# Spectrum Allocation in Cognitive Radio Networks by Hybrid Analytic Hierarchy Process and Graph Coloring Theory

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**Abstract.** In this paper, a graph coloring-based spectrum allocation algorithm in cognitive radio networks combined with analytic hierarchy process is proposed. By analyzing several key factors that affect the quality of the leased spectrum, the algorithm combines the graph algorithm and analytic hierarchy process to assign the optimal spectrum to cognitive users orderly. Simulation results show that the proposed algorithm can effectively improve the network efficiency compared with original algorithms and arose inconspicuous loss to the whole network's fairness. The proposal not only improves the efficiency of spectrum allocation, but also balances the requirements of the overall fairness of cognitive radio networks.

**Keywords:** Cognitive radio  $\cdot$  Graph coloring  $\cdot$  Spectrum allocation Analytic hierarchy process

## 1 Introduction

Spectrum sharing is the key technology in cognitive radio which attracts the increasing interest [1-3]. Graph theory, as a classical optimization theory, has been introduced to solve the difficulty of spectrum allocation in cognitive radio. [4] proposed a list-coloring algorithm based on the graph coloring theory, which includes distributed greedy algorithm and distributed fairness algorithm. In [5], as the list coloring algorithm can allocate only one spectrum once, the authors proposed a spectrum allocation algorithm to assign channels to multiple users at the same time without incurring interference. Based on [5], authors in [6] proposed an improved graph coloring algorithm for spectrum allocation with regards to the maximum weighted independent set to improve the spectrum utilization by combining the power control technology.

In this paper, we apply analytic hierarchy process (AHP), one of the multiobjective decision method to proceed spectrum decision and provide a reasonable spectrum access strategy according to the heterogeneous idle spectrum. The improved proposal takes into account diverse spectral characteristics to meet the actual needs of the spectrum allocation in cognitive radio networks. In addition, by combining the advantages of the methods of AHP and coloring theory, the proposal not only improves the efficiency of spectrum allocation, but also satisfies the requirements of the overall benefits of cognitive radio networks.

## 2 Spectrum Allocation

#### 2.1 Spectrum Selection Based on AHP

In spectrum selection, secondary users always want to switch to the spectrum with high bandwidth, low delay, low jitter and packet loss rate, etc. Therefore, this paper selects four indexes of bandwidth, delay, jitter and packet loss rate as the judgment criterion of spectrum selection. During the course, secondary users should also take their own preferences into account. When the number of considering factors become increasing, secondary users will struggle to make a rational choice by qualitative analysis. We thus introduce the method of AHP to analyze this problem. The selection problem can decomposed into three levels as shown in Fig. 1. In more complex environment, more evaluation criteria can be introduced to make it closer to realize.



Fig. 1. Hierarchical graph of optimal spectrum decision based on AHP algorithm

To compare the impacts of factors  $C_1, C_2, \dots, C_n$  of one layer on a factor  $S_i$  of another layer, such as the importance of different choice criterions on final channel selection, it is essential to make a comparison between only two factors rather than multiple factors at the same time. Selecting two factors  $C_i$  and  $C_j$  each time, we use  $a_{ij}$  to denote the impact ratio of  $C_i$  and  $C_j$  on  $S_i$ , all the comparison results can be expressed in matrix as following

$$A = (a_{ij})_{n \times n}, \quad a_{ij} > 0, \quad a_{ij} = \frac{1}{a_{ji}}.$$
 (1)

We can rewrite (1) as

$$A = \begin{pmatrix} \omega_1/\omega_1 \ \omega_1/\omega_2 \ \cdots \ \omega_1/\omega_n \\ \omega_2/\omega_1 \ \omega_2/\omega_2 \ \cdots \ \omega_2/\omega_n \\ \vdots \ \vdots \ \ddots \ \vdots \\ \omega_n/\omega_1 \ \omega_n/\omega_2 \ \cdots \ \omega_n/\omega_n \end{pmatrix},$$
(2)

where  $\omega = (\omega_1, \omega_2, \cdots)^T$  is the weighted coefficient vector satisfying  $\sum_{i=1}^n \omega_i = 1$ . Thus, as mentioned above, proper selection of the weighted coefficients in the secondary user's utility function is significant. In subsequent simulation tests, we will provide numerical results to testify the effects of different parameter selection in spectrum trading.

#### 2.2 Single Hierarchical Arrangement and Consistency Check

Single hierarchical arrangement refers to the same level of the corresponding factors for the relative importance of the upper level of a factor ranking weight, it can be obtained by normalizing the eigenvector (weighted vector) W of the largest eigenvalue  $\lambda_{max}$  of judgment matrix A. So the essence is to calculate the weight vector. The calculation of the weight vector has the characteristic root method, the sum method, the root method, the power method and so on. In this paper, we use the sum method. The steps of the sun method are as follows:

- 1. Normalize each column vector of  $A : \tilde{W}_{ij} = a_{ij} / \sum_{i=1}^{n} a_{ij}$ .
- 2. Make summation for each row of  $A : \tilde{W}_{ij} = a_i / \sum_{j=1}^n \tilde{W}_{i,j}$ .
- 3. Normalize above matrix vector  $W = \tilde{W}_i / \tilde{W}_{ij} = a_i / \sum_{i=1}^n \tilde{W}_i$ , then obtain  $W = (W, W, W, W, W)^T$ 
  - $(W_1, W_2, \cdots, W_n)^T$  as weight vector.
- 4. Calculate AW.
- 5. Calculate  $\lambda_{max} = \frac{1}{n} \sum_{i=1}^{n} \frac{(AW)_i}{W_i}$ , this is the approximate value of the largest eigenvalue.

The procedure for the consistency check is as follows: **Step 1:** Calculate the consistency index (CI).

$$CI = \frac{\lambda_{max} - n}{n - 1}.$$
(3)

**Step 2:** Seek table to determine the corresponding random index (RI). According to the different order of the judgment matrix, we get the average random index RI.

(4)

Matrix order	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.40

Table 1. Random index RI

Step 3: Calculate the consistency ratio (CR) and make judgments.



Fig. 2. Diagram of greedy spectrum allocation algorithm combined with AHP

When CR < 0.1, the consistency of the judgment matrix is acceptable, when CR > 0.1, it is considered that the judgment matrix does not meet the

consistency requirement, the judgment matrix should be revised again. To test the consistency. We take matrix A as an example. First, we use (3) to calculate the consistency index CI = 0.0083, and then obtain RI = 0.9 via Table 1. Finally, due to the fact that CR = 0.0092 < 0.1, we can conclude that A is verified to pass consistency check.

#### 2.3 Improved Spectrum Allocation Algorithm Model

According to the analysis above-mentioned, by selecting the optimal spectrum and then using the graph algorithm to allocate idle spectrum, we can make full use of spectrum resources, enhance the spectrum utilization and improve the overall efficiency.

Combining the distributed greedy algorithm and the method of AHP, the spectrum decision diagram is shown in Fig. 2.

The improved spectrum allocation algorithm can be described as follows: After initializing the system and updating the node information, according to the measured values of bandwidth, delay, jitter and packet loss rate of each spectrum, the hierarchical structure is established by using AHP; Set up the corresponding comparison matrix, and obtain the spectrum efficiency; Finally, the selected optimal spectrum is allocated using the coloring algorithm.

## 3 Numerical Results

In the simulation process, we suppose that there are six primary users in given region of  $1000 \text{ m} \times 1000 \text{ m}$ , and the initial number of idle channels is 10. In this simulation environment, the total network benefits and user fairness of the two algorithms and their improved algorithms are analyzed and compared. Using



Fig. 3. Network utility curves of the three algorithms in greed mode



Fig. 4. Changing curves of the three algorithms in greedy mode

U(R) to measure the network efficiency, with the variance to measure the fairness between users. In the simulation, randomly generate the network topology diagram. Furthermore, we randomly set the values in available spectrum matrix L, interference matrix C and utility matrix B within [0, 1].

The parameters including bandwidth, delay, jitter and packet loss rate meet the data transmission standard of wireless networks proposed by ITU-T [7]. From Figs. 3 and 4, our proposed method using AHP and distributed greed algorithm (AHP-DGA) is compared with the results obtained by original distributed greedy algorithm (DGA) and weighted distributed greedy algorithm (WDGA). It can be concluded that AHP-DGA can receive high network efficiency and decent network fairness.

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