property. Also, we can use the QFM signal or periodical frequency modulated function as its dictionary.

5 Simulation and Results Analysis

The micromotion of a marine target includes the non-uniform translated motion and three dimension motion, i.e., yaw, pitch, and roll motions. Due to the complex fluctuation of sea surface and motion of target, the m-D signal of a marine target is rather complex and too weak to be detected covered by heavy sea clutter. Therefore, we will employ S-band radar (SSR) dataset with a WaveRider RIB to verify the proposed method.

Descriptions of the SSR dataset is shown in Fig. 2. The specifications of radar as well as the environment parameters can be found in [11]. The dataset under sea state of 4 was chosen for validation. The range-Doppler plot is presented in Fig. 2(b) covering nearly 5 nautical miles. Due to the heavy sea clutter fluctuating downwind, it is rather difficult to separate the target from sea clutter background. Furthermore, high Doppler resolution spectrogram (STFT) of the target's rangebins is plotted in Fig. 3(a1). It can be found that the WaveRider RIB had a narrow Doppler response, with a local disturbance of the sea surface. The time-varying Doppler character indicates a nonuniform motion and Doppler migration, which is a typical m-D signal modeled as a QFM signal. After detection process using a constant false alarm rate (CFAR) detector, the detection result is shown in Fig. 3(a2). However, the true target cannot be figured out due to the heavy sea clutter and poor frequency resolution.

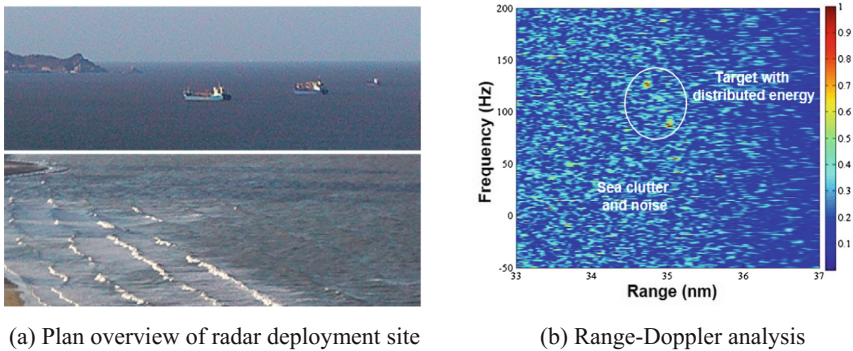


Fig. 2. Description of the S-band radar dataset.

Then, we carry out the ST-SFRAF and compare the detection results with STFT (Fig. 3(b1) and (b2)). The proposed STFD-based method matches up with the micro-motion model better, are more preferable in comparison. In addition, the proposed method has the ability of sea clutter suppression and can accumulate the energy of m-D signal as an obvious peak, which is shown in Fig. 3(b2). Moreover, with the local window, the changes of m-D can be described well. Therefore, the proposed ST-SFRAF method is a good choice for high resolution representation and detection of m-D signal with high-order motion.

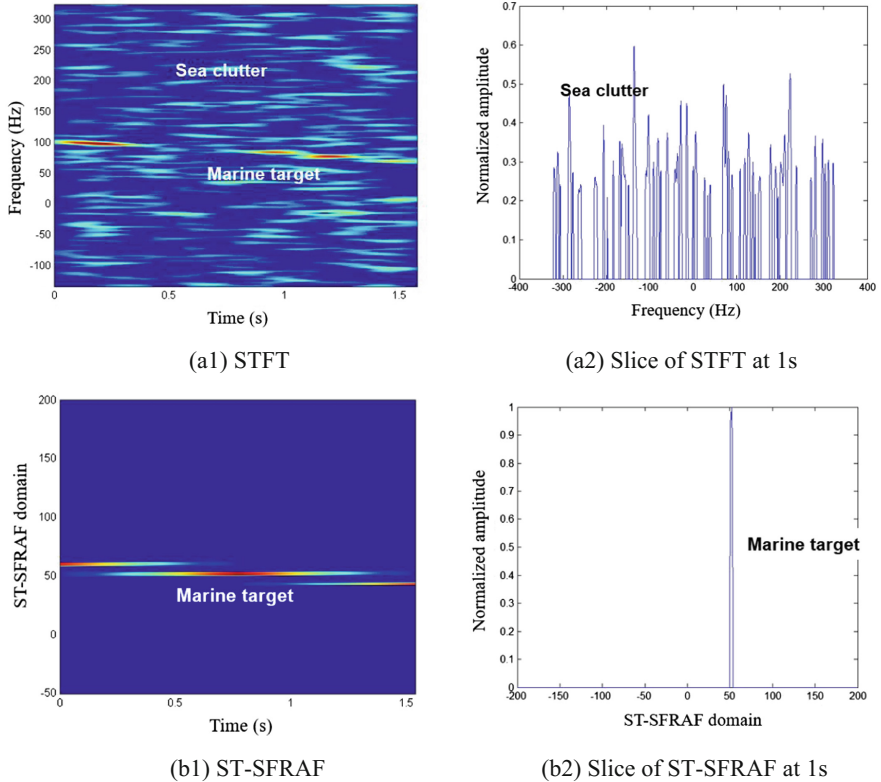


Fig. 3. Comparisons of STFT and ST-SFRAF based m-D extraction methods.

References

1. Simon, W., Luke, R., Stepjen, B., et al.: Doppler spectra of medium grazing angle sea clutter; part 1: characterisation. *IET Radar Sonar Navig.* **10**, 24–31 (2016)
2. Chen, V.C., Li, F., Shen-Shyang, H.O., et al.: Micro-Doppler effect in radar: phenomenon, model, and simulation study. *IEEE Trans. Aerospace Electron. Syst.* **42**, 2–21 (2006)
3. Chen, X., Guan, J., Bao, Z., et al.: Detection and extraction of target with micro-motion in spiky sea clutter via short-time fractional Fourier transform. *IEEE Trans. Geosci. Remote Sens.* **52**, 1002–1018 (2014)
4. Chen, X., Guan, J., Li, X., et al.: Effective coherent integration method for marine target with micromotion via phase differentiation and Radon-Lv's distribution. *IET Radar Sonar Navig. (Special Issue: Micro-Doppler)* **9**, 1284–1295 (2015)
5. Guan, J., Chen, X., Huang, Y., et al.: Adaptive fractional Fourier transform-based detection algorithm for moving target in heavy sea clutter. *IET Radar Sonar Navig.* **6**, 389–401 (2012)
6. Flandrin, P., Borgnat, P.: Time-frequency energy distributions meet compressed sensing. *IEEE Trans. Sig. Process.* **58**, 2974–2982 (2010)
7. Gholami, A.: Sparse time–frequency decomposition and some applications. *IEEE Trans. Geosci. Remote Sens.* **51**, 3598–3604 (2013)

8. Gilbert, A.C., Indyk, P., Iwen, M., et al.: Recent developments in the sparse Fourier transform: a compressed Fourier transform for big data. *IEEE Sig. Process. Mag.* **31**, 91–100 (2014)
9. Liu, S., Shan, T., Tao, R., et al.: Sparse discrete fractional Fourier transform and its applications. *IEEE Trans. Sig. Process.* **62**, 6582–6595 (2014)
10. Chen, X., Dong, Y., Huang, Y., et al.: Detection of marine target with quadratic modulated frequency micromotion signature via morphological component analysis. In: 2015 3rd International Workshop on Compressed Sensing Theory and Its Applications to Radar, Sonar and Remote Sensing (CoSeRa), Pisa, Italy, pp. 209–213 (2015)
11. Chen, X., Guan, J., Liu, N., He, Y.: Maneuvering target detection via radon-fractional Fourier transform-based long-time coherent integration. *IEEE Trans. Sig. Process.* **62**, 939–953 (2014)