Downward-Looking Sparse Linear Array Synthetic Aperture Radar 3-D Imaging Method Based on CS-MUSIC

Fu-fei Gu[1], Le Kang[2,3(✉)], Jiang Zhao[1], Yin Zhang[1], and Qun Zhang[2,3]

[1] China Satellite Maritime Tracking and Control Department, Jiangyin 214430, China
[2] Information and Navigation College, Air Force Engineering University, Xi'an 710077, China
18810495946@163.com
[3] Collaborative Innovation Center of Information Sensing and Understanding, Xi'an 710077, China

Abstract. In this paper, a three-dimensional imaging method for sparse multiple input multiple output (MIMO) synthetic aperture radar (SAR) is proposed. Due to the limitation of the antenna array length in DLSLA 3-D SAR, the cross-track resolution is poor than the resolution in high and along-track direction. To obtain high resolution in cross-track domain, the multiple signal classification (MUSIC) algorithm is introduced into the imaging problem. However, the MUSIC invalid under the condition of less snapshot numbers and presence of coherent sources, which may be caused by data missing or sparse sampling in practice. To overcome these limitations, after the preprocessing such as the range and along-track imaging with ordinary Nyquist based methods, the motion compensation and the quadratic phase compensation, this paper transform the process of cross-track direction into a multiple measurement vectors (MMV) model and applies compressive multiple signal classification (CS-MUSIC) algorithm rather than the conventional method or MUSIC algorithm. Based on CS-MUSIC algorithm, imaging result of high resolution with less snapshot numbers. Compared with the CS-based method, the proposed approach can obtain a better performance of anti-noise. The simulated results confirm the effect of the method and show that it can improve the imaging quality.

Keywords: Three-dimensional synthetic aperture radar · Sparse linear array
Compressive sensing · Multiple-signal-classification
Multiple Measurement Vectors

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1 Introduction

Downward-looking sparse linear array three-dimensional synthetic aperture radar (DLSLA 3-D SAR) obtains range resolution by pulse compression, azimuth resolution by virtual aperture synthesis with platform movement, and cross-track resolution by a linear array antenna [1]. Given the 3-D imaging capacity and downward-looking geometry, the problems in the conventional two-dimensional SAR can be solved by DLSLA 3-D SAR [2] ,which has attracted an increasing interest in recent years. Since ONERA [3] and ARTINO [4], as the real systems of MIMO-SAR, had been developed, a number of traditional 2D SAR imaging algorithms were extended into this 3D imaging mode such as chirp scaling algorithm [5], range migration algorithm [6] and polar format algorithm [7], which are based on matched filter (MF).

Due to the limitation of the space and capacity, the main problem of DLSLA 3-D SAR is that the resolution of cross-track direction is lower than the along-track and range direction. In addition, limited by the length of data, the resolution obtained by traditional MF method will be restricted because of Rayleigh limit To solve this problem, there are mainly two types super-resolution imaging methods, the methods respectively based on compressive sensing (CS) [8–10] and spatial spectrum estimation [11, 12], have been proposed for DLSLA 3-D imaging. However, the CS-based methods requires the sparsity of the target in observation scene and the resolution performance is noise sensitive, which limit the applications of DLSLA SAR. Besides, MUSIC invalids because of the coherence of scatterers in a realistic SAR imaging case, which can be solved by the spatial smoothing method with the reduction of real aperture and the dramatically decreased resolution [11, 12]. In addition, the data missing or sparse sampling resulted in the problem of less snapshot numbers and above-mentioned MUSIC-based method invalids in this case. To obtain the advantages of CS and MUSIC, CS-MUSIC [13] method have been proposed. The method has estimation accuracy under the condition of different snapshots and is robust to noise.

To solve these aforementioned problems of CS and MUSIC in DLSLA SAR, a novel imaging algorithm based on CS-MUSIC is proposed in this paper. The cross-track process is transformed into a Multiple Measurement Vectors (MMV) model, which will enhance the computational efficiency and elevate the performance of anti-noise compared with Single Measurement Vectors (SMV) model [14]. The cross-track location of the target is obtained by CS-MUSIC with constructing a new orthogonal space and searching peaks. The scattering intensity is recovered by the fast Fourier transform (FFT), which can also reduce the range of searching peaks. The super-resolution imaging result under the noise scenarios can be reconstructed by sparse sampling. Finally, we validate our theory by extensive numerical experiments.

2 Geometry and Signal Model

The geometry of DLSLA 3D SAR is shown in Fig. 1. The radar platform flies along the X-axis, at height H with velocity v. The array along the cross-track direction (Y-axis) is composed of N antenna elements with the equal distance d. At slow time t_m, the

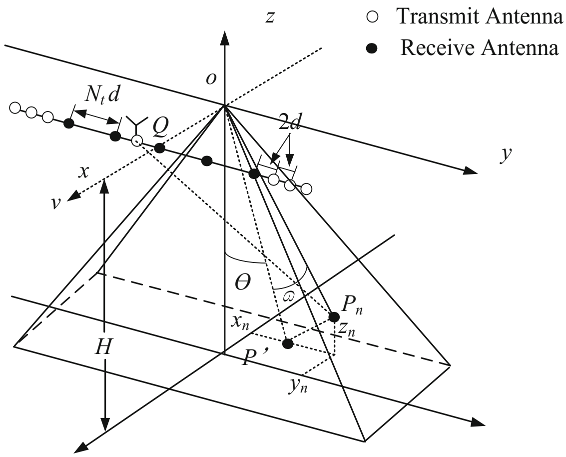

Fig. 1. DLSLA 3-D SAR imaging geometry model.

nth antenna element is located at $P_{mn} = (x_m, y_n, H)$, where $x_m = vt_m, y_n = -L_y d/2 + (n-1)d$. The linear array length is $L_y = (N-1)d$. The point scatterer $P_k(x_k, y_k, z_k)$, the instantaneous distance R between P_k and the nth transmitting antenna can be expressed as

$$R(t_m, y_n) = \sqrt{(vt_m - x_k)^2 + (y_n - y_k)^2 + (H - z_k)^2} \approx R_0 + \frac{x_m^2 - 2vt_m x_k}{2R_0} + \frac{y_n^2 - 2y_n y_k}{2R_0} \tag{1}$$

where R_0 is the projection of the range on the zero-Doppler plane.

The antenna element transmits a linear frequency modulation signal and the echo data can be expressed as

$$S(t, t_m, y_n) = \sum_K \sigma_k \exp\left(j2\pi K_r \left(t - \frac{R(t_m, y_n; K)}{c}\right)^2 - j4\pi \frac{R(t_m, y_n; K)}{\lambda}\right) \tag{2}$$

where σ_k is the backscattering coefficient, c is the light speed and, K_r is the chirp rate and λ is the wave length.

After the received signal are focused into two-dimensional points in the range and along-track domain, the rest phase obtain the cross-track information and two quadratic phase terms, which is about the flight distance and at slowtime t_m and the location of the nth antenna element. Therefore, echo signal can be rewritten as

$$S_n = \sum_K \gamma_k \exp\left(-j\frac{4\pi y_n y_k}{\lambda R_0}\right) + \omega \tag{3}$$

where γ_k is the coefficient of the kth point after two-dimensional focused, ω is the noise with zero mean and variance σ^2.

3 Proposed Imaging Algorithm

In this paper, in order to improve the resolution and enhance the computational efficiency for DLSLA 3-D SAR, a imaging method is proposed, in which the cross-track process is regard as a MMV model and solved by CS-MUSIC.

3.1 MMV Model for Cross-Track Reconstruction

As we can see, the grid points on the cross-track direction in DLSLA 3-D SAR can be discretized as $y_q = q\Delta y$, where $q = 1, 2, \ldots, Q$, Δy is the sampling intervals in the cross-track domain. Assuming $\boldsymbol{s}_n = [s_n(t_1); \cdots; s_n(t_{M_1})] \in \mathbb{C}^{M_1 \times 1}$ is the measurement signal of nth antenna with M_1 sample number in cross-track direction. $\boldsymbol{\gamma}_n = [\gamma_{n1}; \gamma_{n2}; \cdots; \gamma_{nQ}] \in \mathbb{C}^{Q \times 1}$ is the corresponding focused vector of the backscattering coefficient after cross-track focussing. Then, the signal shown in (3) is a linear measurement model, which can be written as

$$\boldsymbol{s}_n = \boldsymbol{\Psi}_c \cdot \boldsymbol{\gamma}_n + \boldsymbol{\omega} \tag{4}$$

where

$$\boldsymbol{\Psi}_c = \left[\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_q \cdots, \boldsymbol{\varphi}_Q\right] \tag{5}$$

where

$$\boldsymbol{\varphi}_q = \left[\exp\left(-j2\pi\left(2y_q/\lambda R_0\right)y_1\right), \cdots, \exp\left(-j2\pi\left(2y_q/\lambda R_0\right)y_N\right)\right] \tag{6}$$

The structure of Eq. (4) is coincidence with the SMV mdoel. So, we can recover the azimuth signal $\boldsymbol{\gamma}$ from measurement vector $\boldsymbol{s}_n$ with CS theory. $\boldsymbol{\Psi}_c$ is the sparse dictionary. Next, we can get the low-dimensional measurement vector through down sampling. We choose the random partial unit matrix as sensing matrix $\boldsymbol{\Phi}_n \in \mathbb{R}^{M \times M_1}$. So the down sampling signal can be expressed as

$$\boldsymbol{s}'_n = \boldsymbol{\Phi}_n \boldsymbol{s}_n = \boldsymbol{\Phi}_n \boldsymbol{\Psi}_c \cdot \boldsymbol{\gamma}_n, \ n = 1, \cdots, N \tag{7}$$

where $\boldsymbol{s}'_n$ is the down sampling signal.

Thus, for cross-track measurement vector, the sparse represent $\boldsymbol{\gamma}$ of signal $\boldsymbol{s}_n$ can be recovered by the following problem

$$\hat{\boldsymbol{\gamma}}_n = \min_{\boldsymbol{\gamma}_n} \|\boldsymbol{\gamma}_n\|_1, \quad \text{s.t.} \quad \left\|\boldsymbol{s}'_n - \boldsymbol{\Phi}_n \boldsymbol{\Psi}_c \boldsymbol{\gamma}_n\right\|_2 < \varepsilon \tag{8}$$

In the reconstruction of cross-track signal, the cross-track direction is vertical with azimuth direction. Meanwhile, at the far field condition, with the length restriction of array antenna, there is no range migration to the same target for all antenna elements. It is implies that the sparse structure of each measurement is the same. Thus, the cross-track direction recovery can be implemented by MMV model. Additional, a same sensing matrix $\mathbf{\Phi}_c \in \mathbb{R}^{M \times M_1}$ should be adopted for the multiple measurement vectors. That is, we can take the $\mathbf{\Phi}_n$ as $\mathbf{\Phi}_c$ for any $n = 1, \cdots, N$.

Thus, the multiple measurement vectors can be denoted as $\mathbf{S} = [\boldsymbol{s}_1, \cdots, \boldsymbol{s}_N]$. M_1 can be regard as the snapshots number. The down sampling signal can be denoted as $\mathbf{\Xi} = \mathbf{\Phi}_a \mathbf{S}$. And the recovery signal can be denoted as $\boldsymbol{\gamma} = [\gamma_1, \cdots, \gamma_N]$. Thus, the problem with MMV model can be reformulated as follows

$$\hat{\gamma} = \min_{\boldsymbol{\gamma}} \|\boldsymbol{\gamma}\|_{2,1} \quad \text{s.t.} \quad \|\mathbf{\Xi} - \mathbf{A}\boldsymbol{\gamma}\|_2 < \varepsilon, \ \mathbf{A} = \mathbf{\Phi}_a \mathbf{\Psi}_c \tag{9}$$

where $\varepsilon \in \mathbb{C}^{M \times N}$. $\|\cdot\|_{2,1}$ is the (2, 1) norm which is defined by $\|\boldsymbol{\gamma}\|_{2,1} = \sum_{i=1}^{N} \|\boldsymbol{\gamma}^i\|_2$ and $\boldsymbol{\gamma}^i$ is the ith row of $\boldsymbol{\gamma}$.

3.2 CS-MUSIC Algorithm

The signal subspace is $R(\mathbf{S})$ and the noise subspace is $R(\boldsymbol{\omega})$, where $R(\cdot)$ denotes the linear space. However, matrix $\mathbf{A}$, the measurement matrix after sparse sampling may not satisfy the above-mentioned property.

The support set of can be denoted by $\text{supp}(\gamma) = \{1 \le q \le Q : \gamma_q \neq 0\}$, where γ_q is the qth raw of $\boldsymbol{\gamma}$. $\mathbf{A}_{I_{K-M_1}} \in \mathbb{C}^{M \times (K-M_1)}$ is composed of the columns of $\mathbf{A}$, and the indexes of the columns are belong to I_{K-M_1}, which is a subset of supp(γ) and $|I_{K-M_1}| = K - M_1$. When $K > M_1$,the array manifold space $R(\mathbf{A}_{I_{K-M_1}})$ and noise subspace $R(\boldsymbol{\omega})$ are not orthogonal. $P_{R(\boldsymbol{\omega})}$ denotes the correlation matrix of noise subspace can be written as $\boldsymbol{\omega}\boldsymbol{\omega}^{\mathbf{H}}$. Thus, the projection space is $R(\boldsymbol{\omega}\boldsymbol{\omega}^{\mathbf{H}}\mathbf{A}_{I_{K-M_1}})$.

According to the above analysis, there are three steps in CS-MUSIC algorithm. Firstly, the indexes set I_{K-M_1} is reconstructed by SOMP or other CS methods. Secondly, the projection space $R(\boldsymbol{\omega}\boldsymbol{\omega}^{\mathbf{H}}\mathbf{A}_{I_{K-M_1}})$ can be obtained by I_{K-M_1} and $\mathbf{A}$. Then the new noise subspace in is $R(P_{R(\boldsymbol{\omega})} - P_{R(\boldsymbol{\omega}\boldsymbol{\omega}^{\mathbf{H}}\mathbf{A}_{I_{K-M_1}})})$.Then the spatial CS-MUSIC spectrum can be expressed as

$$P_{\text{CS-MUSIC}}(y_q) = \frac{1}{\mathbf{A}(y_q)^H \left(P_{R(\boldsymbol{\omega})} - P_{R\left(\boldsymbol{\omega}\boldsymbol{\omega}^{\mathbf{H}}\mathbf{A}_{I_{K-M_1}}\right)} \right) \mathbf{A}(y_q)} \tag{10}$$

3.3 FFT in Cross-track Processing

Due to the SMV model, the spatial smoothing method and searching the peak of the spatial spectral in cross-track domain, the cost of computation in the traditional MUSIC algorithm is unacceptable. MMV model and the CS method have the less burden on peak searching rather than SMV model and spatial smoothing method, respectively. Thus, the FFT can decrease the spectral range of peak searching, which should be adopted.

And the coefficients can be obtained by the fast Fourier transform and be expressed as

$$g(\omega) = \sum_{n=1}^{N} S(y_n) \exp\left(-j2\pi \frac{\omega n}{N}\right), \quad \omega = 1, 2, \ldots N \tag{11}$$

The reduction of the computation can be expressed as

$$\mu = \frac{\cup_{i=1}^{P} \Delta f_i}{f_{all}} \tag{12}$$

where P is the major lobe numbers, Δf_i is the width of the peak lobe, $\cup_{i=1}^{P} \Delta f_i$ denotes the union of the 3 dB of each major lobe and f_{all} is the spatial spectral range in cross-track domain.

4 Simulations

To do further analysis, the simulations are provided. The parameters of platform and antenna, which are referenced the ARTINO system [4], are shown in Table 1.

Table 1. Parameters of platform and antenna.

Parameters	Value	Parameters	Value
Carrier frequency f_c (GHz)	37.5	Space distance of adjacent EPC d (m)	0.004
Bandwidth B_r (MHz)	300	Number of transmitting antenna N_t	10
Height of platform H (m)	500	Number of receiving antenna N_r	11
Velocity of platform v (m/s)	15	Cross-track resolution (m)	4.5
Pulse duration Tr (us)	0.1	Azimuth resolution (m)	0.5
Pulse repeat frequency PRF (Hz)	1000		

The algorithm is designed for 3D distributed scene in real use. So, 3D distributed imaging scene simulation must be added to make the imaging scene less sparse and check the performance. Figure 2 shows the imaging results of the L_1-CS method, MUSIC method and CS-MUSIC method. As we can see, the imaging result in Fig. 2(b) is complete and clear, the Fig. 2(c) misses some scattering points. That is caused by the scene is not sparse enough, which means the application of CS-based method is limited. The cross-track resolution in Fig. 2(d) is lower than the resolution in Fig. 2(b) and (c) because of the reduction of aperture in smoothing algorithm. The validity of the proposed method has been verified.

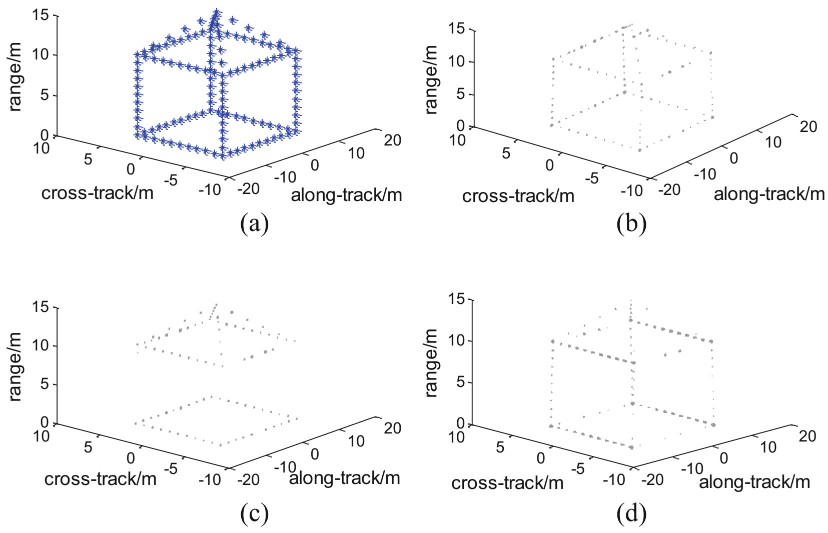

Fig. 2. Three dimensional imagery with SNR = 10 dB. (a) The distribution of the 3D imaging scene, (b) reconstructed by CS-MUSIC algorithm, (c) reconstructed by L_1-CS algorithm, (d) reconstructed by MUSIC algorithm.

To verify the anti-noise performance, the effects of signal-to-noise ratio (SNR), the point interval, and the probability of resolution are provided. In Fig. 3, it can be seen that the probability of resolution are improved with the point interval raised.

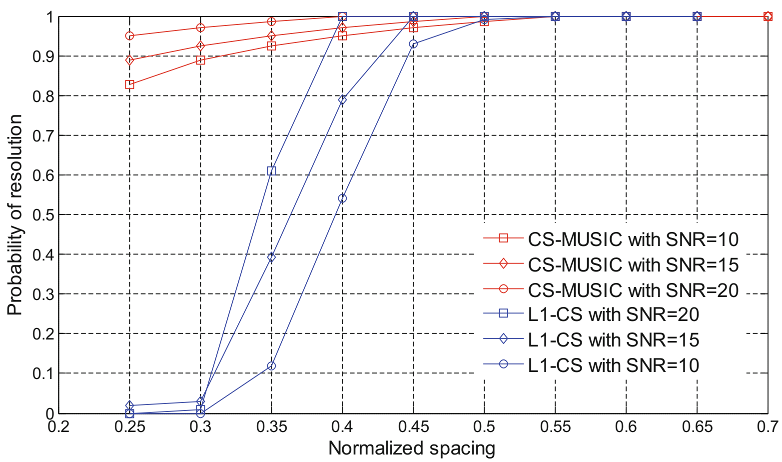

Fig. 3. Probability of separation versus different point interval. Two points are provided at (0, 0, 0) and (0, 0, $0.25\rho_y$–$1.2\rho_y$) with interval $0.05\rho_y$. The times of Monte Carlo simulation is 500.

The spatial spectrum of proposed CS-MUISC-based method, the ordinary spatial smoothing and nearby spatial smoothing method are given. Figure 4 shows five points located at −7, −4, 0, 5, and 9 m in cross-track domain. The spatial spectrum of proposed CS-MUISC-based method has five peaks, however, the spatial spectrum of nearby spatial smoothing method has only four peaks, and the spatial spectrum of the ordinary spatial smoothing method has only three peaks.

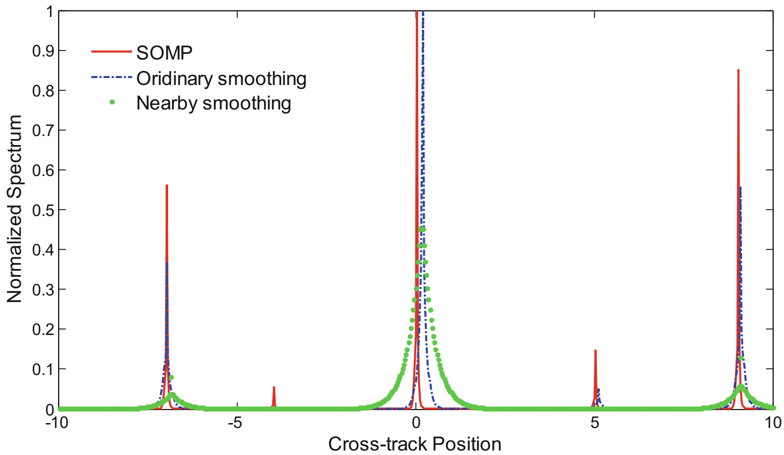

Fig. 4. Spatial spectrum in cross-track direction.

5 Conclusion

In this paper, we exploit the CS-MUSIC method for DLSLA 3-D SAR imaging at cross-track direction. The cross-track resolution can be improved compared with the conventional MUSIC-based imaging method, and the proposed method can obtain a better performance of anti-noise compared with the CS-based method. Finally, we validate our theory by extensive numerical experiments.

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