# Learning Algorithm for Tracking Hypersonic Targets in Near Space

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**Abstract.** With the development of hypersonic vehicles in near space such as X-51A, HTV-2 and so on, tracking for them is becoming a new task and hotspot. In this paper, a learning tracking algorithm is introduced for hypersonic targets, especially for the sliding jump maneuver. Firstly the algorithm uses the Sine model, which makes the tracking model more close to the particular maneuver, next two Sine models different in angular velocity are used into IMM algorithm, and it learns the target tracking error characteristics to adjust the sampling rate adaptively. The algorithm is compared with the single accurate model algorithm and general IMM algorithms with fixed sampling rate. Through simulation experiments it is proved that the algorithm in this paper can improve the tracking accuracy effectively.

**Keywords:** Learn  $\cdot$  Target tracking  $\cdot$  Near space  $\cdot$  Interacting multiple models Sampling rate

### 1 Introduction

Near space is the air space from ground 20–100 km, also known as suborbital space or aerospace transition zone. It is near space where the near space vehicle voyages and completes the specific tasks such as attacking, reconnoitre, communication, early warning, navigation and so on [1]. It has very important military value and significance.

The hypersonic vehicle has high speed, strong maneuver and periodic jumping motion, and its flight process can be simplified into 3 stages: boost section, cruise section and attack section. Sliding jump flight is adopted in the cruise section and this kind of trajectory is not easy to be detected and intercepted with strong penetration capability.

In view of the above characteristics, radar tracking for near space targets is still in the exploratory stage. Based on the relationship between the position estimation value and the acceleration, the literature [2] proposed a modified CS model which can be adjusted adaptively and used it into the IMM algorithm. The literature [3] applied IMM algorithm with CV and CA models in unscented Kalman filter. The algorithms mentioned above are based on the interacting multiple model algorithm. Although the interaction models are different, they adopt the existing maneuvering models which are not close to the sliding jump flight. With the idea of current statistical model, the target angular velocity is corrected in the literature [4], and it was combined with the extended Kalman filter, but the maneuvering frequency and maximum acceleration should be set artificially which means poor adaptive ability. The paper [5] establishes a specific target motion model for the jumping maneuver, but it needs speed information and cannot be applied into general phased array radar.

Because of the extremely complex motions of near space targets, it is difficult to establish accurate models [6]. Therefore, this paper introduces the concept of learning algorithm and constructs a learning tracking system, which makes the tracking algorithm adaptive to complex motion situation.

### 2 Learning Tracking Algorithm

### 2.1 Framework

The system diagram of this algorithm is as follows (Fig. 1):



Fig. 1. Learning tracking algorithm framework.

After the system gets the observations in accordance with the corresponding sampling period T, data will be processed in IMM-Kalman filter based on 2 Sine models with different angular velocity, and transmitted to the evaluation system. The evaluation system determines T in the next time according to the target tracking error and adjustment rules.

### 2.2 Model

At present, the most common movement models for maneuvering targets tracking are Singer model, CS model, Jerk model and corresponding improved model. In the tracking for linear moving targets, these models have good tracking accuracy. But for the non-ballistic trajectory of near space hypersonic targets, particularly the sliding jump maneuver, the former models have low matching degree. This algorithm adopts Sine model [7].

Sine model's state equation is as follows:

$$X(k+1) = F(T, w_0)X(k) + W(k)$$
(1)

Where, *k* represents time;  $X = \begin{bmatrix} x & \dot{x} & \ddot{x} \end{bmatrix}^T$  represents state vector, including target position, velocity, acceleration and jerk; *F* is state transition matrix; *T* is sampling period, and *W* is Gauss white noise whose covariance is *Q*.

$$Q(k) = \frac{\sigma_w^2}{\pi} \begin{pmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ q_{41} & q_{42} & q_{43} & q_{44} \end{pmatrix}$$
(2)

Where,  $\sigma_w^2$  represents acceleration variance.

Measurement equation is as follows:

$$Z(k) = H(k)X(k) + V(k)$$
(3)

In the equation, Z represents measurement vector after unbiased transformation, H is the measurement matrix, and V is Gauss white noise.

#### 2.3 Residual and Norm

The learning method of this algorithm is constructed based on the residual sequence, and the following two residual vectors are defined [8]:

$$V(k+1) = H\hat{X}(k+1|k+1) - Z(k+1)$$
(4)

$$\bar{V}(k+1) = H\hat{X}(k+1|k) - Z(k+1)$$
(5)

The information represented by the two is different: the residual vector V(k+1) is determined by the filtered value of the corresponding measurement information that has been fused at the k+1 moment, and the predicted residual vector  $\overline{V}(k+1)$  is determined by the predicted state of the k moment. If the measurement information is reliable, the value of the residual vector indicates the reliability of the X(k+1|k), so the predicted residual vector can reflect the disturbance of the dynamic system better than the residual vector.

Define the norm of the predicted residuals as follows:

$$d(k+1) = \overline{V}^T(k+1)S^{-1}(k+1)\overline{V} \quad (k+1)$$
(6)

In the equation, S is the innovation covariance matrix.

#### 2.4 Adjustment for Sampling Period

The norm reflects the tracking effect of the target, so it is considered as the basis for adjusting the sampling period.

Sampling period allocation method: when the target is in the non-maneuvering state, the innovation norm d(k) obeys the  $\chi^2(m)$  distribution (*m* is the observation dimension). Now take d(k) as standard to judge whether the target is maneuvering, and set the false alarm rate of the decision as  $\alpha$ , according to the distribution table, we can find the corresponding threshold  $d_{\alpha}$  [9]. The  $\alpha(n)$  false alarm rates are selected as the key node, and the corresponding value sequence  $d_{\alpha}(n)$  is obtained by  $\chi^2(m)$  table, so

that the sequence data can be compared with the sequence value, and the new sampling period can be determined by comparing the results.

Taking node number N = 3 as an example, a specific rule is given.

When  $d_{\alpha}(n) < d(k) < d_{\alpha}(n+1)$ ,

If  $\alpha(n) < 10\%$  we think that the target is likely to be maneuvering, then the maximum data rate is allocated for the target, such as 0.1 s;

If  $10\% < \alpha(n) < 90\%$  we consider the accidental flight disturbance or observation outliers to make the norm larger, and then assign a lower data rate for the target, such as 0.2 s;

If  $90\% < \alpha(n)$  we think the target is non motorized, then the target data rate is the lowest value, such as 0.5 s.

If the *N* takes a larger value, the adaptive sampling period tracking algorithm would work better in theory.

In the IMM algorithm, there are many d because of the presence of multiple models. At this point, the d used to decide the sampling interval at the next time is computed by adding products of multiplying the d of each model with the corresponding model probability.

$$d = \sum_{i=1}^{j} d_i * u_i \tag{7}$$

Where, j is the number of interactive models,  $u_i$  is the probability of the i model.

### **3** Simulation and Analysis

#### 3.1 Simulation Settings

Refering to some basic test data for X-51 released by the U.S. government in May 2013, this paper simplifies the complicated mathematical model [10] (including dynamics model, engine thrust model, aerodynamic model, atmospheric model and so on). According to the primary characteristics of the near space vehicle (including flight height and velocity), an analog trajectory with time length of 300 s is set as follows and its angular velocity is 0.06 rad/s (Fig. 2). In fact, the target does a uniform motion at different speeds in the X and Y axis (for this reason, the following simulation only shows the results in the Y axis), with a sinusoidal motion whose period is 0.06 rad/s in the Z axis. It is assumed that the sampling period of ground-based radar is 0.2 s, the radial distance error is 100 m, the azimuth and pitch angle errors are both 0.1°. The data processing method is an unbiased conversion measurement Kalman algorithm.

### 3.2 Experiment One

Taking into account that the parameter w of the Sine model is to set artificially in advance, and the aircraft's sliding jump trajectory in the actual situation is not known exactly, so we need to consider the influence on the tracking effect when the angular speed w is set different values. In the experiment we set 3 different angular velocity of



Fig. 2. Trajectory simulation of hypersonic target

0.06 rad/s, 0.05 rad/s, and 0.07 rad/s, among whom 0.06 rad/s matches with the simulated trajectory's angular velocity. And then we use the Kalman filter algorithm to achieve Monte-Carlo simulation for 100 times. The standard examining tracking effect is the position tracking mean square error in the direction of Z axis. The analysis process of the other two axes is similar, no more details.

As can be seen from the Figs. 3 and 4, since the Y axes move at the constant speed, while the Z axis makes sinusoidal acceleration motion, the root mean square curves of the filtering error in the X and Y axis are similar, while that of the Z axis is slightly different. But when different angular velocities are compared, we can find that the error curve when the angular velocity matches is stable and convergent. In time of about first 50 s the other two curves have high coincidence degree with the w = 0.06 rad/s curve, this is because the filter time is not long, the difference is small, then it begins to appear big shock as time goes, the error caused by the mismatch of angular velocity increases and shocks gradually. The mean square error of each axis of the three angular velocities is statistically averaged, as shown in the following Table 1. From this experiment it can be concluded that the Sine model can indeed track high speed targets in sliding jump maneuver with good tracking accuracy, but the premise is the angular velocity set should match with the actual, if there are some errors, tracking will be unstable, the filtering error will appear concussion.

### 3.3 Experiment Two

It is a good method to make use of several Sine models with different angular speed for interactive tracking under the condition of uncertain target's actual motion parameters. In this experiment, two sine models with different angular velocities w = 0.05 rad/s, and w = 0.07 rad/s are used interactively to track the simulated trajectories of the above w = 0.06 rad/s aircraft.



Fig. 3. Position root mean square error in Y axis in different w



Fig. 4. Position root mean square error in Z axis in different w

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Table I.	Statistical	average	ot	position	root	mean	error	1n	Z 2X1S
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w/ (rad/s)	Root mean square error in X/m	Root mean square error in Y/m	Root mean square error in Z/m
0.06	60.2149	63.4617	85.3250
0.05	101.8573	123.0269	1291.1
0.07	104.8638	134.3005	1623.2

The tracking root mean square error is compared between the algorithm with adaptive sampling rate (IMM-AT), and general IMM algorithms whose sampling period are 0.1 s, 0.2 s and 0.5 s.

The filtered root mean square error curve of the IMM algorithm with three different fixed sampling intervals is drawn in Figs. 5 and 6. Overall, although the angular velocity of two Sine models in the interaction differs from the real's, the filtering curve is relatively stable without substantial concussion, and the error is generally much less than single model's error when w = 0.05 rad/s or w = 0.07 rad/s. On the other hand, in either direction axis, the root mean square error decreases as the sampling interval decreases. This shows that, to a certain extent, the filtering results can be improved by reducing the sampling interval. From the statistical error of the following table, when T = 0.5 s, the error of the IMM algorithm on the X and Y axis is less than w = 0.06single model algorithm, but the filtering effect of the Z axis is not as good as that of the w = 0.06 single model algorithm. This is because the target in the Z axis is accelerated by sinusoidal motion, and the matching of angular velocity has a great influence on its filtering. However, the Z axis error of the IMM algorithm when T = 0.2 s is less than the error from single model algorithm when w = 0.06. So it is possible to reduce the error when the error is large, especially the Z axis error, by changing the sampling interval. The root mean square error of the IMM-AT algorithm in the last line of the Table 2 verifies the feasibility of the algorithm.



Fig. 5. Position root mean square error in Y axis in different T



Fig. 6. Position root mean square error in Z axis in different T

Sampling/s	Root mean square error	Root mean square error	Root mean square error		
	in X/m	in Y/m	in Z/m		
0.1	37.2816	27.0823	64.1143		
0.2	56.6592	39.7474	96.5491		
0.5	68.4965	50.5756	132.8217		
IMM-AT	48.2112	34.6849	83.3211		

Table 2. Statistical average of position root mean error in z axis

### 4 Conclusion

Aimed at sliding jump maneuver of near space hypersonic vehicle, an interactive multiple model algorithm based on the Sine model with adaptive sampling rate is proposed. This is a kind of algorithm that can learn and adjust according to the feedback of the system. Compared with the single accurate model algorithm and the IMM algorithm with different fixed sampling rate, it proves the feasibility and practicability of the learning algorithm in this paper.

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## References

- 1. Chen, W., Wu, X., Tang, Y.: Application of thrust vectoring control technology in near space vehicle. Winged Missiles J. (5), 64–70 (2013)
- Li, C., Bi, H., Zhang, B., Xiao, S.: An improved tracking algorithm for hypersonic targets. J. Air Force Eng. Univ. (Nat. Sci. Edn.) 13(5), 50–54 (2012)
- Qin, L., Li, J., Zhou, D.: Tracking for near space target based on IMM algorithm. Syst. Eng. Electron. 36(7), 1243–1249 (2014)
- Xiao, S., Tan, X., Li, Z., Wang, H.: Near space hypersonic target MCT tracking model. J. Projectiles Rockets Missiles Guidance 33(1), 185–194 (2013)
- Cao, Y., Li, Y.: State estimation algorithm based on high speed-acceleration target in near space. Modern Defence Technol. 41(6), 97–101 (2013)
- 6. Guo, X., Liu, C., Zhang, Y., Wei, G., Wang, G.: Tracking algorithms for near space hypersonic target. Command Control Simul. **38**(5), 8–12 (2016)
- Wang, G., Li, J., Zhang, X., Wu, W.: A tracking model for near space hypersonic slippage leap maneuvering target. Acta Aeronautica et Astronautica Sinica 36(7), 2400–2410 (2015)
- Liu, Y., Feng, X., Ye, Y., Wang, Y.: Improved current statistical model and adaptive tracking algorithm. Sci. Technol. Eng. 13(22), 6464–6468 (2013)
- 9. Shi, L., Wang, X., Xiao, S.: Adaptive data rate tracking of phased array radar based on residue norm. Shipboard Electron. Countermeasure **28**(5), 45–47 (2005)
- Xiaohua, N., Yiming, X.: Flight trajectory modeling and simulation for target tracking on NSHV. Comput. Simul. 33(3), 41–46 (2016)