

An Optimized Method for Turbocharged Diesel Engine EGR Performance Evaluation

Xiang-huan Zu, Chuan-lei Yang^(✉), He-chun Wang,
and Yin-yan Wang

College of Power and Energy Engineering, Harbin Engineering University,
Harbin 150001, China

zuhuan0815@163.com,

{yangchuanlei, wanghechun}@hrbeu.edu.cn,

wyzzxm@sina.com

Abstract. The purpose of this study is to propose a multi-objective decision making optimization method based on subjective and objective empowerment optimization. This paper intend to achieve the evaluation of turbocharged diesel engine EGR performance. First, the main diesel engine EGR parameters were selected as decision target and corresponding experimental data are used as effect sample matrix to establish the initial multi-objective-making model. The characteristics and optimization requirements of turbocharged diesel engine EGR are considered. Secondly, the expert scoring, grey correlation analysis are used to solve the optimized weight vector. Finally, the optimized decision-making model was established to explore the intrinsic objective relationship of EGR evaluation index parameters and give the best evaluation and optimal decision. The results show that the optimized method can successfully solve the turbocharged diesel engine EGR performance evaluation and optimal decision problem, which can provides theoretical support and reference for the further optimization of EGR.

Keywords: Data mining · Performance evaluation · Diesel engine
Exhaust gas recirculation · Optimized model

1 Introduction

As one of the effective ways to reduce NO_x pollutants emissions, exhaust gas recirculation (EGR) has been widely used in marine turbocharged diesel engines. The main process is to introduce a part of exhaust gas into the intake pipe, mixed with fresh air and enter the cylinder to re-enter the combustion process [1–3]. The key to the EGR technology is to introduce enough exhaust gas into the intake pipe and give the best EGR rate according to the different operating conditions of the engine [4, 5]. Due to the effect of different EGR rates on diesel engine performance and emissions is different. Therefore, the power, economy and emission performance of diesel engine must be taken into account in determining the optimal EGR rate. The basic principle of EGR is to reduce NO_x emissions as much as possible, while have a minimal impact of other pollutants emissions.

At present, the general approach is to obtain the operating parameters of the engine through a large number of tests, then analyzing the test results synthetically to specify the corresponding determination principle. Such as Shuai [4], Zhang [5], taking 13 working point particles do not exceed the principle of the original machine. In the determination of the best EGR rate. Because of the subjective judgment and the purpose of the decision-maker is different, the current decision-making principles are not the same. There is a common shortcomings that is subjective factors are too strong and lack a clear theoretical support and guidance. Therefore, it is inevitable that the subjective judgment will bring the error and affect the final selection result. Obviously the EGR performance evaluation can be seen as a typical multi-objective decision-making problems. Therefore, the multi-objective grey situation decision-making method is used. As an important branch of grey theory which is a classic artificial intelligence method, the multi-objective grey situation decision making theory has unique advantages in decision-making problems for selecting the best scheme for a number of programs [6], which has been widely used in aerospace, electronic and other fields because of its low computational complexity and high recognition effect [7–12]. Currently, more and more scholars have considered to seek the optimization of decision-making model [11–13], to improve the reliability of decision-making results, However, different optimization methods are only suitable for some specific issues. By reviewing the relevant information, we have not found the clear literature on the EGR performance evaluation and the optimal EGR rate of the turbocharged diesel engine, and the related theoretical guidance is also less, so it is necessary to explore the subject of this study.

In summary, in order to solve the EGR performance evaluation and optimal decision-making of the turbocharged diesel engine, an optimized multi-objective grey situation decision-making method is proposed. This method makes use of the advantages of traditional grey decision, grey relational analysis, while combining the characteristics and optimization requirements of diesel engine EGR performance, which can explore the intrinsic association between different EGR performance parameters and the ranking of different EGR schemes can be obtained. The results show the this approach can successfully applied to EGR performance evaluation problem and the evaluation results is reasonable, Which has certain theoretical reference and guidance significance for the optimization of turbocharged diesel engine EGR performance.

2 Preliminary Knowledge

2.1 Multi-objective Grey Decision-Making

The main components of the traditional multi-objective grey decision model include event set, strategy set, situation set, decision goal and decision weight.

First, Construct the corresponding set of situations according to the event set and the strategy set. Assume that $A = \{a_1, a_2 \cdots a_n\}$ is the event set, the strategy set is $B = \{b_1, b_2 \cdots b_m\}$, the situation set is $s = \{s_{ij} = (a_i, b_j) \mid a_i \in A, b_j \in B\}$, and the $u_{ij}^{(k)}$ ($i = 1, 2 \cdots n; j = 1, 2, \cdots m$) is the effect sample value of the situation under the target.

Secondly, Choose targets and each target needs to determine its effectiveness measure:

$$r_{ij}^{(k)} = \frac{u_{ij}^{(k)}}{\max_i \max_j \{u_{ij}^{(k)}\}} \tag{1}$$

called upper effect measure, which mainly used to measure the degree of albino value deviated from the maximum whitening value;

$$r_{ij}^{(k)} = \frac{\min_i \min_j \{u_{ij}^{(k)}\}}{u_{ij}^{(k)}} \tag{2}$$

called lower effect measure, which mainly for the degree of albino value deviation from the lower limit;

$$r_{ij}^{(k)} = \frac{u_{i_0j_0}^{(k)}}{u_{i_0j_0}^{(k)} + |u_{ij}^{(k)} - u_{i_0j_0}^{(k)}|} \tag{3}$$

called medium effect measure, where $u_{i_0j_0}^{(k)}$ is the moderate effect of the specified effect under the target.

These three measures are applicable to different occasions: If you want the situation the bigger the better, you can use the upper effect measure; if you want the smaller the better the loss of the situation, then the lower effect measure can be chose, if you want the effect to be near a specified value, use a medium effect measure.

Thirdly, solve the consistent effect measure matrix of situation set according to the effect measure of each target.

$$R^{(k)} = (r_{ij}^{(k)}) = \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \cdots & r_{1m}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \cdots & r_{2m}^{(k)} \\ \vdots & \vdots & \cdots & \vdots \\ r_{n1}^{(k)} & r_{n2}^{(k)} & \cdots & r_{nm}^{(k)} \end{bmatrix} \tag{4}$$

where, $r_{ij}^k = (r_{ij}^{(1)}, r_{ij}^{(2)}, \dots, r_{ij}^{(3)})$ is called the consistent effect measure vector of situation s_{ij} under target k.

The fourth step is to Establish decision weight $\eta_k (k = 1, 2, \dots, s)$, where $\sum_{k=1}^s \eta_k = 1$ and solve integrated effect measure r_{ij} and integrated effect measure matrix of situation s_{ij}

$$r_{ij} = \sum_{k=1}^s \eta_k \bullet r_{ij}^{(k)} \tag{5}$$

$$R = (r_{ij}) = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \quad (6)$$

At last, if $\max_{1 \leq j \leq m} \{r_{ij}\} = r_{ij_0}$, then called b_{j_0} is the optimal strategy to event a_i ; if $\max_{1 \leq j \leq m} \{r_{ij}\} = r_{ij_0}$ then called a_{i_0} is the optimal event to strategy b_j ; if $\max_{1 \leq j \leq m} \{r_{ij}\} = r_{ij_0}$ then called $s_{i_0j_0}$ is optimal situation.

2.2 Grey Correlation Analysis

Grey relational analysis theory is an important branch of grey system theory [13, 14]. The linearly interpolated method is used to transform the discrete behavior observations of the system factors into the polylines of segmented readings, then construct the model of measure degree according to the geometric feature of the polyline.

The basic steps of the grey relational model are as follows:

Step 1: the original sequence

$$X_0(t) = \{x_0(1), x_0(2), \cdots, x_0(n)\}$$

is specifies the reference data sequence, also called the parent sequence.

$$X_i(t) = \{x_i(1), x_i(2), \cdots, x_i(n)\}$$

is the sequence of data to be compared, also known as the sub sequence;

Step 2: make $\xi_i(k)$ is the correlation coefficient for sequence $X_0(t)$ and $X_i(t)$ at time k :

$$\xi_i(k) = \frac{\min_i \min_k |x_0(k) - x_i(k)| + 0.5 \max_i \max_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + 0.5 \max_i \max_k |x_0(k) - x_i(k)|} \quad (7)$$

where 0.5 is the resolution factor, usually between 0–1.

Step 3: Calculate the average of the correlation coefficients at each time of sequence $X_i(t)$, i.e. the degree of correlation of the subsequence $X_i(t)$ to the parent sequence $X_0(t)$:

$$r_i = \frac{1}{N} \sum_{k=1}^N \xi_i(k) \quad (8)$$

3 Optimization of Decision-Making Target Weights

In the traditional multi-objective grey decision model, the target weight is usually determined by the subjective weighting method. Although the method can play the expertise or experience of experts or technical staff and has a certain degree of professionalism, it will have an impact on the evaluation decision results because of its great subjectivity and arbitrariness. Therefore, the optimization of the target weight has become the focus of this paper.

3.1 Evaluation Target Selection

First, it is need to establish select the evaluation target. Due to the effect of different EGR rates on the diesel engine combustion and emissions is different, the selection of evaluation indicators should be take into account the combustion and emission performance of the diesel engine as much as possible. In this paper, the fuel consumption rate, in-cylinder explosion pressure, NO_x , smoke and CO were selected as the evaluation targets. Since the main purpose of EGR is to minimize the emission of NO_x pollutants, so define NO_x as the main decision-making target, the other four indicators for the secondary decision-making target.

The determination of the optimal EGR rate is essentially the search for the best compromise between diesel engine combustion and emissions performance, and how this compromise is reflected in the optimization model is the primary consideration. Considering the important role of target weights in decision-making model, in the optimization model, a compromise between diesel engine combustion and emission performance can be achieved By adjusting the target weight vector $\eta_k (k = 1, 2, 3, 4, 5)$, where k respectively on behalf of the fuel consumption rate, cylinder burst pressure, NO_x , smoke and CO.

3.2 Establishment of NO_x Index Weight

Taking into account the main purpose of EGR, which is to effectively reduce the NO_x emissions. Therefore, the expert scoring is used firstly to customize the target weight of NO_x according to the different conditions of the diesel engine. By repeating the trial and reviewing the information, the rules are as follows:

- I. When the diesel engine is at a low load conditions (this article defines $\leq 50\%$ load), NO_x emission concentration is low and in order to ensure the stability and economy of diesel engine, it is suitable to choose a lower EGR rate, thus make the NO_x weight $\eta_3 = 0.4$.
- II. When the diesel engine is at a high load (this article defines $\geq 50\%$ load), the NO_x emission concentration is high and in order to ensure the necessary emissions, it is suitable to adopt a higher EGR rate, thus make the NO_x weight = 0.5.

3.3 Establishment of All Index Weight

Based on the importance of other indicators and NO_x , this paper introduces the gray relational analysis method to solve the other index weights. The corresponding NO_x values (including the original machine value) at different EGR rates were used as the parent sequence, while the other four evaluation indicators corresponding to the value (including the original machine value) as a sub-sequence, then the correlation coefficient $r'_i (i = 1, 2, 3, 4)$ between the other four evaluation indexes and NO_x index was solved by grey relational analysis. At last, the correlation coefficient between the primary and secondary decision goals can be obtained:

$$r_i = \frac{r'_i}{\sum_{i=1}^4 r'_i} \quad (9)$$

Solve the initial subjective weight vector. Known as η_3 and r_i , the other four decision-making target weight value $\eta_k (k = 1, 2, 4, 5)$ are solved by the formula $r_i(1 - \eta_3)$, and then the initial subjective weight vector is constituted.

4 Establishment of Optimization Model

For EGR performance evaluation and decision making:

Event set $A = \{a_1\}$, i.e. the event is the best EGR rate decision.

Strategy set $B = \{b_1, b_2 \dots b_m\}$ consist of m decision-making program and b_m represent different EGR rate.

The decision-making evaluation targets are the fuel consumption rate, in-cylinder explosion pressure, NO_x , smoke and CO and their corresponding weights are, $\eta_1, \eta_2, \eta_3, \eta_4$ and η_5 .

The situation of each EGR rate is carried out under the same experimental conditions, and $u_{ij}^{(k)}$ is represent the measurement value corresponding to each decision objective under different conditions for different EGR rates. That is, the experimental value of different parameters. As far as EGR performance evaluation indicators are concerned, fuel consumption, cylinder burst pressure, NO_x , CO and soot are the smaller the better, so choose the lower effect measure.

Specific decision modeling steps are as follows:

Step 1: Develop the effect sample matrix $u_{ij}^{(k)} (i = 1, 2 \dots n; j = 1, 2, \dots m)$, which is composed of the experimental data corresponding to different EGR rates at different working conditions. And solve the consistent effect measure matrix according to (1)–(3).

Since the event $n = 1$ in this article, the effect sample matrix under different targets can be merged into a new matrix:

$$(u_{ij}) = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ u_{n1} & u_{n2} & \cdots & u_{nm} \end{bmatrix} \quad (10)$$

where, The abscissa i represents each target and ordinate j represents different EGR rates.

Step 2: Solve the optimized weight vector $\eta_k (k = 1, 2, 3, 4, 5)$.

Step 3: Substituting η_k into (5) to obtain the corresponding comprehensive effect measure matrix.

Step 4: According to the principle of optimal decision, the advantages and disadvantages of different EGR schemes are sorted and the optimal EGR rate is obtained.

5 Test Validation and Result Analysis

5.1 Acquisition of Test Data

In order to verify the effectiveness of the optimization method, taking a certain type of turbocharged diesel engine as the research object.

The main technical parameters of diesel engine are shown in Table 1.

Table 1. Main technical parameters of TBD234V12

Project	Parameter
Power/kW	444 (1800 r/min)
Cylinderbore/mm \times stroke/mm	128 \times 140
Compression ratio	15:1
Cylinder arrangement	V-shaped 12-cylinder 60° angle
Combustion chamber type	Direct injection w type

The test included low, medium and high three speed test and each speed in turn selected 25%, 50%, 75% load, a total of 9 working conditions, Part of the operating point test data is shown in Table 2. cgr, fc, co, no, soot and cbp represents EGR rate, fuel consumption, CO, NO_x, soot and cylinder burst pressure in Table 2.

5.2 Analysis of Results

I. low load conditions

OP1 and OP2 are the low speed 50% load and medium speed 25% load, respectively. Taking OP1 as an example, EGR rates were 2.2%, 4.6%, 7.5%, 9.8% and 11.5%. The effect sample matrix $u_{ij}^{(k)}$ is base on experimental data under different EGR rates:

Table 2. Part of the operating point test data

OP	cgr	fc	co	no	soot	cbp
OP1	0	236.3	309	1093	0.045	7.6462
	2.2	241.6	316.57	1104.5	0.063	7.2545
	4.6	242.7	335.53	1002.6	0.088	7.2108
	7.5	243.9	366.7	943.5	0.084	7.1393
	9.8	246.9	427.84	890.65	0.12	7.0167
	11.5	248.7	503.62	783.6	0.27	6.9568
OP2	0	230.1	188	825	0.035	6.6859
	1.5	230.5	196.2	783.2	0.041	6.5428
	4.5	234.6	211.3	743.2	0.049	6.3595
	7.8	236.1	229.4	669.1	0.053	6.3052
	9.5	242.9	273.5	543.5	0.09	6.0485
	12.6	244.3	380.6	497.4	0.27	5.9274
OP3	0	212.9	268	1438	0.108	9.5091
	1.7	214.7	286.53	1432	0.11	9.1568
	4.2	217.9	304.76	1351.6	0.13	9.0763
	7.4	218.2	329.89	1185	0.15	8.8016
	9.1	220.8	366.54	1069.4	0.21	8.7569
	11.8	224.3	426.71	994.2	0.33	8.5597
OP4	0	200.4	160	2186	0.093	10.8
	1.6	199.8	156.4	2101	0.1	10.5505
	3.9	202.3	164.2	1894	0.13	10.4165
	7.5	205	172.2	1653	0.148	10.2256
	9.7	209.2	206	1521	0.165	10.0584
	11.1	212.2	312.3	1465	0.32	9.8568

$$(u_{ij}^{(5)}) = \begin{bmatrix} 241.60 & 242.70 & 243.90 & 246.90 & 248.70 \\ 7.2545 & 7.2108 & 7.1393 & 7.0167 & 6.9568 \\ 1104.5 & 1002.6 & 943.50 & 890.65 & 783.60 \\ 0.0630 & 0.0880 & 0.0840 & 0.1200 & 0.2700 \\ 316.57 & 335.53 & 366.70 & 427.84 & 503.62 \end{bmatrix}$$

Among them, the abscissa represents the fuel consumption rate, CO, NO_x, soot and in-cylinder burst pressure respectively. Ordinate j represents different EGR rates, such as the first column representing an EGR rate of 2.2%.

Step 1: first solve the consistent effect measure matrix:

$$(r_{ij}^{(5)}) = \begin{bmatrix} 1.0000 & 0.9955 & 0.9906 & 0.9785 & 0.9715 \\ 0.9590 & 0.9648 & 0.9744 & 0.9915 & 1.0000 \\ 0.7095 & 0.7816 & 0.8305 & 0.8798 & 1.0000 \\ 1.0000 & 0.7159 & 0.7500 & 0.5250 & 0.2333 \\ 1.0000 & 0.9435 & 0.8633 & 0.7399 & 0.6286 \end{bmatrix}$$

Step 2: solve the initial subjective weight vector. Since OPI belongs to low load operating point, $\eta_3 = 0.4$. Determine the grey association sequence:
 Mother sequence:

$$X_0 = [1093 \quad 1104.5 \quad 1002.6 \quad 943.50 \quad 890.65 \quad 783.60]$$

Subsequence:

$$X_1 = [236.3 \quad 241.60 \quad 242.70 \quad 243.90 \quad 246.90 \quad 248.70]$$

$$X_2 = [7.6462 \quad 7.2545 \quad 7.2108 \quad 7.1393 \quad 7.0167 \quad 6.9568]$$

$$X_3 = [0.045 \quad 0.0630 \quad 0.0880 \quad 0.0840 \quad 0.1200 \quad 0.2700]$$

$$X_4 = [309 \quad 316.57 \quad 335.53 \quad 366.70 \quad 427.84 \quad 503.62]$$

The correlation coefficient between the other four evaluation indexes and the NO_x index are respectively:

$$r_i = [0.9684 \quad 0.9926 \quad 0.7038 \quad 0.7398]$$

the initial subjective weight vector:

$$\eta_k = [0.1707 \quad 0.1749 \quad 0.4000 \quad 0.1240 \quad 0.1304]$$

Step 3: The comprehensive effect measure matrix is solved and the advantages and disadvantages are sorted according to the optimal principle:

$$R = [0.8766 \quad 0.8631 \quad 0.8573 \quad 0.8539 \quad 0.8516]$$

It can be seen from the result of OPI operating conditions, the performance ranking of different EGR rate is:

$$2.2\% > 4.6\% > 7.5\% > 9.8\% > 11.5\%$$

It is shown that the optimal EGR rate is 2.2% for this condition, and when the EGR rate is higher, the comprehensive performance evaluation value decreases obviously, so a smaller EGR rate should be adopted.

Similarly, the performance ranking of different EGR rate at OP2 conditions can be obtained:

$$R = [0.8384 \quad 0.8219 \quad 0.8332 \quad 0.8125 \quad 0.8102]$$

$$1.5\% > 7.8\% > 4.5\% > 9.5\% > 12.6\%$$

which indicates that the optimal EGR rate is 1.5% at this condition, and when the EGR rate is higher, the comprehensive performance evaluation value is obviously decreased.

It can be seen from the result of OP1 and OP2 that a lower EGR rate should be adopted when diesel engine in low or medium speed, low load conditions. Meanwhile, the comprehensive evaluation value is obviously decreased while the EGR rate is higher. Analysis of the reasons is that when at low load, the NO_x pollutant emissions is low, part of the dynamic performance of diesel engines will be consumed if the EGR rate is too high. In order to ensure the economy and power of diesel engines, it is appropriate to reduce the EGR rate.

II. High load conditions

OP3 and OP4 represent the medium speed 75% load and high speed 75% load, respectively. Let $\alpha = 0.5$ and through the simulation calculation, thus the comprehensive effect measure matrix of OP3 and OP4 are as follows:

$$\text{OP3: } [0.8324 \quad 0.8323 \quad 0.8675 \quad 0.8821 \quad 0.8886]$$

$$\text{OP4: } [0.8392 \quad 0.8470 \quad 0.8890 \quad 0.9024 \quad 0.8604]$$

the performance ranking of different EGR rate at OP3 and OP4 conditions can be obtained:

$$\text{OP3: } 11.8\% > 9.1\% > 7.4\% > 1.7\% > 4.2\%$$

$$\text{OP4: } 9.7\% > 7.5\% > 11.1\% > 3.9\% > 1.6\%$$

As can be seen from the results of OP3 and OP4, with the increase of EGR rate, the comprehensive evaluation value increases, which indicates that the higher EGR rate should be adopted under high speed and high load conditions.

In summary, it can be seen from the above assessment and decision-making results. When working at the low or medium speed, low load conditions, due to the lower NO_x emission concentration, it is appropriate to use a smaller EGR rate to balance the power and economy of diesel engines. While working at high load conditions, the NO_x emission concentration is high and in order to ensure the emissions performance, a higher EGR rate should be adopt. With the increase in speed and load, it is appropriate to increase the EGR rate. This is consistent with the characteristics of EGR performance of the current turbocharged diesel engine and also shows the effectiveness of the optimization method.

Although the optimization model can be effectively implemented and successfully applied to the EGR performance evaluation and decision making of turbocharged diesel engine, it needs to be further improvement. Such as limited by the test conditions, it is temporarily unable to obtain more work points corresponding to the data, So that there are some defects in the optimization model, such as method of determining the initial

subjective weight still need more data points to amend etc., which is also the next step to continue to study.

6 Conclusion

Aiming at the problem of EGR performance evaluation and optimal decision-making of the turbocharged diesel engine, an optimized multi-objective gray situation decision-making method is proposed. This method can integrate the EGR operating characteristics of the turbocharged diesel engine into the optimization model, which makes the final decision result more reasonable.

The results show that when the turbocharged diesel engine is at a low load, the difference between the comprehensive evaluation values of different EGR rates is not significant when the EGR rate is less than 9%, and with the EGR rate increased, the comprehensive evaluation value decreased obviously when the EGR rate is greater than 10%, thus it is appropriate to use a lower EGR rate. When the diesel engine is at high load, when the EGR rate increases to about 7%, the corresponding comprehensive evaluation value increases more significant, thus it is appropriate to use a higher EGR rate.

The results show that the decision result of this method is basically consistent with the performance characteristics of EGR, as well as the current best EGR rate determination principle. So it can be successfully applied to the decision-making problem of optimal EGR rate under different conditions of turbocharged diesel engine.

Acknowledgement. The authors would like to thank the reviewers for their constructive comments. This work is supported by National Science & Technology Plan Projects (2015BAG16B01).

References

1. Asad, U., Zheng, M.: Exhaust gas recirculation for advanced diesel combustion cycles. *Appl. Energy* **123**, 242–252 (2014)
2. Luján, J.M., Galindo, J., Vera, F., et al.: Characterization and dynamic response of an exhaust gas recirculation venturi for internal combustion engines. *Proc. Inst. Mech. Eng. Part D J. Automob. Eng.* **221**, 497–509 (2007)
3. Chen, G.-S., Wu, W., Shen, Y.-G., et al.: Influence of different EGR cycle modes on combustion and emission characteristics of heavy-duty diesel engine. *Chin. Intern. Combust. Engine Eng.* **35**(2), 20–26 (2014)
4. Shuai, Y., Xiuyuan, L., Qijia, Y., et al.: EGR rates optimization rule and experimental study about influence of EGR rates on diesel engine. *Trans. Chin. Soc. Agricult. Mach.* **37**(5), 30–33 (2006)
5. Zhang, Z.-D., Fang, Y.-B., Chen, Z.-T.: Research and experiments of EGR rates effect on a turbocharged diesel engine. *Chin. Intern. Combust. Engine Eng.* **37**(5), 30–33 (2006)
6. Alonso, S., Herrera-Viedma, E., Chiclana, F., Herrera, F.: A web based consensus support system for group decision making problems and incomplete preferences. *Inf. Sci.* **180**, 4477–4495 (2010)

7. Zhou, H., Wang, J., Zhang, H.: Grey stochastic multi-criteria decision-making approach based on prospect theory and distance measures. *J. Grey Syst.* **29**(1), 15–33 (2017)
8. Zhang, Y., Wang, W., Bernard, A.: Embedding multi-attribute decision making into evolutionary optimization to solve the many-objective combinatorial optimization problems. *J. Grey Syst.* **28**(3), 124–143 (2016)
9. Liu, S.F., Yuan, W.F., Sheng, K.Q.: Multi-attribute intelligent grey target decision model. *Control Decis.* **25**, 1159–1163 (2010)
10. Ou, J., Zhang, A., Zhong, L.: Maintenance scheduling of aircrafts based on multi-criteria optimization and preference programming. *Syst. Eng.-Theory Pract.* **35**(5), 1347–1350 (2015)
11. Yang, B.H., Fang, Z.G., Zhou, W., et al.: Incidence decision model of multi-attribute interval grey number based on information reduction operator. *Control Decis.* **27**(2), 182–186 (2012)
12. Wang, Y., Du, J., Wang, H., et al.: Grey decision making theory approach to the turbocharged diesel engine. In: *Proceedings of 2007 IEEE International Conference on Grey Systems and Intelligent Services*, pp. 784–788. IEEE Press, Nanjing (2007)
13. Liu, S.F., Lin, Y.: *Grey Information: Theory and Practical Applications*. Springer, London (2006). <https://doi.org/10.1007/1-84628-342-6>
14. Wu, L.F., Wang, Y.N., Liu, S.F.: Grey convex relation and its properties. *Syst. Eng.-Theory Pract.* **32**(7), 1501–1506 (2012)