A Novel Channel Extraction Method Based on Partial Orthogonal Matching Pursuit Algorithm

Wang Xiangjun(✉), Dou Zheng, and Lin Yun

Harbin Engineering University, Harbin 150001, HLJ, China
wangxiangjun0216@hrbeu.edu.cn

Abstract. Channelization has proven to be very successful in digital receivers application, and it is a critical component of reducing the sampling rate process. Compressed sensing has been widely applied to reconstruct sparse signals sampled at sub-Nyquist rate. In this paper, a stable and fast algorithm termed Partial Orthogonal Matching Pursuit (POMP) is proposed for a channelized digital receiver. It is suitable for sparse channels in wide bandwidth. The novel POMP algorithm is analyzed and compared with the conventional channelization method based on polyphase filters, and numerical simulations demonstrate that the POMP detection not only achieves the basic functions of a channelizer, but also outperforms a polyphase channelizer. Moreover, the POMP algorithm is an efficient method to suppress the aliasing and leaking between channels.

Keywords: POMP algorithm · Channelizer · Channel extraction
Compressed sensing

1 Introduction

As the frequency spectrum distribution in the communication environment becomes increasingly complex, digital receivers are required to have a larger bandwidth, and a simple yet efficient channelizer is required to separate the band-of-interest (BOI) at the same arriving time. The original channelization structure consists of a bank of mixers and low-pass filters structure [1], which results in much computational consumption. To reduce computational complexity, Harris researched a polyphase channelizer which is an efficient implementation of the conventional channelizer, and reviewed the procession how a conventional channelizer is converted to a standard polyphase channelizer [2]. Harris summarized the advantages of a multichannel polyphase filter bank: simultaneously performing the uncoupled tasks of down conversion, bandwidth limiting and sampling rate change. Due to the above advantage, the polyphase channelizer structure has been extensively studied. Chen established a polyphase analysis channelizer and a polyphase synthesis channelizer utilizing analysis filter banks and synthesis filter banks respectively [3]. Recently, Kim proposed an

G. Sun and S. Liu (Eds.): ADHIP 2017, LNICST 219, pp. 204–212, 2018.
https://doi.org/10.1007/978-3-319-73317-3_25

efficient channelizer based on polyphase filter banks and the channels can be arbitrarily resampled to any desired rate [4].

Even though the polyphase channelizer outperforms the traditional channelizer and shows its effectiveness, there inevitably exist aliasing and leaking between channels due to filters' non-rectangular coefficient. Moreover, there are not many signals exist at the same time within a certain bandwidth, especially for some military bands and electronic reconnaissance bands. Hence many channels do not exist any signals, causing many processes in a channelizer to be wasted.

The compressed sensing (CS) theory can exploit the sparseness of the spectrum distribution, and reduce the useless work of the spectrum channelization. The core idea of CS is to recover a lot of useful information with a few linear measurements. The well-known approach has been extensively studied these years in the field of communications, such as sparse channel estimation [5,6], sparse targets detection [7], and cognitive radio [8]. Many algorithms have been proposed to recover sparse signals, and the Orthogonal Matching Pursuit (OMP) algorithm has been widely used due to its implementation simplicity and low computational complexity. The OMP algorithm is a greedy algorithm for sparse approximation in the field of compressed sensing [9].

In this paper, we propose a novel Partial Orthogonal Matching Pursuit algorithm (POMP). As the name implies, we first reconstruct one channel which contains the strongest signal using several atoms in sensing matrix. Then we remove the channel from the whole frequency band. After that we find the next strongest signal and reconstruct the channel contains it, and then remove the second channel. The process is recycled until there is no signal detected. In each cycle the intermediate frequency (IF) signals are equivalent to down-convert to baseband, and the sampling rate is also reduced. The POMP algorithm not only achieves the effect of down-conversion, down-sampling and bandwidth limiting, which are core functions of a polyphase channelizer, but also avoids aliasing and leaking phenomena between channels.

2 Background Knowledge

In this section, we suggest to utilize the location relationship between the signals distributed in spectrum and the corresponding columns in sensing matrix to achieve channel extraction.

The Polyphase Channelizer. The basic functions of a polyphase channelizer are down-conversion, bandwidth limiting, and down-sampling. However, due to the non-ideal characteristic of the filter, there inevitably exist aliasing or blind spot between channels, as shown in Fig. 1.

The channel aliasing will cause the error detection of signals, while the blind spot between channels may lead to the loss of signals' detection. Neither of the phenomena is expected to exist.

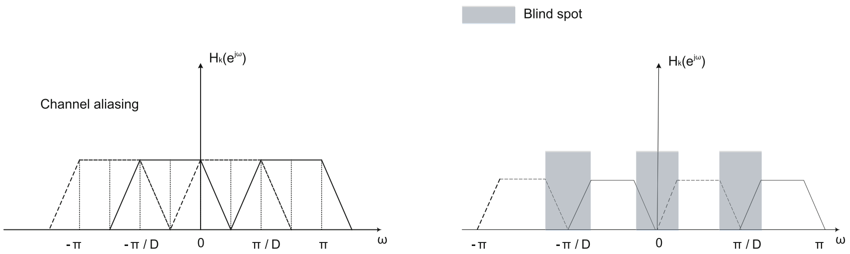

Fig. 1. The aliasing and blind spot phenomena between channels. Both phenomena are caused by the non-rectangular feature of filters.

The OMP Theory. The fundamental problem in compressed sensing (CS) is to recovery a high-dimensional vector x from a small number of linear measurements y [10], the linear measurements are given by:

$$y = \Phi \cdot x = \Phi \cdot \Psi \cdot \theta \tag{1}$$

where $x \in C^{N\times 1}$ denotes the original signal in the time domain in the field of communication, and x is K-sparse under the DFT dictionary matrix Ψ, so $x = \Psi \cdot \theta$. θ is the sparse express of x under the DFT sparse bases. $y \in C^{M\times 1}$ represents the vector obtained by the linear measurements of x, Φ represents the measurement matrix. The sensing matrix A is defined here:

$$A = \Phi \cdot \Psi \tag{2}$$

where $A \in R^{M\times N}$ is also known as the dictionary which contains the atoms that we need in the procession of OMP, and A satisfies the Restricted Isometry Property (RIP) of order $2K$. The core idea of the OMP algorithm is to find a biggest projection of measurement y from the dictionary A. In other words, the atom correlated to the maximum inner product value between sensing matrix A and the current residual needs to be singled out in each iteration. The residual is updated in each iteration, which can be expressed as follows:

$$r_k = y - A\hat{\theta}_k \tag{3}$$

where k represents the iteration times which equals to the sparsity of the signal, and the non-zero items in θ stand for the position where signals exist in the frequency band. We generally use the least square method to minimize the residual r_k. In the process of OMP, we find the fact that,the location of signals, non-zero items in θ and the corresponding atoms in dictionary A is one to one correspondence.

3 POMP Algorithm Model

Based on the theoretical analysis and the issues raised above, this section describes the process of POMP algorithm that we propose for extracting channels containing signals from the BOI. As the projection of y on the dictionary

A corresponds to the position of the nonzero terms in θ. We can use partial atoms in A to extract several channels respectively, instead of reconstructing the whole BOI.

For a BOI within an intermediate frequency band, we first find the position of the strongest signal in the BOI via calculating the inner product values between columns in A and the current residual. The process can be expressed by formula (4).

$$pos = argmax| < A_{pos}, r_k > |, (pos = 1, 2, \cdots, N) \tag{4}$$

where r_k represents the current residual at the kth iteration, and the initial residual is y. A_{pos} denotes the posth atom of the sensing matrix A. $| <, > |$ is the sign of inner product. After recording the atom corresponding to the maximum inner product value, we use partial columns of A to recovery signals in a channel range with the strongest signal as the center of the channel. In other words, the pursuiting range of the OMP algorithm in each iteration is reduced to one channel, rather than the entire BOI. The formulas (5), (6) and (7) are executions for reconstructing one channel, and are operated repeatedly until all signals in a channel are fully recovered, the iteration times is equal to the sparsity of the channel.

$$\begin{gathered} A_\lambda = argmax| < A_\lambda, r_{k-1} > |, \\ ((pos - \frac{N}{2 \times chan}) \le \lambda \le (pos + \frac{N}{2 \times chan})) \end{gathered} \tag{5}$$

$$\hat{\theta}_k = argmin \parallel y - A_\Lambda \theta_k \parallel_2 \tag{6}$$

$$r_k = y - A_\Lambda \hat{\theta}_k \tag{7}$$

The iteration range is determined in formula (5), where $chan$ denotes the amount of the channels, and $\frac{N}{2\times chan}$ represents half-channel range. The signal is estimated by the least squares method in formula (6), where $\parallel \cdot \parallel_2$ is the sign of L2-norm. $A_\Lambda = A_\Lambda \cup A_\lambda$ represents the support set which is expanded in each iteration. The signification of formula (6) is to make the reconstructed measurement closer to the initial measurement y. Residual is updated in each iteration, as shown in formula (7), the subscript k denotes the iteration time. The iteration ends when k equals to the sparsity k_{max} , where k_{max} represents the amount of signals in each channel.

After recovering a channel, the signals contained in this channel is subtracted from the BOI x, as shown in line (15) in Algorithm 1. The corresponding measurement y has also been updated. The atoms within a channel range is screened out, we set these columns to zero for the purpose of simplicity, so we get a new sensing matrix A, as shown in line (17) in Algorithm 1, where M denotes the measurement times. Then we find the position of the strongest signal from the remained signals, and recover the second channel using partial columns of A , with the strongest signal lies in the center of it. Recycling like this, until the amplitude of the strongest signal we find is less than the threshold Th (Th represents the detection threshold, which is not analyzed here in detail), the whole reconstruction procession ends. We summarize the process in Algorithm 1.

Algorithm 1. POMP algorithm model

input: ϕ, y,x,A,k_{max},Th,$chan$
channel number t
initial: $t = 1$, $A_{\Lambda} = \emptyset$
while $\max |< A_{pos}, y >| > Th$ $(pos = 1, 2, \cdots, N)$ **do**
  $k = 1$
  $r_0 = y$
  $pos = argmax|< A_{pos}, y >|$ $(pos = 1, 2, \cdots, N)$
  while $k < k_{max}$ **do**
    find: $A_{\lambda} = argmax|< A_{\lambda}, r_{k-1} >|$,
    $((pos - \frac{N}{2 \times chan}) \leq \lambda \leq (pos + \frac{N}{2 \times chan}))$
    enlarge: $A_{\Lambda} = [A_{\Lambda}, A_{\lambda}]$
    $\hat{\theta}_{t_k} = argmin \parallel y - A_{\Lambda}\theta_{t_k} \parallel_2$
    update: $r_k = y - A_{\Lambda}\hat{\theta}_{t_k}$
    $k = k + 1$
  end while
  update: $x = x - \Psi\hat{\theta}_{t_k}$
  update: $y = \phi \times x$
  update: $A(:, ((pos - \frac{N}{2 \times chan}) : (pos + \frac{N}{2 \times chan}))) = zeros(M, \frac{N}{chan})$
  $t = t + 1$
end while
output:the frequency vector per channel θ_t

Based on the algorithm mentioned above, we only need partial columns instead of all columns in sensing matrix A to extract channels which contain signals, as long as the BOI is sparse in frequency. So the iteration number is less than the number required for full band reconstruction. Certainly, with the number of signals increase, and the distribution of them is relatively uniform, the number of channels and iterations will increase accordingly.

4 Numerical Simulations

The POMP algorithm and a ten-channels polyphase channelizer are implemented and simulated in MATLAB. In order to compare the performance of the two methods, we use them to process the same signal which is sparse in frequency domain, and the original signal can be demonstrated in time domain and frequency domain as Fig. 2.

In POMP algorithm, the measurement times M is 64, which is determined by $K \log(N/K)$, and the iteration time in each channel is determined by the sparsity K. In channelization method, there are fifty-percent aliasing between channels. The passband and the stopband of the prototype filter are $f_s/(2 \times 10)$ and $f_s/(2 \times 5)$ respectively, which causes the down-sampling time to be half of the channel numbers. The taps of the polyphase filters are polyphase components of the prototype filter's taps, and the order of the prototype filter is 100.

We process the signal in Fig. 2 utilizing a polyphase channelizer and the POMP method respectively. Then we get results shown in Figs. 3 and 4.

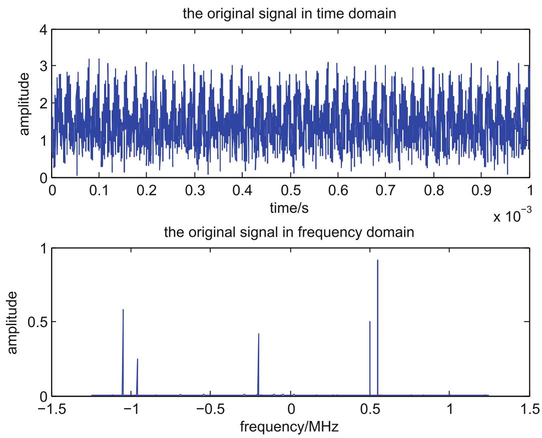

Fig. 2. The original signal in time domain and in frequency domain. The signal is noise-free, complex valued and generated in time domain. The centre frequency is zero, and the sampling rate is 2.5 MHz. The original signal consists of a few single-frequency signals which are generated randomly.

Figure 3 shows the division of the original signal by a ten-channel polyphase channelizer. Due to the aliasing phenomenon, the sampling rate per channel is 0.5 MHz, which is one-fifth of the original sampling rate. We can also notice that the aliasing occurs in the sixth and the ninth channels, which will cause misjudgment to the subsequent signal processing.

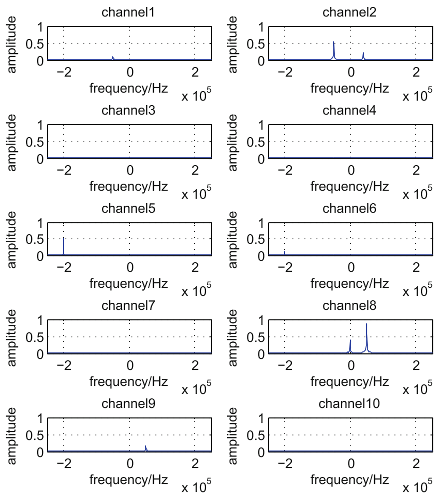

Fig. 3. The division of the original signal by a ten-channel polyphase channelizer. The signals in channel 9 and channel 6 don't exist actually, it is caused by the aliasing between channels.

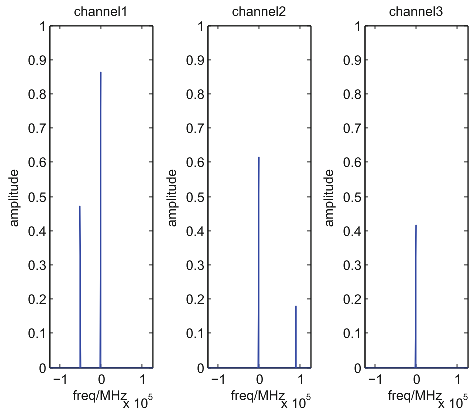

Fig. 4. The channel extracting using POMP algorithm. The amount of channels depends on the number and the intensive degree of signals. Here we need three channels to extract all the signals. The strongest signal in each channel lies in the center of the channel. The three channels are arranged according to the intensity of the center signal.

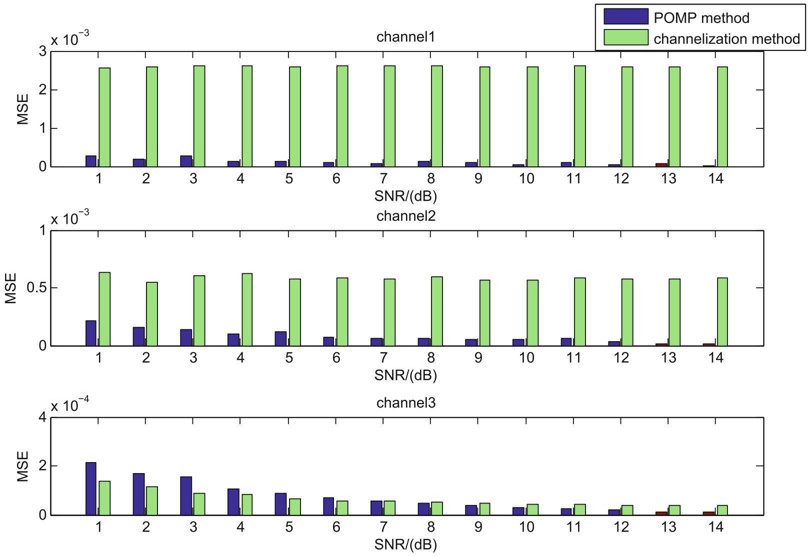

Fig. 5. The first figure contrasts the channel 8 in the channelizer and channel 1 in POMP algorithm. The second figure contrasts the channel 5 in the channelizer and the channel 3 in POMP. The third figure contrasts the channel 2 in the channelizer and the channel 2 in POMP. From the overall reconstruction effect, the POMP algorithm performs better.

Figure 4 shows the extracting of the channels using POMP algorithm. For this simulation experiment, we only need three channels to extract all the signals.

The sampling rate per channel is 0.25 MHz, which is one-tenth of the original sampling rate. The signal locates at the center of each channel are sorted in descending order of energy. We can notice that the POMP algorithm can also achieve the tasks of down-sampling rate, down-conversion, and band limiting. Besides, the aliasing phenomenon has been good to avoid.

We compare the mean square error (MSE) of the signal extracted by the two methods respectively with the original signal at different SNRs. The results are shown in Fig. 5. We notice that the POMP performs significantly better in the first and the second figures. The effect of POMP reconstruction is relatively poor at low SNRs as shown in the third figure, however, with the SNRs increase, the POMP reconstruction errors drop relatively rapid.

These results demonstrate that the performance of the POMP algorithm is better than a polyphase channelizer. We further compare the efficiency of the two methods by calculating the CPU time that the two methods take to reconstruct the whole BOI at different SNRs. The average CPU execution time of the POMP algorithm is 1.618240 s, while the channelizer is 2.086837 s, which demonstrate that the POMP algorithm is more efficient.

5 Conclusion

We study the position relationship among the signal distribution, non-zero items in θ and the corresponding columns in sensing matrix A, and propose the POMP algorithm for extracting channels contain signals from BOI. The proposed algorithm is implemented and simulated, and compared with a ten-channel polyphase channelizer by processing the same signal. The numerical results demonstrate that when the BOI is sparse, the POMP algorithm performs better than a polyphase channelizer. The proposed algorithm not only achieves the tasks of down-convertion, down-sampling and band-limiting, which are core functions for a channelizer, but also avoids the aliasing or leaking that a channelizer may cause.

Acknowledgment. This work is supported by the Key Development Program of Basic Research of China (JCKY2013604B001), the National Natural Science Foundation of China (61301095), the Funda-mental Research Funds for the Central Universities (GK2080260148 and HEUCF1508).This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation.Meantime, all the authors declare that there is no conflict of interests regarding the publication of this article.We gratefully thank of very useful discussions of reviewers.

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