A Precoding Scheme Based on SLNR for Downlink MU-MIMO Systems

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Abstract. The precoding scheme plays an important role in suppressing co-channel interference for downlink multi-user (MU) multiple-input-multiple-output (MIMO) communication systems. The effects of noise are not ignored in precoding scheme based on Signal-to-leakage-and-noise ratio (SLNR) and there are no limits on the number of transmit antennas. In this paper, a modified SLNR-based precoding scheme is presented, which can balance the channel gain for each stream per user by diagonalizating leakage-and-noise and the user's channel matrices simultaneously. Simulation results show that better BER performance can be obtained by the proposed scheme as compared with zero-forcing (ZF) precoding and conventional SLNR solution.

Keywords: Downlink · MU MIMO · SLNR-based precoding · ZF

1 Introduction

In multi-user MIMO systems, precoding technique plays a major role in improving the system performance. The core idea is pre-processing the data to be transmitted by the use of the channel state information (CSI) at the base station, to reduce the co-channel interference (CCI) among users and achieve a higher performance gain. Linear precoding is widely used due to its low complexity [1].

Signal-to-interference-plus-noise ratio (SINR) is always applied as a measure of performance, but it is also a challenge for its coupled problem with precoding matrix and the number of users. In previous studies, so as to solve this problem, zero-forcing (ZF) precoding is proposed to cancel the CCI [2, 3]. The shortcomings of these schemes are the restriction on the number of the receive antennas (RA) and transmit antennas (TA). Furthermore, zero-forcing scheme does not take the effects of noise into consideration [4].

Another criterion, SLNR criterion is first proposed by Sadek et al. [5, 6], which cleverly solved the precoding matrix design problem under the SINR criterion [7]. In view of the SLNR criterion, the precoding scheme is expected to make the received signal power of each active user as large as possible, while the sum of the noise power and the interference power leaked from other users is as small as possible. As a criterion, SLNR is better than SINR [8], for the reason that the SLNR of any user depends only on its own precoding matrix, and has nothing to do with other users' coding matrix. Therefore, it is possible to avoid the nesting problem of optimizing the

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precoding matrix of each user, and derive the optimal closed solution of each user's precoding matrix directly. In addition, the precoding scheme which is based on SLNR criterion is no longer constrained by the quantity of system antennas, and thus has a wider application space.

Conventional SLNR-based linear precoding pursues the maximization of SLNR by the use of the generalized eigenvalue decomposition (GED) of the leakage-channel-and-noise covariance matrix among users and the channel covariance matrix [9]. Whereas, in a real communication environment, different users experience different channel fading, and it is difficult to balance the SINR of each user's received signal. The SINR of the users which experience severe channel fading will be much lower [10], thus affecting the overall system performance.

In this paper, the fairness of communication among users in downlink MU-MIMO system is considered, and the precoding scheme is improved. The core idea is reducing the maximum value of SLNR slightly so as to balance the SINR of each user with multiple data streams.

2 System Model

2.1 Downlink MU-MIMO System

The block diagram of a downlink multi-user MIMO system is presented in Fig. 1. The vector $\mathbf{s}_k(n)$ represents the transmitted data of user k at nth time instant, and $\mathbf{w}_k$ represents precoding matrix. Assuming that the downlink MU-MIMO system with K users has N TAs at the base station, and each user in the block diagram has M_K RAs. Then, at time instant n the overall transmit matrix could be expressed as

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{w}_k \mathbf{s}_k(n) \tag{1}$$

For convenience, the data vector $\mathbf{s}_k(n)$ and precoding matrix $\mathbf{w}_k$ are subject to

$$E|\mathbf{s}_k(n)|^2 = 1, \quad \|\mathbf{w}_k\|^2 = L_k \quad k = 1, 2 \ldots K \tag{2}$$

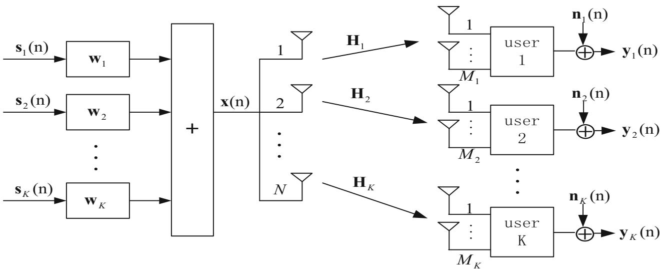

Fig. 1. Downlink MU-MIMO system model

The channel is assumed to be frequency flat faded

$$\mathbf{H}_k = \begin{bmatrix} h_k^{(1,1)} & \cdots & h_k^{(1,N)} \\ \vdots & \ddots & \vdots \\ h_k^{(M_k,1)} & \cdots & h_k^{(M_k,N)} \end{bmatrix}_{M_k \times N} \tag{3}$$

where $h_k^{(r,t)}$ is channel impulse response between the tth$(t = 1, 2, \cdots, N)$ TA and the rth $(r = 1, 2, \cdots, M_k)$ RA at user. Assume that $h_k^{(r,t)}$ obey the complex Gaussian distribution with mean = 0 and variance = 1, i.e., the channel is Rayleigh faded. In this case, the kth$(k = 1, 2, \cdots, K)$ user's received signal at time instant n is

$$\mathbf{y}_k(n) = \mathbf{H}_k\mathbf{x}(n) + \mathbf{n} = \mathbf{H}_k \sum_{j=1}^{K} \mathbf{w}_j\mathbf{s}_j(n) + \mathbf{n}_k(n) \tag{4}$$

where $\mathbf{n}_k(n)$ denotes the additive white Gaussian noise with $\sigma^2 - variance$. So SNR at each RA is

$$SNR = 1/\sigma^2 \tag{5}$$

For the convenience of research, we assume that CSI $\mathbf{H}_k(k = 1, 2, \cdots, K)$ is known, and $\mathbf{s}_k(n)$, $\mathbf{H}_k$ and $\mathbf{n}_k(n)$ are assumed to be independent of one another.

2.2 SLNR-Based Precoding System Model

We can expand the expression in (4) into the following form

$$\mathbf{y}_k(n) = \mathbf{H}_k\mathbf{w}_k\mathbf{s}_k(n) + \mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{w}_j\mathbf{s}_j(n) + \mathbf{n}_k(n) \tag{6}$$

The first term is the signal that the receiver actually needs to receive, while the second term includes the interference signal which is going to leak to other users. SINR of the user k is defined as (omit the time index n for convenience)

$$SINR_k = \|\mathbf{H}_k\mathbf{w}_k\|^2 / (\mathbf{M}_k\sigma^2 + \sum_{j=1, j\neq k}^{K} \left\|\mathbf{H}_k\mathbf{w}_j\right\|^2) \tag{7}$$

Choosing SINR expression as the criterion of performance cannot avoid coupled problem with K and $\mathbf{w}_k$ [6]. In order to solve it, zero-forcing precoding has been proposed in previous paper. The basic idea of zero-forcing schemes is to cancel CCI.

$$\mathbf{H}_k\mathbf{w}_j = \mathbf{0} \quad for\, all \;\; j, k = \{1, 2, \cdots, K\}, j \neq k \tag{8}$$

In this way, although the CCI is completely removed, the noise power does not accordingly decrease, and may even be amplified. In addition, in order to let the expression in (8) hold, the following relationship must be satisfied

$$N > \max_{i}\left\{\sum_{k=1,k\neq i}^{K} M_k\right\} \tag{9}$$

Because SINR-based scheme is subject to the above condition, it is necessary to apply a new criterion which takes noise into consideration and will not be limited by constraint condition in (9). SLNR-based scheme can satisfy the above requirements.

SLNR-based precoding system model is illustrated in Fig. 2, where $\left\|\mathbf{H}_k\mathbf{w}_j\mathbf{s}_j\right\|^2$ $(k = 1,\cdots,j-1,j+1,\cdots,K)$ is defined as leakage from user j to other users. In the above we assume that $E|\mathbf{s}_k(n)|^2 = 1$, so SLNR of user k can be defined as

$$SLNR_k = \|\mathbf{H}_k\mathbf{w}_k\|^2/(M_k\sigma^2 + \sum_{j=1,j\neq k}^{K}\left\|\mathbf{H}_j\mathbf{w}_k\right\|^2) \tag{10}$$

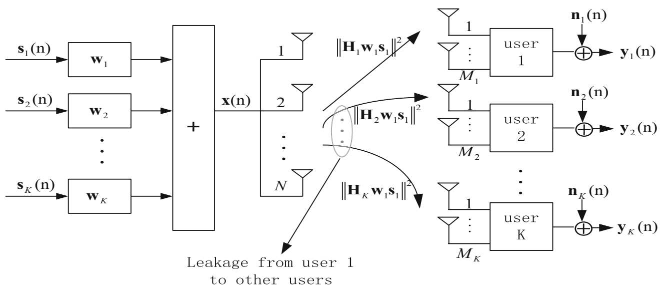

Fig. 2. SLNR-based precoding system model

Compared with SINR, the SLNR-based scheme calculates the interference of user k to other users, rather than the interference of other users to user k. To maximize every user's SLNR, the precoding matrix $\mathbf{w}_k$ should satisfy

$$\|\mathbf{w}_k\|^2 = L_k \quad k = 1,2\ldots K \tag{11}$$

where L_k denotes the data stream of the user k. In the following, the data stream of every user is assumed equal, and uniformly expressed as L. Then

$$SLNR_k = \frac{Tr(\mathbf{w}_k^H\mathbf{H}_k^H\mathbf{H}_k\mathbf{w}_k)}{Tr(\mathbf{w}_k^H(\frac{M_k\sigma^2}{L}\mathbf{I}_N + \tilde{\mathbf{H}}_k^H\tilde{\mathbf{H}}_k)\mathbf{w}_k)} \tag{12}$$

$$\tilde{\mathbf{H}}_k = [\mathbf{H}_1,\cdots,\mathbf{H}_{k-1},\mathbf{H}_{k+1},\cdots\mathbf{H}_K]^T \tag{13}$$

The optimization problem of $\mathbf{w}_k$ can be expressed as

$$\mathbf{w}_k^{opt} = \arg \max_{w_k \in \mathbb{C}^{N \times L}} \frac{Tr(\mathbf{w}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_k)}{Tr(\mathbf{w}_k^H (\frac{M_k \sigma^2}{L} \mathbf{I}_N + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k) \mathbf{w}_k)} \tag{14}$$

Since the base station simultaneously transmits L spatially multiplexed data streams to all users, co-channel interference includes inter-user interference (IUI) among users and inter-stream interference (ISI) among multiple data streams. So the signal recovered at the receiver of user k is

$$\hat{\mathbf{s}}_k = \frac{(\mathbf{H}_k \mathbf{w}_k)^H}{\|\mathbf{H}_k \mathbf{w}_k\|^2} \mathbf{y}_k = \frac{(\mathbf{H}_k \mathbf{w}_k)^H}{\|\mathbf{H}_k \mathbf{w}_k\|^2} \mathbf{H}_k \mathbf{w}_k \mathbf{s}_k + \frac{(\mathbf{H}_k \mathbf{w}_k)^H}{\|\mathbf{H}_k \mathbf{w}_k\|^2} (\mathbf{H}_k \sum_{j=1, j \neq k}^{K} \mathbf{w}_j \mathbf{s}_j + \mathbf{n}_k) \tag{15}$$

where the first term of the equation is the desired signal and the other term includes noise and the interference. In order to achieve the decoupling of multiple data streams at the receiver of user, the optimization of the precoding matrix problem in (14) needs to satisfy the constraint condition

$$\mathbf{w}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{w}_k = \mathbf{D}_k \tag{16}$$

where $\mathbf{D}_k$ is a diagonal matrix. Both $M\sigma^2/L\mathbf{I}_N + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k$ and $\mathbf{H}_k^H \mathbf{H}_k$ are Hermitian matrices, and the former is positive definite. It is known from the characteristics of the generalized eigenvalue decomposition (GED) that there must be an invertible matrix $\mathbf{Q}_i \in \mathbb{C}^{N \times N}$ satisfying the conditions of

$$\mathbf{Q}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{Q}_k = diag(\lambda_1, \cdots, \lambda_N) \tag{17}$$

$$\mathbf{Q}_k^H [(M\sigma^2/L)\mathbf{I}_N + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k] \mathbf{Q}_k = \mathbf{I}_N \tag{18}$$

With $\{\lambda_i\}_{i=1}^N$ being sorted in descending order. The column vector of $\mathbf{Q}_i$ and $\{\lambda_i\}_{i=1}^N$ are respectively the generalized eigenvectors and eigenvalues of the matrix $\left\{\mathbf{H}_k^H \mathbf{H}_k, M\sigma^2/L\mathbf{I}_N + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k\right\}$. It has been proved in [3] that the precoding matrix $\mathbf{w}_i$ maximizing the $SLNR_k$ in (14) is given by

$$\mathbf{w}_k^{opt} = \alpha \mathbf{Q}_k \begin{bmatrix} \mathbf{I}_L \\ \mathbf{0} \end{bmatrix} \tag{19}$$

where α is a scalar in order to satisfy $\|\mathbf{w}_k\|^2 = L$, and according to (20), we obtain

$$SLNR_k^{\max} = \sum_{j=1}^{L} \frac{\lambda_j}{L} \tag{20}$$

3 Proposed SLNR-Based Precoding Scheme

In the real communication environment, signals of different users experience different channel fading. And, when is larger than 1, it is difficult to balance the SINR of each user. The severer the channel fading becomes, the lower is the SINR at the receiver, which will affect the overall system performance. In this section, the fairness of communication among users in downlink MU-MIMO systems is considered, and an improved precoding scheme is proposed in the following. In this section, we diagonalize two matrices simultaneously. There must be a full rank matrix which satisfies the conditions in (22) and (23)

$$\mathbf{T}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{T}_k = diag(\beta_1, \beta_2, \cdots, \beta_N) \tag{21}$$

$$\mathbf{T}_k^H (\frac{M\sigma^2}{L} \mathbf{I}_N + \tilde{\mathbf{H}}_k^H \tilde{\mathbf{H}}_k) \mathbf{T}_k = diag(\gamma_1, \gamma_2, \cdots, \gamma_N) \tag{22}$$

where $\{\beta_i\}_{i=0}^{M}$ are sorted in descending order from 1 to 0, and $\{\beta_i\}_{i=M+1}^{N} = 0$. In the meanwhile, $\{\gamma_i\}_{i=0}^{M}$ are sorted in ascending order from 0 to 1, $\{\gamma_i\}_{i=M+1}^{N} = 1$. Furthermore, elements in $\{\gamma_i + \beta_i\}_{i=1}^{N}$ are all 1. The precoding matrix is then given by

$$\mathbf{w}_{kp}^{opt} = \varphi \mathbf{T}_k \begin{bmatrix} \mathbf{I}_L \\ \mathbf{0} \end{bmatrix} \tag{23}$$

where φ is a scalar in order to satisfy $\|\mathbf{w}_k\|^2 = L$. And in this way, $SLNR_k$ can be calculated by

$$SLNR_k = \sum_{a=1}^{L} \beta_a / \sum_{a=1}^{L} (1 - \beta_a) \tag{24}$$

In contrast with expression in (21), $SLNR_k$ here is a little smaller. But when the number of data streams ≥ 2, *SINR* of each stream will be more balanced. Denote the *SINR* of stream l as η'_l, then

$$\eta'_l = \varphi^4 \beta_l^2 / \varphi^2 \sigma^2 \beta_l = \varphi^2 \beta_l / \sigma^2 \tag{25}$$

Furthermore, assume that $l > m$, the ratio of SINRs between two data streams l and m can be written as

$$\eta'_l / \eta'_m = \beta_l / \beta_m \tag{26}$$

While in the conventional scheme

$$\eta_l / \eta_m = \lambda_l / \lambda_m \tag{27}$$

Since both $\{\lambda_i\}_{i=1}^{N}$ and $\{\beta_i/\gamma_i\}_{i=1}^{N}$ are generalized eigenvalues of $\left\{\mathbf{H}_i^H\mathbf{H}_i, M\sigma^2/L\mathbf{I}_N + \tilde{\mathbf{H}}_i^H\tilde{\mathbf{H}}_i\right\}$, there must be

$$\{\beta_i/\gamma_i\}_{i=1}^{N} = \{\lambda_i\}_{i=1}^{N} \tag{28}$$

We can easily find that

$$(\eta_l'/\eta_m') < (\eta_l/\eta_m) \tag{29}$$

This means that in the proposed scheme, SINR of any two streams is more balanced than that in the original scheme. And the overall BER performance will get better, which will be proved by simulations in the next section.

4 Simulation Results

In this section, first the BER in cases of single user, zero forcing scheme and SLNR-based solution in downlink MU-MIMO systems are simulated. Parameters of simulations are given in Table 1. All simulations are run on the basis of a quasi-static MIMO channel mode. We assume that the CSI is known and the additive white Gaussian noise is subject to σ^2-variance. Furthermore, we assume that the number of RAs belonging to different users is equal. Simulation results are shown in Fig. 3. As known in Sect. 2, when $L = 1$, there is no ISI. Similarly, when the number of users is 1, there is no any IUI. Therefore, there is no doubt that single user scheme's BER performance is the best. Although ZF scheme can reduce CCI to zero, it cannot equalize noise. The SLNR-based scheme takes into account both the noise matrix and the channel impulse response matrix. So its BER performance is better than ZF schemes. In addition, when the number of RAs is 3, i.e. it does not satisfy the expression in (9) so that the worst BER performance is obtained.

In the following simulations, we assume that TA = 8, User = 2 and RA = 3, Fig. 4 illustrates the sum rate and BER performance of the original and the proposed scheme when $L = 1, 2, 3$, where P represents "proposed" and O represents "original". As we can see in the following figures, the sum rate of system with the proposed scheme is less than that of the original scheme. With the increase of L, the gap inbetween becomes gradually smaller, and BER performance gets worse. It is because the interference among data streams increases with L. In summary, with the proposed scheme, the improved over-all error bit performance is obtained on the expense of the sum rate.

Table 1. Simulation parameters

Precoding scheme	L	TA	User	RA of user	Modulation
Single user	1	8	1	3	QPSK
SLNR-based	1	8	3	3	QPSK
ZF, RA = 2	1	8	3	2	QPSK
ZF, RA = 3	1	8	3	3	QPSK

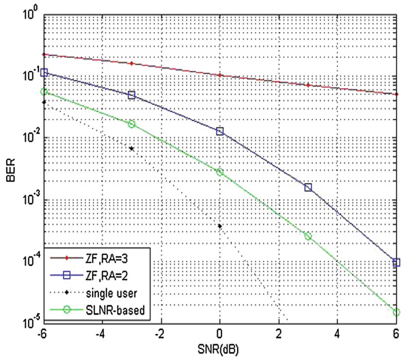

Fig. 3. The BER performance of single user, zero forcing and SLNR-based solution

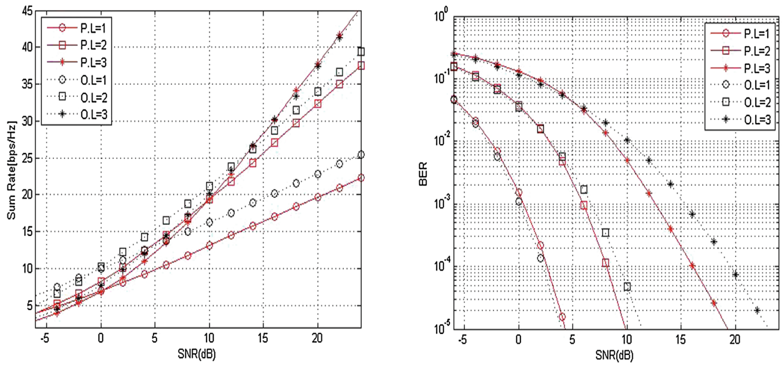

Fig. 4. The BER and sum tate performance of original and proposed schemes

5 Conclusion

The downlink multi-user MIMO system and the traditional SLNR-based system have been detailed in this paper. On this basis, an improved SLNR-based scheme was proposed, which makes the overall BER performance better. The simulation results show that when the number of data streams increase, the BER performance of the proposed scheme is superior to the original scheme, and its sum rate performance gradually catches up with that of original scheme.

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