Computationally Efficient 2D DOA Estimation for Cylindrical Conformal Array

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Abstract. A computationally efficient two-dimensional (2D) direction of arrival (DOA) estimation method based on cylindrical conformal antenna array is investigated in this paper. By dividing the entire array into several sub-arrays and transforming every sub-array to virtual uniform rectangular array (URA) via interpolation technique, the generalization propagator method (GPM) without eigen-decomposition is employed to estimate the noise subspace accurately and quickly. Furthermore, in order to lower the computational complexity of the 2D spectral peak searching, a rank reduction (RARE) method based on URA is utilized to solve the 2D DOAs by successive 1D spectrum functions. At last, some numerical simulations verified the superiority of the proposed method.

Keywords: Conformal antenna array · Direction of arrival
Interpolation technique · Generalization propagator method
Rank reduction algorithm

1 Introduction

The conformal array is usually referred to an array amounted with sensors on the curvature surface [1]. The conformal array has many advantages that contains reduction of aerodynamic drag, wide-angle coverage, space saving, reduction of radar cross-section and so on [2]. Due to this flexibility, conformal array has many promising applications in a variety of fields such as radar, sonar, airborne, ship-borne and wireless communication [3].

Among various of techniques for conformal array, the direction of arrival (DOA) estimation has attracted a lot of interests. However, in contrast to the ordinary array, the distinct electromagnetic characteristics of conformal array leads to an tough problem of DOA estimation owing to the curvature of the carrier surface. As a result, the DOA algorithm such as multiple signal classification (MUSIC) [4] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [5] and other conventional methods are not suitable for conformal array directly. Besides, because of the "shadow effect" of the metallic

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cylinder, not all of the sensors can receive the signal and which will degrade the detection performance dramatically. In view of these problems, many DOA algorithms have been investigated recently. [6] proposed a higher accuracy DOA estimation via parallel factor analysis (PARAFAC). A general transformation procedure based on geometric algebra is proposed in [7] and the author estimated the parameters by ESPRIT too. [8] introduced a new perspective to shadowing effect and utilized rank reduction (RARE) method to obtain better estimation performance. In order to detect more signals than sensors, [9] firstly utilized the array extension character of the nested array to improve the degree of freedom (DOF) of the array. However, the references mentioned above need eigen-decomposition or 2D spectral peak searching, which will bring much computational complexity to the actual system.

The contribution of this paper is developing a fast DOA estimation method based on cylindrical conformal array. By dividing the whole array into sub-arrays, the interpolation technique is exploited to map each sub-array to an uniform rectangular array (URA). Then, the DOAs are estimated based on the efficient Generalization propagator method (GPM) [10] algorithm without any eigen-decompositon. Besides, the proposed method only requires several 1-D spectrum peaking searchings to estimate the 2D DOAs. Moreover, the estimated parameters are automatically paired together without extra operation.

2 The Signal Model of the Conformal Array

Consider D narrowband far-field signal sources that impinge on an arbitrary 3D conformal array of M directional sensors. Assume $k_0 = 2\pi/\lambda$ and λ is the wavelength of the signal source, the snapshot data model is established in [11], and the corresponding array steering vector is given by

$$\mathbf{a}(\theta, \phi) = [r_1 e^{-jk_0 \mathbf{p}_1 \cdot \mathbf{u}}, r_2 e^{-jk_0 \mathbf{p}_2 \cdot \mathbf{u}}, ..., r_M e^{-jk_0 \mathbf{p}_M \cdot \mathbf{u}}]^T \tag{1}$$

$$\begin{aligned} r_i &= (g_{i\theta}^2 + g_{i\phi}^2)^{1/2}(k_{i\theta}^2 + k_{i\phi}^2)^{1/2} cos(\theta_{igk}) = |g_i||p_l| cos(\theta_{igk}) \\ &= \mathbf{g}_i \cdot \mathbf{q}_l = g_{i\theta} k_\theta + g_{i\phi} k_\phi \end{aligned} \tag{2}$$

where θ and ϕ are the elevation and azimuth angles, respectively. $\mathbf{p}_i = [x_i, y_i, z_i]$, i=1,2,...M denotes the position vector of the ith sensor, $\mathbf{u} = [sin\theta cos\phi, sin\theta sin\phi, cos\phi]^T$ denotes the propagation vector. r_i is the response of unit signal by the ith element in the global coordinate system. As shown in Fig. 1(b), $\mathbf{u}_\theta$ and $\mathbf{u}_\phi$ are unit vectors, k_θ and k_ϕ are the polarisation parameters of signal, $\mathbf{g}_i$ is the pattern of the ith element, $\mathbf{q}_i$ is the direction of the electric field, θ_{igk} denotes the angle between vector $\mathbf{g}_i$ and vector $\mathbf{q}_i$. The critical step is the transform from global coordinate to local coordinate and More details can be found in [11]. Thus, the snapshot data model of conformal array antenna can be expressed as

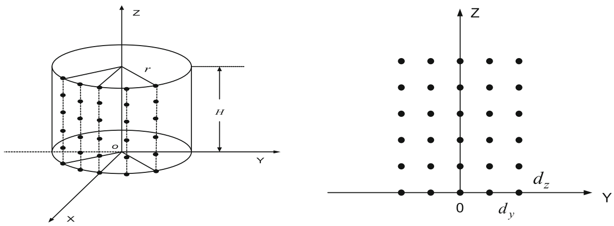
$$\begin{aligned} \mathbf{X}(n) &= \mathbf{G} \cdot \mathbf{A}\mathbf{S}(n) + \mathbf{N}(n) = (\mathbf{G}_\theta \cdot \mathbf{A}_\theta \mathbf{K}_\theta + \mathbf{G}_\phi \cdot \mathbf{A}_\phi \mathbf{K}_\phi)\mathbf{S}(n) + \mathbf{N}(n) \\ &= \mathbf{B}\mathbf{S}(n) + \mathbf{N}(n) \end{aligned} \tag{3}$$

where $\mathbf{B} = \mathbf{G} \cdot \mathbf{A}$, $\mathbf{G}$ is the antenna response matrix, $\mathbf{A}$ is the $M \times D$ full-rank steering matrix, $\mathbf{S}(n)$ denotes the $D \times 1$ source waveforms and $\mathbf{N}(n)$ is the $M \times 1$ additive noise which is spatially white and statistically independent from the signal source.

3 The Proposed Method

3.1 Cylinder Conformal Array Structure and the 2D Array Interpolation Technique

The configure of the cylindrical conformal antenna array is given in Fig. 1(a), where the sensors are uniformly distributed over the surface of the cylinder. Due to the "shadow effect", which could degrade the DOA estimation performance dramatically because of the incomplete steering vector, the sub-array divided technique is utilized in this paper. Firstly, we divide the whole array into 6 sub-arrays, and each sub-array covers a sector of $\pi/3$. Thus, we could always find a sub-array and all the sensors of this sub-array can receive the signal from any direction. Then, the combination of all sub-arrays will cover all the possible impinging signals. Because the array structure and the DOA estimation process are the same for each sub-array, only one sub-array shown in Fig. 1(a) is considered throughout this paper.



(a) (b)

Fig. 1. (a) Cylindrical conformal array. (b) Interpolated array.

The 2D interpolation technique is used here to mitigate the effects of imperfect conformal array response, such as the diversity of the sensor's response, mutual coupling effect and so on. The principle of interpolated array is to transform the true conformal array response $\mathbf{B}$ to the desired virtual array respoone $\tilde{\mathbf{A}}$ by define an interpolation matrix $\mathbf{T}$ within the field of view [12], as shown in Fig. 1(b). This process can be expressed as

$$\mathbf{T}^H(\mathbf{G} \cdot \mathbf{A}) = \tilde{\mathbf{A}}. \tag{4}$$

Obviously, we cannot calculate a pert $\mathbf{T}$ because the solution of (4) is not close-form. Thus, The equation of $\tau = \left\| \tilde{\mathbf{A}} - \mathbf{T}^H(\mathbf{G} \cdot \mathbf{A}) \right\| / \|\mathbf{G} \cdot \mathbf{A}\|$ is used to evaluate the interpolation accuracy. When τ is small enough, for example, 0.01, then $\mathbf{T}$ can be accepted. A stable and accurate method to solve the $\mathbf{T}$ is proposed in [13]. It is observed that it will cost much time in calculating the interpolation matrix $\mathbf{T}$ if the size of the sector $[\Theta, \Phi]$ and number of interpolation are very large. However, we only need to calculate the matrix $\mathbf{T}$ once and it also can be done off-line and stored in the system so that it won't increase the computation burden when estimating the DOA parameters.

3.2 DOA Estimation Method Based on GPM and RARE

In the last section, we transform the conformal array to a virtual URA firstly, as shown in Fig. 1(b). The URA has $M = M_y \times M_z$ sensors, and the sensor's space along y axis and z axis are d_y and d_z, respectively. For simplicity, we define $d_y = d_z = \lambda/2$. Then, the steering vector of the URA $\tilde{\mathbf{A}}$ is given by

$$\tilde{\mathbf{A}}(u, v) = [\tilde{\mathbf{a}}(u_1, v_1), \tilde{\mathbf{a}}(u_2, v_2), ..., \tilde{\mathbf{a}}(u_D, v_D)] \tag{5}$$

where $\tilde{\mathbf{a}}(u, v) = \tilde{\mathbf{a}}_y(u) \otimes \tilde{\mathbf{a}}_z(v)$, $\tilde{\mathbf{a}}_y(u) = [1, e^{-j(2\pi/\lambda)d_y u}, ..., e^{-j(2\pi/\lambda)d_y(M_y-1)u}]^T$, $\tilde{\mathbf{a}}_z(v) = [1, e^{-j(2\pi/\lambda)d_z v}, ..., e^{-j(2\pi/\lambda)d_z(M_z-1)v}]^T$. And $u = sin\theta sin\phi$ and $v = cos\theta$ are the direction variables relative to the y-axis and z-axis, respectively. Then, according to (3), the original conformal array manifold $\mathbf{B}(u, v)$ can be rewritten as

$$\mathbf{B}(u, v) = \mathbf{G} \cdot \mathbf{A} = \mathbf{F}\tilde{\mathbf{A}}(u, v) \tag{6}$$

where $\mathbf{F} = (\mathbf{T}^H)^{-1}$. Denote $\mathbf{B}(u, v)$ as $\mathbf{B}$ which is decomposed as

$$\mathbf{B} = [\mathbf{B}_1 \mathbf{B}_0 \mathbf{B}_2]^T \tag{7}$$

where $\mathbf{B}_1 \in \mathbb{C}^{L \times D}$, $\mathbf{B}_0 \in \mathbb{C}^{D \times D}$, $\mathbf{B}_2 \in \mathbb{C}^{(M-L-D) \times D}$, $L = 0, ..., M - D - 1$. Note that $\mathbf{B}_0$ is a nonsingular matrix, so two propagator matrices exist and satisfy $\mathbf{B}_1 = \mathbf{P}_{1L}^H \mathbf{B}_0, \mathbf{B}_2 = \mathbf{P}_{2L}^H \mathbf{B}_0$. Define a block matrix $C_L^H \in \mathbb{C}^{(M-D) \times M)}$, which is given by

$$\mathbf{C}_L^H = \begin{bmatrix} -\mathbf{I}_L & \mathbf{P}_{1L}^H & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{2L}^H & -\mathbf{I}_{M-L-D} \end{bmatrix} \tag{8}$$

It is easy to know that $\mathbf{C}_L^H \mathbf{B} = 0$. Then, divide the array output matrix $\mathbf{X}(t)$ as

$$\mathbf{X}(t) = [\mathbf{X}_1 \mathbf{X}_0 \mathbf{X}_2]^T. \tag{9}$$

Assume the noise is zero in (3), we can get $\mathbf{X}_1 = \mathbf{P}_{1L}^H \mathbf{X}_0, \ \mathbf{X}_2 = \mathbf{P}_{2L}^H \mathbf{X}_0$. Then, the covariance matrix of $\mathbf{X}(t)$ can be expressed as

$$\mathbf{R} = E(\mathbf{X}\mathbf{X}^H) = E(\mathbf{X}[\mathbf{X}_1^H \ \mathbf{X}_0^H \ \mathbf{X}_2^H]) = [\mathbf{D} \ \mathbf{E} \ \mathbf{F}] \tag{10}$$

where $\mathbf{D} \in \mathbb{C}^{M\times L}$, $\mathbf{E} \in \mathbb{C}^{M\times D}$, $\mathbf{F} \in \mathbb{C}^{M\times(M-L-D)}$. And we have

$$\mathbf{D} = E(\mathbf{X}\mathbf{X}_1^H) = E(\mathbf{X}\mathbf{X}_0^H)\mathbf{P}_{1L} = \mathbf{E}\mathbf{P}_{1L} \tag{11}$$

$$\mathbf{F} = E(\mathbf{X}\mathbf{X}_2^H) = E(\mathbf{X}\mathbf{X}_0^H)\mathbf{P}_{2L} = \mathbf{E}\mathbf{P}_{2L} \tag{12}$$

where $\mathbf{E} = E(\mathbf{X}\mathbf{X}_0^H)$. However, when considering the noise, the propagator matrix can be estimated by the following minimization problem

$$J_1 = min \left\|\mathbf{D} - \mathbf{E}\mathbf{P}_{1L}\right\|_F^2, \quad J_2 = min \left\|\mathbf{F} - \mathbf{E}\mathbf{P}_{2L}\right\|_F^2. \tag{13}$$

The optimal solution of J_1 and J_2 are given by

$$\mathbf{P}_{1L} = (\mathbf{E}^H\mathbf{E})^{-1}\mathbf{E}^H\mathbf{D}, \mathbf{P}_{2L} = (\mathbf{E}^H\mathbf{E})^{-1}\mathbf{E}^H\mathbf{F}. \tag{14}$$

The DOAs can be obtained by solving the following spectrum function,

$$f(u,v) = (\mathbf{B}(u,v)^H\mathbf{C}_L\mathbf{C}_L^H\mathbf{B}(u,v)) = 0 \tag{15}$$

In order to utilize the full information contained in the received data, reconstruct $\mathbf{C} = [\mathbf{C}_0, ..., \mathbf{C}_{i-1}, \mathbf{C}_{M-D-i+1}, ..., \mathbf{C}_{M-D}]$, where $1 \le i \le \left\lfloor \frac{M-D+1}{2} \right\rfloor$ [10]. Therefore, (15) can be rewritten as

$$f(u,v) = (\mathbf{B}(u,v)^H\mathbf{C}\mathbf{C}^H\mathbf{B}(u,v)) = 0 \tag{16}$$

In fact, the larger the i, the DOA estimation performance will be better, but it will consume much time too. As a result, we prefer to choose the value according practical need. However, it can be seen from (16) that the function requires to perform an exhaustive 2D spectrum speak searching of both u and v, which leads to very high computational complexity. Hence, in order to reduce computation burden, the spectral RARE technique is employed here. It follows from (6) that we can easily get

$$\mathbf{B}(u,v) = \mathbf{F}[\tilde{\mathbf{a}}_y(u) \otimes \tilde{\mathbf{a}}_z(v)] = \mathbf{F}[\mathbf{I}_y \otimes \tilde{\mathbf{a}}_z(v)]\tilde{\mathbf{a}}_y(u). \tag{17}$$

The Eq. (16) can be rewritten as

$$f(u,v) = \tilde{\mathbf{a}}_y^H(u)\mathbf{Z}(v)\tilde{\mathbf{a}}_y = 0 \tag{18}$$

where

$$\mathbf{Z}(v) = [\mathbf{I}_y \otimes \tilde{\mathbf{a}}_z(v)]^H\mathbf{F}^H\mathbf{C}\mathbf{C}^H\mathbf{F}[\mathbf{I}_y \otimes \tilde{\mathbf{a}}_z(v)]. \tag{19}$$

Since $\tilde{\mathbf{a}}_y(u) \neq 0$, (18) holds true only if $\mathbf{Z}(v)$ reduces rank. Generally speaking, $\mathbf{Z}(v)$ is a full-rank matrix, but the $\mathbf{Z}(v)$ will reduce rank when v is the true DOA. As a result, $\mathbf{Z}(v)$ will reach a minimum value when v coincides with the true DOA v_i. Therefore, we could have

$$f(v) = 1/min(\mathbf{Z}(v)). \tag{20}$$

Utilizing the $\{\hat{v}_i\}_{i=1}^{D}$ estimated by (20), the corresponding angles $\{\hat{u}_i\}_{i=1}^{D}$ can be estimated by searching the highest peaks of the following function

$$f(u) = \frac{1}{\|[\mathbf{F}(\tilde{a}_y(u) \otimes \hat{a}_z(v_i))]^H \mathbf{C}\|^2}, \; i = 1, 2, ..., D. \tag{21}$$

At last, the elevation and azimuth angles $(\theta_i, \phi_i)_{i=1}^{D}$ can be obtained by

$$\theta_i = cos^{-1}(v_i), \quad \phi_i = sin^{-1}(\frac{u_i}{sin(\theta_i)}). \tag{22}$$

Remark: Compared with the traditional 2D spectral spectrum algorithm based on subspace decomposition, the method proposed in this paper avoids eigen-decomposition that reduces much computational burden. Besides, we replace the exhaustive 2D spectrum peak searching with successive 1D searchings, which further reduces the calculation cost significantly. In addition, the estimated azimuth and elevation can be paired automatically.

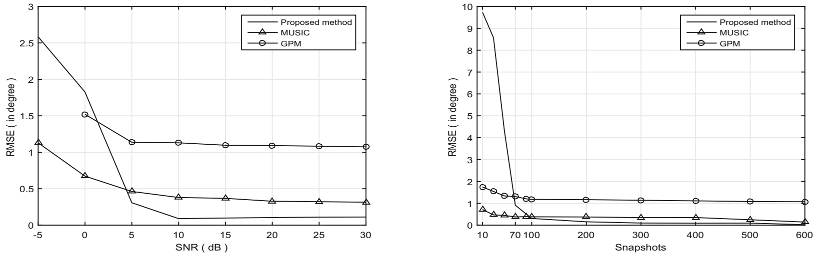
4 Simulations and Results

In order to illustrate the performance of the proposed method, some simulations are taken out in comparison with the traditional MUSIC algorithm and GPM. Because of the symmetry of the array, only one sub-array of the conformal array is utilized in the following simulations. As is shown in Fig. 1(a), the sub-array consists of 30 elements located uniformly and covers a sector of $\pi/3$. Therefore the angle between each two adjacent sensors is $\pi/6$, the height of the cylinder array $H = 5\lambda/2$ and the radius $r = 2\lambda$. As shown in Fig. 1(b), the corresponding interpolated array is composed of 6 rows and 5 columns with $d_x = d_y = \lambda/2$ and the interpolation error $\tau = 0.18$. We take 100 numbers of Monte Carlo trials in the following figures.

As shown in Fig. 2, the root-mean-square-error (RMSE) varying with the SNR and the snapshots are investigated. Assume there are two incident sources, and the DOAs (θ, ϕ) are $(52°, -8°)$ and $(68°, 8°)$, respectively. Assume the snapshots are 500, the RMSE curve versus SNR is plotted in Fig. 2(a) of three methods. Then, let SNR is 10 dB, the RMSE versus snapshots is given in Fig. 2(b). The RMSE values are obtained by running 100 Monte Carlo simulations. It can be seen from the Fig. 2, the performance of the three methods are improved with increased SNR and snapshots. But the estimation accuracy of the proposed method is not as good as MUSIC and PM with low SNR and small snapshots. However, when the SNR is greater than 5 dB and the snapshots is larger than 70, the performance of the proposed algorithm outperforms MUSIC and PM. That's because we adopt the interpolation technique that transforms the original conformal array to a more clear planar array only if the interpolation error τ is small enough.

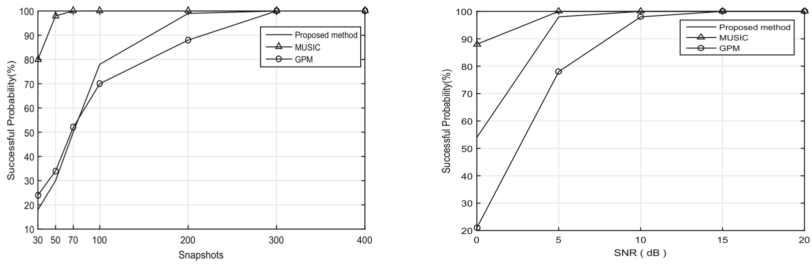
Assume the DOAs are given by $(60°, 0°)$ and $(65°, 0°)$, the resolution performance is examined by the probability of successful detection in Fig. 3. Let the snapshots are 500, the resolution performance varying with SNR is given in

Fig. 3(a), and the resolution performance varying with snapshots when SNR is 10 dB is given in Fig. 3(b). From the Fig. 3, we can see that the proposed method is better than GPM but worse than MUSIC algorithm with low SNR and small snapshots. However, with the SNR and snapshots is larger, the resolution performance of the MUSIC and proposed algorithm are contiguous.



(a) (b)

Fig. 2. (a) The RMSE versus SNR. (b) The RMSE versus snapshots.



(a) (b)

Fig. 3. (a) The successful probability versus snapshots. (b) The successful probability versus SNR.

5 Conclusion

In this paper, we have demonstrated an efficient 2D DOA algorithm for cylindrical conformal array. In this method, in order to avoid "shadow effect" of the metallic cylinder, the array is divided into several sub-arrays. Then, the interpolation technique is applied to transform the curved arrays to URA with omni-directional elements. Next, by combining the GPM and RARE method together, the signal's DOAs could be solved accurately and quickly. Rather than estimate the DOAs by the 2D spectrum peak searching, the proposed algorithm can obtain the 2D DOAs by several 1D peak searching, which reduce the computational complexity greatly. Moreover, the azimuth and elevation can be paired automatically.

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