Analysis of Passive Intermodulation Effect on OFDM Frame Synchronization

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Abstract. Passive intermodulation can lead to a decrease in the performance of frame synchronization for the orthogonal-frequency-division multiplexing (OFDM) systems. In this paper, the Schmidl&Cox algorithm of frame synchronization is simplified by difference calculation to avoid overly complicated analysis. The statistical properties of time metric function in the presence of passive intermodulation interference are obtained by Gaussian distribution fitting. The closed form of false and missing detection probabilities are derived to evaluate the frame synchronization performance. Finally, simulations are conducted to demonstrate the validity of the analysis results.

Keywords: Passive intermodulation · Frame synchronization
Orthogonal-frequency-division multiplexing (OFDM) · Statistical properties

1 Introduction

As the demand of propagation rate rises, multicarrier modulation has been used in wireless communication systems. The orthogonal-frequency-division multiplexing (OFDM) system is one of the most successful implementations of multicarrier modulation. But when the system transmits the multiple carriers, the carriers which pass through the passive device can generate the combination products of the multi-frequencies due to nonlinearity [1]. Thus the passive intermodulation (PIM) products are formed. PIM has become a threat for these multicarrier systems, especially for the OFDM system with high transmitting power [2, 3].

PIM can lead to a degradation of the sensitive receivers when falling into the receiving band. The degradation in the performance of the communication systems can be quantified by bit error rate (BER) and synchronization probabilities. The PIM effects on the BER of M-PSK modulations were investigated in [4]. However, there is no complete model to characterize the PIM effect on synchronization. The OFDM frame synchronization is to find the start position of every frame in OFDM systems. The classical Schmidl&Cox synchronization algorithm was proposed for OFDM system by Schmidl and Cox in 1997 [5]. Based on the Schmidl&Cox algorithm, the influence of narrowband interference on timing synchronization was investigated by Marey and Steendam [6]. However, the broadband characteristic of PIM interference brings difficulty to the analysis on frame synchronization [7, 8].

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Motivated by the above observations, the frame synchronization performance in the presence of PIM using the simplified Schmidl&Cox method was investigated in this paper. To avoid complicated calculation of analysis, the Schmidl&Cox algorithm is simplified by the difference calculation at first. Among various models of PIM, the non-analytic behavioral model proposed by Jacques Sombrin is used for its simple and effective property [9]. The statistical properties of the time metric function falling into or outside the cyclic prefix (CP) are analyzed in presence of PIM. According to the predefined decision threshold, the false and missing detection probabilities analysis of OFDM systems interfered by PIM are obtained. Simulation results demonstrate the validity of the approximations with the analysis.

This paper is organized as follows. In Sect. 2, we describe the simplified Schmidl&Cox algorithm. In Sect. 3, the statistical properties of the time metric function are analyzed under the influence of PIM. We discuss the false and missing detection probabilities of OFDM systems in presence of PIM in Sect. 4. Finally conclusions are given in Sect. 5.

2 Frame Synchronization Algorithm of OFDM Systems

Here we adopt the typical frame structure of the OFDM system from the Schmidl&Cox algorithm, which uses two training sequences as the frame header [5]. It is shown in Fig. 1. Due to the fact that the first training symbol is used for frame synchronization at the receiver [5], we only focus on the first training symbol in this paper.

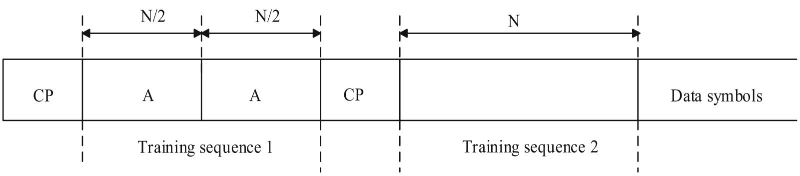

Fig. 1. The typical frame structure of OFDM in Schmidl&Cox algorithm

The first training symbol can transmit sequence PN_1 on the even frequency, and transmit "0" on the odd frequency. Through IFFT, we can obtain the training symbols with two same parts in the time domain.

Since the characteristics of the training symbol in time domain basically remain unchanged at the receiver, we adopt difference method to find the start position of the frame based on the accordance of the two parts of the training symbol.

Define the time metric function as

$$M(d) = \frac{1}{N}\sum_{m=0}^{N/2-1} [r(d+m) - r(d+m+N/2)]^2 \tag{1}$$

where $r(d)$ is the demodulated signal sample, N is the length of training symbol, d is the time indication of the first sample in the N-sample window. The window slides in the time domain to search for the first training symbol.

The iteration of the time metric function is

$$M(d+1) = M(d) + \frac{1}{N}[r(d+N/2) - r(d+N)]^2 - \frac{1}{N}[r(d) - r(d+N/2)]^2 \quad (2)$$

When $M(d)$ is minimum, the start position d is correctly acquired. Therefore, the start position of the signal is estimated as

$$\hat{d} = \arg\min M(d). \quad (3)$$

3 Statistical Properties of Time Metric in the Presence of Passive Intermodulation

The statistical properties of the time metric function in presence of PIM are the basis for analyzing the false or missing probability of the OFDM system. If the locating start position of the signal is behind the actual start position, it will inevitably introduce intersymbol interference (ISI). On the contrary, if the locating position falls into CP, the data can also be correctly received after amendment [10]. Therefore, it is generally considered that the frame can be correctly grabbed when falling into CP. Motivated by the above observations, we will elaborate the statistical properties of time metric function in two cases, i.e., falling into and not falling into CP.

If s_m is the sampled useful signal with the variance σ_s^2, and z_m is the PIM interference with the variance σ_z^2, the signal at the receiver can be expressed as $r_m = s_m + z_m$. The signal-to-interference power ratio SIR at the receiver is σ_s^2/σ_z^2.

For convenience, we define $(r_{d+m} - r_{d+m+N/2})^2$ as P_m.

3.1 Statistical Analysis of Time Metric Function Falling into Cyclic Prefix

When falling into the CP, P_m is denoted as P_{m_in}, and it only contains PIM interference z_m, then the time metric function $M(d_{in})$ can be expressed as

$$M(d_{in}) = \frac{1}{N}\sum_{m=0}^{N/2-1} P_{m\_in} = \frac{1}{N}\sum_{m=0}^{N/2-1} \left[z_{d+m} - z_{d+m+N/2}\right]^2 \quad (4)$$

According to the CLT, when N is larger, the time metric function $M(d_{in})$ follows the Gaussian distribution. Assuming that each sampling point is independent of each other and the mean value is 0, the mean of P_{m_in} is $2\sigma_z^2$ and the variance is

$$\begin{aligned}\mathrm{var}(P_{m\_in}) &= E[P^2_{m\_in}] - E^2[P_{m\_in}] = E[(z_{din+m} - z_{din+m+N/2})^4] - 4\sigma_z^4 \\ &= 2E[z_m^4] + 6E^2[z_m^2] - 2\sigma_z^2 = 2E[z_m^4] + 2\sigma_z^4\end{aligned} \tag{5}$$

We consider the PIM interference model by non-analytic model as $z_m = \alpha x_m |x_m|^{0.6}$ [9], where x_m is the input signal of passive device. In OFDM system, x_m can be verified by simulation to approximately follow the Laplace distribution. Considering $E[z_m^2] = \sigma_z^2$, we have $E[z_m^4] = E[\alpha^4 x_m^4 |x_m|^{2.4}] \approx 20\sigma_z^4$ through high order moment of Laplace distribution. Then the variance of P_{m_in} is about 42 σ_z^4. By the CLT, M (d_{in}) follows the Gaussian distribution with the mean σ_z^2 and the variance $21\sigma_z^4/N$.

Therefore, the probability density function (PDF) of $M(d_{in})$ can be given by

$$f_{Md\_in}(x) = \frac{1}{\sqrt{42\pi\sigma_z^2/N}} \exp\left(-\frac{N(x-\sigma_z^2)}{42\sigma_z^4}\right) \tag{6}$$

When N = 1024 and $\sigma_z^2 = 1$, the theoretical and simulated PDFs are shown in Fig. 2.

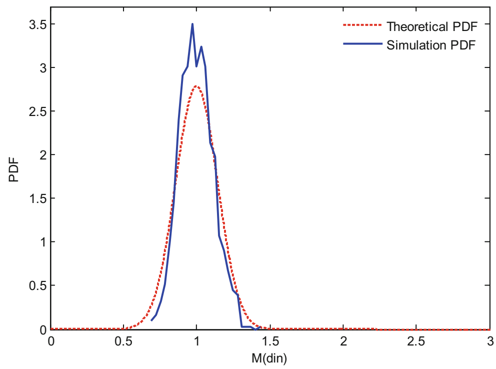

Fig. 2. Simulation and theoretical PDFs of M (d) falling into the cyclic prefix

Figure 2 reflects that the PDF of M (d_{in}) is approximately consistent with the Gaussian distribution when falling into the CP. In general, the simulation and theoretical curves are both Gaussian distributions with mean value σ_z^2, but the variances exist little difference, which is caused by the approximation when solving M (d_{in}).

3.2 Statistical Analysis of Time Metric Functions Outside Cyclic Prefix

When the time metric function is outside the CP, P_m does not only contain the PIM interference z_m, but also includes the useful signal s_m. Replace P_m with P_{mout}, the time metric function M (d_{out}) can be written as

$$M(d_{out}) = \frac{1}{N}\sum_{m=0}^{N/2-1} P_{out} = \frac{1}{N}\sum_{m=0}^{N/2-1} \left((s_{d+m} + z_{d+m}) - (s_{d+m+N/2} + z_{d+m+N/2})\right)^2 \quad (7)$$

Similar to Sect. 3.1, the time metric function $M\ (d_{out})$ follows Gaussian distribution when N is larger by CLT. Here we assume that each sampling point is independent of each other, the useful signal and PIM interference are independent of each other, and both the mean values of z_m and s_m are 0. Then the mean of P_{mout} is $2\sigma_s^2 + 2\sigma_z^2$.

Based on the fact that $E[\mathrm{s}_m^2] = \sigma_{\mathrm{s}}^2$, and s_m approximately follows Gaussian distribution, which has been verified by the simulation, the variance of P_{mout} can be given by

$$\begin{aligned} \mathrm{var}(P_{mout}) &= E[P_{mout}^2] - E^2[P_{mout}] = 2E[r_m^4] + 6(\sigma_s^2 + \sigma_z^2)^2 - (2\sigma_s^2 + 2\sigma_z^2) \\ &\approx 8\sigma_s^4 + 16\sigma_s^2\sigma_z^2 + 42\sigma_z^4 \end{aligned} \quad (8)$$

$M\ (d_{out})$ follows the Gaussian distribution by CLT, and its mean is $\sigma_s^2 + \sigma_z^2$ and variance is $(4\sigma_s^4 + 8\sigma_s^2\sigma_z^2 + 21\sigma_z^2)/N$. Then the PDF of $M\ (d_{out})$ is

$$f_{Mdout}(x) = \frac{1}{\sqrt{2\pi/N \cdot (4\sigma_s^4 + 8\sigma_s^2\sigma_z^2 + 21\sigma_z^4)}} \exp\left(-\frac{N[x - (\sigma_s^2 + \sigma_z^2)]^2}{8\sigma_s^4 + 16\sigma_s^2\sigma_z^2 + 42\sigma_z^4}\right) \quad (9)$$

When N = 1024, $\sigma_s^2 = 1$, and $\sigma_z^2 = 1$ (that is, the SIR is 0 dB), the theoretical and simulated PDF are shown in Fig. 3.

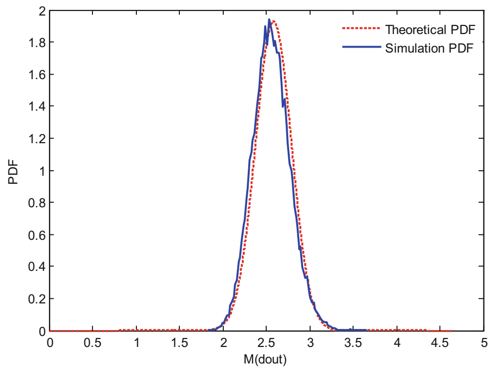

Fig. 3. Simulated and theoretical PDFs of $M\ (d)$ outside cyclic prefix

From Fig. 3, when $M\ (d)$ is outside the CP, the PDF is consistent with Gaussian distribution with the mean value $\sigma_s + \sigma_z$, the variance is also approximately equal. Therefore, we can conclude that the statistical property model for $M\ (d)$ is reasonable.

4 Probability of Missing/False Detection in Presence of Passive Intermodulation

During the process of frame capture, we predefined a threshold λ. When the time metric function $M(d)$ is larger than λ, we judge that the correct start position is not reached and the capture is not completed. Otherwise, we conclude that the correct start position has been found and the process of capture is completed.

From Eqs. (6) (9), the time metric functions $M\ (d_{in})$ and $M\ (d_{out})$ both follow the Gaussian distribution. When $M\ (d)$ is no more than the predefined threshold λ, but $M\ (d)$ does not fall into the CP, it causes a false detection [10]. The false probability can be expressed as

$$P_{false} = \int_{-\infty}^{\lambda} f_{Mout}(m)dm = \int_{-\infty}^{\lambda} \frac{1}{\sqrt{2\pi}\sigma_{out}} \exp\left[-\frac{(m-u_{out})^2}{2\sigma_{out}^2}\right]dm = \frac{1}{2} + \frac{1}{2} erf\left(\frac{\lambda - u_{out}}{\sqrt{2}\sigma_{out}}\right) \quad (10)$$

When $M\ (d)$ falls into the CP, but $M\ (d)$ is greater than the predefined threshold λ, a missing detection occurs [10]. The missing probability can be expressed as

$$P_{miss} = \int_{\lambda}^{\infty} f_{M\_in}(m)dm = \int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{in}} \exp\left[-\frac{(m-u_{in})^2}{2\sigma_{in}^2}\right]dm = erfc\left(\frac{\lambda - u_{in}}{\sqrt{2}\sigma_{in}}\right) \quad (11)$$

Since $M\ (d_{in})$ and $M\ (d_{out})$ both follow the Gaussian distribution, the decision threshold λ is constructed by the variance ratio of the two Gaussian distributions.

$$\lambda = \sigma_z^2 \left[1 + \frac{SIR}{1 + \sqrt{(8SIR^2 + 16SIR + 42)/42}}\right] \quad (12)$$

For the sake of brevity, we normalize the power of PIM as 1. Therefore, the false detection probability is

$$P_{false} = \frac{1}{2} + \frac{1}{2} erf\left(\frac{\lambda - 1 - SIR}{\sqrt{(8SIR^2 + 16SIR + 42)/1024}}\right) \quad (13)$$

The missing detection probability is

$$P_{miss} = \frac{1}{2} erfc\left(\frac{\lambda - 1}{\sqrt{42/1024}}\right) \quad (14)$$

In the case of the influence of PIM, based on the synchronization method in Sect. 2 and the threshold λ predefined above, the false and missing detection probabilities are respectively shown in Figs. 4 and 5.

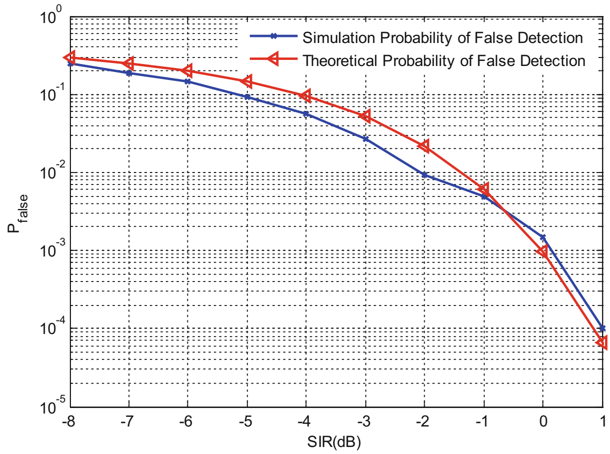

Fig. 4. The theoretical and simulated curves of false detection probability

In Figs. 4 and 5, the theoretical analysis is consistent with the simulation. And for the same false or missing detection probability, there is a less than 2 dB SIR gap between the theoretical and the simulation curves, which illustrates the rationality of the analysis model.

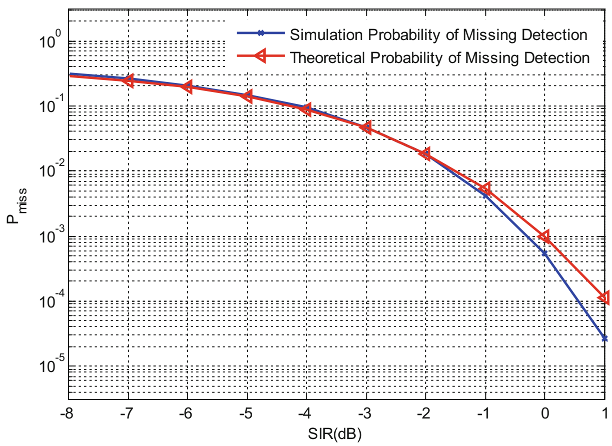

Fig. 5. The theoretical and simulated curves of missing detection probability

5 Conclusion

In this paper, we have analyzed the effects of PIM on the performance of OFDM frame synchronization. Based on the Schmidl&Cox algorithm, difference calculation method has been used to simplify analysis computation. The probabilities of both missing and false detections of a training sequence are derived by the statistical properties of time metric function. As the results shown, the theoretical analysis model of frame synchronization performance in the presence of PIM approximates the simulation result within 2 dB bias, which can provide useful guidance for the design of OFDM systems.

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