A Fast Cyclic Spectrum Detection Algorithm for MWC Based on Lorentzian Norm

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Abstract. In order to solve the problem of high sampling rate in the wideband spectrum sensing of cognitive radio, this paper studies the method of cyclic spectrum detection based on the modulation wideband converter (MWC). A novel fast cyclic spectrum detection algorithm of MWC based on Lorentzian Norm is proposed to deal with the influence of some non-ideal factors on the performance of the existing MWC system reconstruction algorithm in physical implementation. Firstly, the objective function for sparse optimization is build based on smoothed L0-norm constrained Lorentzian norm regularization. Then a parallel reconstruction method is implemented in a unified parametric framework by combining the fixed-step formula and the conjugate gradient algorithm with sufficient decent property. Simulation results demonstrate that the proposed algorithm can not only improve the recovery probability of sparse signal, but also has a higher detection probability in low SNR environment compared with traditional reconstruction algorithms.

Keywords: Cognitive radio · Cyclic spectrum detection
Modulated wideband converter · Signal recovery · Impulsive noise

1 Introduction

Ensuring the normal communication of the primary user (PU) is a prerequisite for cognitive radio, therefore, spectrum sensing is very important. Fast and accurately sensing of the whole frequency domain information is the target of spectrum sensing, and it is still a huge challenge for spectrum sensing. Different with the spectrum sensing problem of traditional narrowband systems, cognitive radio needs to complete the dynamic access to broadband. The cyclic spectrum feature detection has stronger ability to resist the uncertainty of the noise power, and can better distinguish the noise and signal than energy detector, so it has a good application prospect [1]. However, traditional detection methods are based on the Nyquist theorem for sampling, such a high-speed ADC design and mass information processing in the broadband spectrum sensing is difficult to achieve [2].

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Compressed sensing (CS) is a new kind of compressed sampling technology [3]. Tian and Giannakis introduced CS technology into the broadband spectrum sensing of cognitive radio [4]. Professor Eladar's research group proposed a modulated wideband converter (MWC) with parallel multi-branch structure [5]. Modulated wideband converter can theoretically use existing devices to sample the continuous frequency sparse multiband signal with sub Nyquist sampling and accurately reconstruct the original signal.

In literature [6], it is pointed out that the signal reconstruction of MWC system can be transformed into multiple measurement vectors, which is a generalized form of single measurement vector (SMV) model in CS theory. Eldar and Rauhut demonstrated that the MMV model can significantly improve the probability of successful reconstruction of unknown sparse signals relative to the SMV model algorithm in [7] and [8]. The number of MWC system channels determines the hardware complexity of the device. The literature [9] proposed a reconstruction algorithm based on random projection idea, which reduces the minimum number of channels required for high probability reconstruction. However, there are still large gaps in the performance of the MWC reconstruction algorithm between theory and practice, and it is assumed that different measurement columns meet the joint sparse characteristics. In addition, the reconstruction model does not consider the effect of some non-ideal factors on the performance of the system.

In order to solve the above problems, a fast cyclic spectrum detection algorithm for MWC based on Lorentzian norm (MWC-FCSD) is proposed. For the beginning, the objective function for sparse optimization was built based on matrix smoothed L0-norm. The Lorentzian norm is used to fit the error term of the noise, which effectively suppresses the singular values in the measurement vector and improves the reconstruction precision and robustness. Then, the conjugate gradient method with fixed step is used to solve the parallel optimization problem under the unified parameter framework, which reduces the matrix storage and operation, and improves the convergence speed and efficiency of the algorithm. Finally, the algorithm is applied to the cyclic spectrum detection. The simulation results show the effectiveness of the proposed algorithm.

2 Cyclic Spectrum Estimation Based on Compressive Sensing

2.1 MWC Compressed Sampling

The block diagram of MWC system is shown in Fig. 1. The system uses a parallel multi-channel structure, and each channel consists of a pseudo-random sequence generator, a mixer, a low-pass filter and a low-speed sampler. Different channels of the MWC system are mixed with different pseudo-random $\pm$ 1 waveform functions with the same period T_p, so that each frequency band is weighted with different Fourier coefficients to ensure that all the frequency band information can be obtained by low speed sampling.

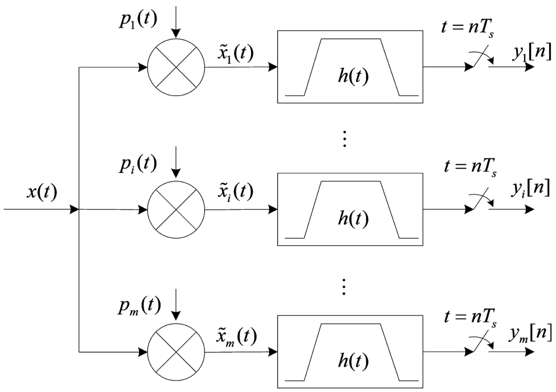

Fig. 1. MWC sampling system block diagram.

According to Fourier analysis, we can get the relationship between $Y_i(e^{j2\pi fT_s})$ and $X(f - lf_p)$:

$$Y_i(e^{j2\pi fT_s}) = \sum_{l=-L_0}^{L_0} c_{il} X(f - lf_p) \,, \tag{1}$$

where $f \in F_s = [-f_s/2, f_s/2]$. c_{il} is the coefficients of Fourier expanding series of $p_i(t)$. To facilitate the analysis of subsequent signal reconstruction process, the combination of all m samplers, we can obtain the following matrix form:

$$\boldsymbol{Y}(f) = \boldsymbol{\Phi z}(f), f \in F_s \,, \tag{2}$$

where $\boldsymbol{\Phi}_{i,j} = c_{i,j-L_0-1} \in \mathrm{R}^{m \times L}$, $Y_i(f) = Y_i(e^{j2\pi fT_s})$, $z_l(f) = X(f - lf_p), f \in F_s$.

2.2 Cyclic Spectrum of Compressed Sampling Signal

To calculate the cyclic spectrum of $x(t)$, we must derive the linear relationship between the cyclic spectrum and the reconstructed signal. The mean of the sequence $x(n)$ is zero and the cyclic spectrum is stable, so the autocorrelation function can be defined as

$$r_x(n, v) = E\{x(nT_s)x^*(nT_s + vT_s)\} = E\{x[n]x^*[n + v]\} \,, \tag{3}$$

where $r_x(n, v) = r_x(n + kP, v)$, the integer P means the cyclic period

The Fourier coefficient of $r_x(n, v)$ is called Cyclic Autocorrelation Function (CAF). For the sampling length N is limited, so the estimation of CAF can be represented as

$$\tilde{r}_x^{(c)}(a, v) = \{\frac{1}{N} \sum_{n=0}^{N-1-v} r_x(n, v) \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}an}\} \mathrm{e}^{-\mathrm{j}\frac{\pi}{N}av} \,, \tag{4}$$

where $a \in [0, N-1]$. Based on continuous signal processing, the CAF of the discrete cyclic stationary signal increases the correction factor $\mathrm{e}^{-\mathrm{j}\pi av/N}$. Although this expression is biased, its estimated variance is less than other unbiased estimates [10].

The cyclic spectrum can be obtained by the Fourier transform of CAF which is represented as

$$s_x^{(c)}(a,b) = \sum_{v=0}^{N-1} \tilde{r}_x^{(c)}(a,v)\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}bv}, \tag{5}$$

where $b \in (0, N-1]$ is the digital form of spectral frequency $f = (b/N)f_s$.

3 Problems in MWC Signal Reconstruction

3.1 Arbitary Sparse Structure Model

MWC compressed sampling is equivalent to the projection process as shown in Fig. 2. The spectral shift step f_p determines the final position of each frequency band of $\boldsymbol{X}(f)$ in $\boldsymbol{z}(f)$. So, the original sparse multi-band signal can be reconstructed by tracking the sparsest solution of MMV problem and performing spectral shift.

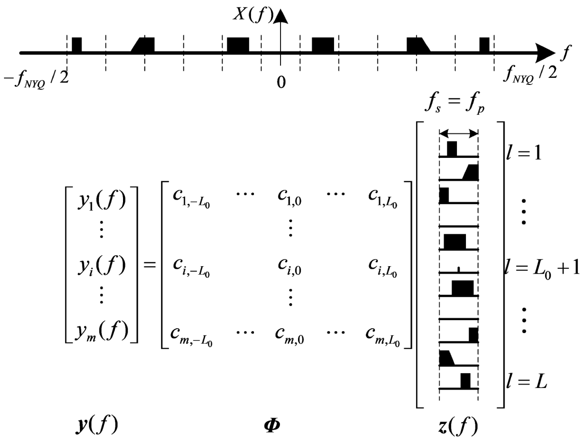

Fig. 2. Illustration for the spectrum of MWC system.

MMV model can be represented below:

$$\boldsymbol{Y} = \boldsymbol{\Phi Z}, \tag{6}$$

where $\boldsymbol{Z} = [\boldsymbol{z}^{(1)}, \cdots, \boldsymbol{z}^{(L)}], \boldsymbol{z}^{(l)} \in \mathrm{R}^{N\times 1}$ is consist of L numbers of sparse column vectors, and it is assumed that these vectors have K numbers of common nonzero rows, which means joint K sparse.$\boldsymbol{Y} = [\boldsymbol{y}^{(1)}, \cdots, \boldsymbol{y}^{(L)}], \boldsymbol{y}^{(l)} \in \mathrm{R}^{M\times 1}$ is the sampled value matrix. L is the total column number of the measured vectors. MMV reconstruction problem is essentially to obtain the sparsest solution by solving the optimization problem with sparse constraints.

$$\arg\min \sum_{l=1}^{L} ||\boldsymbol{z}^{(l)}||_{l_0} \text{ st.}\boldsymbol{y}^{(l)} = \boldsymbol{\Phi}\boldsymbol{z}^{(l)}\, l = 1, \cdots, L\,, \tag{7}$$

where $z^{(l)}$ represents the l-th column vector of matrix $\boldsymbol{Z}$ and $\boldsymbol{y}^{(l)}$ represents the l-th column vector of matrix $\boldsymbol{Y}$. $|| \bullet ||_{l_0}$ represents the l_0 norm. Solving MMV model can be regarded as solving a series of SMV problems with sparseness constraints, which belong to the typical combinatorial optimization problems.

However, in MWC compressed sampling system, the sparseness of different measured columns are arbitrary and the locations of nonzero elements were not consistent, and the joint sparseness assumption of traditional MMV model cannot accurately describe the sparseness of such signals. In this paper, it is assumed that MMV model has arbitrary sparse structure (MMV of Arbitrary Sparse Structure, ASS-MMV), which means the sparseness and support sets of different column vectors matrix do not require the same, conforming the frequency sparseness features of vector $z(f)$ in MWC system.

3.2 Effect of Analog Low Pass Filter

In MWC system, in order to retain the low frequency $f \in [-f_s/2, f_s/2]$ after mixing to achieve low rate sampling, it is required analog low pass filters to complete the anti-aliasing filtering. The relationship between the Fourier transformation of the output sequence $y_i(n)$ and the original signal $x(t)$ in Eq. (5) is established in the case of ideal filtering. However, the actual analog low pass filters have some non-ideal condition, such as the transition band and the passband fluctuation, as showed in Fig. 3.

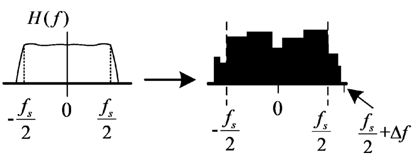

Fig. 3. Influence of transition band of filter on baseband.

Equation (7) represents the MMV model of compressed sensing without noise. However, during the compressed sampling process in MWC system, the measured value matrix can be influenced by noise and interface, because of the aliasing frequency component in filter transition band and the distortion in passband. So the MMV model with noise can be expressed as:

$$Y = \Phi Z + W \ , \tag{8}$$

where $\mathcal{W} = [w^{(1)}, \cdots, w^{(L)}], w^{(l)} \in \mathrm{R}^{M \times 1}$, W represents additive noise.

The residual frequency components, located in transition band $[f_s/2, f_s/2 + \Delta f]$ in Fig. 3, were superimposed on the original components in the form of discrete frequency after A/D sampling. So a number of singular points were added to the compressed sample value matrix, and each element in the sample value matrix was directly related to the original sparse signal. In the framework of compressed sensing, noise W was divided into two categories: Gaussian white noise and non-Gaussian impulse noise. In MWC system, the aliasing distortion caused by analog low pass filter transition band sampling belongs to the latter.

4 Cyclic Spectrum Detection Algorithm for MWC

4.1 Sparse Optimization Objective Function

For the MMV solution problem in the noise model, the Eq. (8) can be modified as follows:

$$\arg\min \sum_{l=1}^{L} ||z^{(l)}||_{l_0} + \lambda \mathrm{loss}(y^{(l)} - \Phi z^{(l)}) \ , \tag{9}$$

where $\mathrm{loss}(y^{(l)} - \Phi z^{(l)})$ denoting error term, and $\lambda \geq 0$ is regularization parameters, which controls the balance between allowable error and sparseness. When the W is Gaussian noise, the norm can be used to fit the error term. In this case, (5) has the following expression:

$$\arg\min \sum_{l=1}^{L} ||z^{(l)}||_{l_0} + \lambda ||y^{(l)} - \Phi z^{(l)}||_2^2 \ . \tag{10}$$

It can be seen from the analysis in [11] that the minimum mean square error of the signal under the compressed sensing optimal reconstruction is proportional to the variance of the noise. When the W is the impact noise, as it is characterized by a large variance, elements with larger value will appear in the error term. Because there are discrete points, the l_2 norm will linear amplify the impact of residual.

The literature [12] pointed out that when the Lorentzian norm is used to fit the error term, due to the bounded soft-return characteristic of the derivative function, the penalty for the element with large amplitude in the error term is heavier and its effect is the same as the l_1 norm; The penalty of the element with smaller amplitude in the error term is lighter, and its function is the same as the l_2 norm. So it is possible to robustly reduce the effect of the outliers on the reconstruction results. The Lorentzian norm is defined as follows:

$$||\boldsymbol{u}||_{LL_2,\gamma} = \sum_{m=1}^{M} \log(1+\gamma^{-2}\boldsymbol{u}_m^2)\,,\ \gamma > 0\ , \quad ((11))$$

where $\boldsymbol{u} \in \mathrm{R}^{M\times 1}$ is a column vector, and $||\bullet||_{LL_2,\gamma}$ denotes the Lorentzian norm of $\boldsymbol{u}$, γ is the scale parameter of the Lorentzian norm and determines the robustness of the LL_2 norm to the error term outliers.

In this paper, the Lorentzian norm is used to replace the norm in (10) to fit the error term. The solution of MMV model under impact noise can be expressed as follows:

$$\arg\min \sum_{l=1}^{L} ||\boldsymbol{z}^{(l)}||_{l_0} + \lambda||\boldsymbol{y}^{(l)} - \boldsymbol{\Phi}\boldsymbol{z}^{(l)}||_{LL_2,\gamma}\ , \quad (12)$$

where $||\boldsymbol{y}^{(l)} - \boldsymbol{\Phi}\boldsymbol{z}^{(l)}||_{LL_2,\gamma}$ denotes the Lorentzian norm of the i-th column reconstruction error term of the sampling matrix.

4.2 ASS-MMV Fast Reconstruction Algorithm

The signal reconstruction of MMV model can be summarized as an optimization problem:

$$L(\boldsymbol{X}) = \sum_{l=1}^{L} ||\boldsymbol{z}^{(l)}||_{l_0} + \lambda||\boldsymbol{y}^{(l)} - \boldsymbol{\Phi}\boldsymbol{z}^{(l)}||_{LL_2,\gamma}\ . \quad (13)$$

In the formula (13), the norm is pseudo-norm, which is highly discontinuous and cannot be solved by analytic method. It belongs to the NP-Hard problem. The SL0 algorithm approximates the l_0 norm by a class of smooth Gaussian functions, and solves the minimum problem directly by analytic method. This method not only improves the reconstruction probability, but also greatly shortens the computing time. The optimal objective function based on smooth norm and Lorentzian norm is

$$L(\boldsymbol{Z}) = \sum_{l=1}^{L} F_\sigma(\boldsymbol{z}^{(l)}) + \lambda||\boldsymbol{y}^{(l)} - \boldsymbol{\Phi}\boldsymbol{z}^{(l)}||_{LL_2,\gamma}\ , \quad (14)$$

where $F_\sigma(\boldsymbol{z}^{(l)}) = N - \sum_{i=1}^{N} f_\sigma(z_i^{(l)})$, $f_\sigma(z_i^{(l)})$ denotes Standard Gaussian function

$$f_\sigma(\boldsymbol{s}) = \exp(-\boldsymbol{s}^2/2\sigma^2)\ , \quad (15)$$

where σ is used to measure the relationship between the accuracy and smoothness of the l_0 norm of the vector $\boldsymbol{s}$. By the properties of the Gaussian function

$$\lim_{\sigma \to 0} F_\sigma(z^{(l)}) = ||z^{(l)}||_0 \,. \tag{16}$$

The fast reconstruction algorithm of ASS-MMV model uses the characteristics of SL0 algorithm which converge to the vicinity of the optimal value at each σ value, and set the initial least squares solution of sparse vector: $\boldsymbol{Z}_0 = \boldsymbol{\Phi}^{\mathrm{H}}(\boldsymbol{\Phi}\boldsymbol{\Phi}^{\mathrm{H}})^{-1}\boldsymbol{Y}$. The algorithm reconstructs the multi-vector in parallel with the numerical optimization algorithm under the unified parameter setting framework, and realizes the parallel reconstruction of the MMV signal model with arbitrary sparse structure.

The iterative method is used to solve the optimal solution as (16):

$$z_{k+1} = z_k + a_k \boldsymbol{d}_K, \tag{17}$$

where z_k is the k-th iteration point, $\boldsymbol{d}_k$ is the k-th search direction, a_k is the k-th iterative step. In order to overcome the slow convergence rate of steepest descent method, and high computational complexity and large storage of newton method used in SL0 algorithm, a conjugate gradient algorithm based on fixed step size is adopted in this paper [13].

5 Numerical Experiment

We performed simulations to demonstrate the effectiveness of the MWC-FCSD algorithm. Firstly, the influence of the transition band of analog low-pass filter on the reconstruction performance of MWC-FCSD algorithm, OMPMMV algorithm and MSL0 algorithm are analyzed. Secondly, the reconstruction time of each algorithm is compared. Finally the performance of cyclic spectrum detection is verified. For all the experiments we create sparse multiband signals, which is BPSK modulation signal with different energy E_n, carrier frequency f_n and bandwidth B_n. The carriers f_n for very signal are chosen uniformly at random in $[-f_{NYQ}/2, f_{NYQ}/2]$ with f_{NYQ} = 10 GHz. In order to use MATLAB to simulate the sampling process of the analog signal, the sampling rate $10f_{NYQ}$ is used to simulate the analog signal.

The parameters of MWC are configured as follows, $f_s = f_p = 51.28\,\text{MHz}$, $M = 195$, $L = 2L_0 + 1 = 195$, $E_n \in [1, 3]$, τ_n determined randomly in the effective observation time.

The following simulations are repeated 500 times for each set of parameters setting. The ratio of the number of successes to the number of experiments is taken as the reconstruction success rate which is defined as the recovered support sets is the same as the actual support sets.

A. Influence on performance of transition band of LP filter

In order to evaluate the influence of analog low-pass filter on signal reconstruction performance, we adjust the rectangle coefficients of low-pass filter, and compare the reconstruction success rate of OMPMMV algorithm, MSL0 algorithm and MWC-FCSD algorithm, as shown in Fig. 4. The multiband signals consist of $N = 4$ pairs of bands, and the channel number takes one of the two choices: 30 or 50.

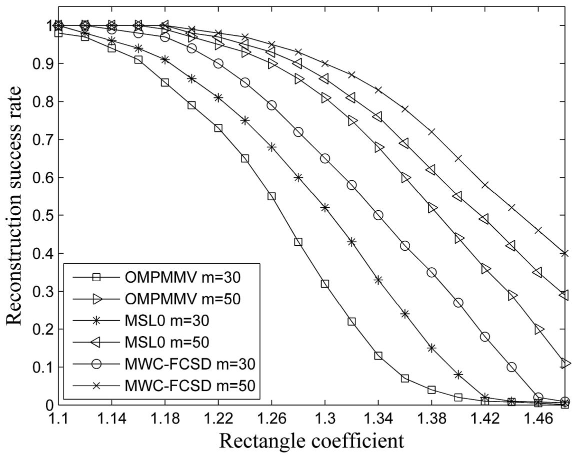

Fig. 4. Reconstruction success rate comparison under different transition bandwidth.

As can be seen from Fig. 4, the larger the rectangular coefficient of the low-pass filter, the smaller the success rate of the reconstruction. This is because the wider the transition band of the low-pass filter, the higher the frequency aliasing of the baseband signal after sampling, that is, the greater the impact noise. Due to the use of the Lorentzian norm fitting error term in MWC-FCSD algorithm, the singular value in the observation vector can be robustly suppressed. It can be seen from Fig. 4 that the reconstruction success rate can be improved by increasing the number of channels. In order to achieve high probability reconstruction (more than 90%), the number of channels and the rectangular coefficient required by MWC-FCSD algorithm, OMPMMV algorithm, MSL0 algorithm, respectively, are $m = 30$ and $r = 1.3$, $m = 50$ and $r = 1.26$, $m = 50$ and $r = 1.26$. In conclusion, our algorithm can effectively improve the reconstruction ability of the MWC system, reduce the number of hardware channels, and the design requirements of the analog low-pass filter transition bandwide.

B. Comparison of reconstruction time under different channels

In this section, we add a set of simulation data of SL0 algorithm based on SMV model to verify the advantages of MMV model in reconstruction speed. The average operation times of the four algorithms are given in Table 1, and the number of channels is set to 24, 26, 28, 30, 32, 34. In addition to $N = 6$, the other parameters are consistent with experiment A.

Table 1. Reconstruction times comparison of several algorithms.

Channel	OMPMMV	MSL0	MWC-FCSD	SMV-SL0
24	0.8361	1.6279	1.8294	19.276
26	0.8527	1.6764	1.8846	19.985
28	0.8704	1.7302	1.9403	20.515
30	0.8918	1.7824	1.9971	21.132
32	0.9174	1.8395	2.0572	21.760
34	0.9352	1.9047	2.1163	22.376

As shown in Table 1, the reconstruction time increases with the number of channels. Based on the MMV model, the reconstruction times of MWC-FCSD algorithm, OMPMMV algorithm, MSL0 algorithm are always much smaller than the of SMV-SL0 algorithm, because the SMV-SL0 algorithm needs to be reconstructed one by one. OMPMMV algorithm has the fastest reconstruction speed, which is a greedy iterative algorithm. Compared with the MSL0 algorithm, although the computational complexity of objective function gradient and search step size of MWC-FCSD algorithm is slightly larger, the reconstruction times are close.

C. Cyclic spectrum detection performance of MWC compression sampling

The detection signal consists of three channels which are occupied at the same time. The cycle spectrum of detection signal reconstructed by our algorithm while $m = 50$ and SNR = 0 dB is shown in Fig. 5. The signals PU1, PU2, PU3 have significant spectral peaks at their cyclic frequencies. The peak and its position information can be used for signal detection and signal modulation recognition. The cyclic spectrum estimation based on compressed sensing makes use of the sparsity of the cyclic spectral domain, which reduces the requirement of sampling rate ($f_{\sum} = mf_s$).

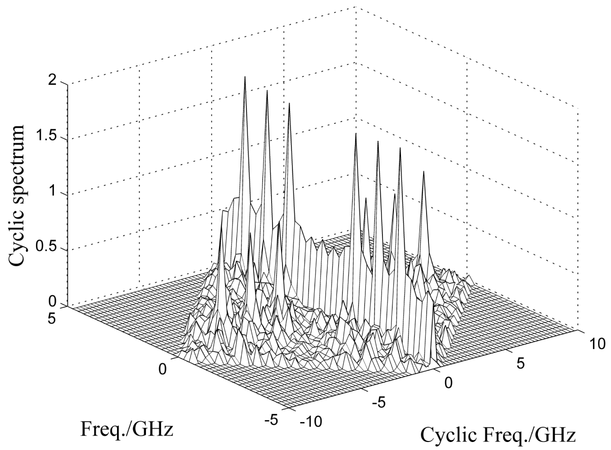

Fig. 5. Cyclic spectrum of the reconstruction signal on m = 50.

Figure 6 shows the detection probability curves for various numbers m of channels and various SNRs. When SNR = 20 dB, m = 36, the detection probability Pd is close to 1. When the SNR is lower than 10 dB, the detection probability decreases sharply, which is due to the decline of the cyclic spectrum sparsity under low SNR and lead to deterioration of the reconstruction performance. Meanwhile, the sampling rate can be adjusted according to the SNR, and channel number can be reduced when the SNR is high. The detection probability of the detection probability curve in Fig. 6 is obtained by 500 Monte-Carlo simulations.

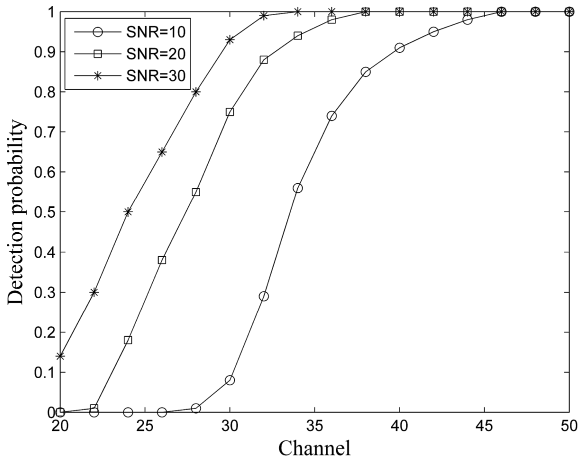

Fig. 6. Cyclic spectrum detection performance of MWC-FCSD algorithm.

6 Conclusion

In this paper, a fast cyclic spectrum detection algorithm for MWC based on Lorentzian norm is proposed. Our algorithm solves the problem that the performance of the existing MWC sub-Nyquist sampling reconstruction algorithm is easy to be influenced by non-ideal factors. Simulation and experimental results show that the algorithm proposed in this paper has the advantages of good reconfiguration performance and few reconstruction channels compared with the existing algorithms. It is not only achieves efficient reconstruction of MWC compression samples with arbitrary sparse structure, but also can effectively reduce the influence of non-ideal factors such as filter transition.

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