

Two-Dimensional Fractal Dimension Feature Extraction Algorithm Based On Time-Frequency

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Abstract. Digital signal modulation recognition is the technology of signal recognition. In the non-cooperative communication field, the technology is used to process signal and extract feature, and recognize the signal. Because of small distance and intersection between feature classes of digital signal fractal box dimension, its difficult to recognize the signal. This paper proposes a new algorithm. This algorithm is based on time frequency image of two dimensional fractal box dimension feature extraction.

Keywords: Modulation recognition · Fractal box dimension
Feature extraction

1 Introduction

In recent years, with the rapid development of software radio technology, communication reconnaissance, confrontation and other modern information technology research has become research hotspots [1]. Radio technology plays an important role both in the civil and military fields. Electronic reconnaissance technology is of great importance to the promotion of civil technology and the enhancement of national defense. Wireless communication uses spatial electromagnetic radiation to transmit images, text, sound and other information. The open channel environment makes communication reconnaissance possible [2]. Signal recognition technology has become an important means of electronic reconnaissance. The signal recognition technology realizes the signal identification by extracting the characteristic parameters of the received signal according to the difference between the different signal characteristics. At present, signal recognition algorithm can be divided into two categories [3]. The first category is based on test method of decision theory, the second category is based on statistical model of identification methods [4].

The signal modulation recognition method based on decision theory can theoretically guarantee the optimal recognition results under the Bayes minimum criterion, but this kind of method has very obvious shortcomings. This method requires too much parameter information and has large computational complexity [5]. So it is not suitable for real-time classification of signal modulation recognition. The signal recognition method based on statistical mode can be regarded as a mapping relation. By extracting the effective and stable characteristics of

the signal, the signal is mapped from the time domain space to the feature space. Then the signal is classified and identified by the difference between the signal characteristics. The appropriate classification features and decision criteria have to implement easily. So the algorithm based on statistical mode is more extensive used in the actual project. Fractal theory has been studied in depth in recent years. Fractal theory are applied in many fields such as protein sequences, kinetic structures, liquid structures, DNA sequence analysis, fault diagnosis [6], image processing [7] and so on [8]. In this paper, we propose a two-dimensional box dimension feature extraction method based on time-frequency image. Simulation results show that method proposed in this paper is superior to one-dimensional signal box dimension feature and is suitable for the classification and recognition of multiple signals [9].

2 Research on Fractal Feature of Digital Signal

2.1 Fractal Box Dimension Theory

Fractal dimension is a tool to describe the dimension of the object. Among the many fractal dimension calculation methods, the box dimension algorithm is simple and can calculate the fractal dimension of the object well [10].

Let (X, d) be an object space, M is a non-empty compact set family of X , A is a non-empty compact set in X . For each positive number ε , the number of boxes covering A can be represented by $N(A, \varepsilon)$, the box length is ε , then [7]:

$$N(A, \varepsilon) = \{M : A \subset \sum_{i=1}^M N(x_i, \varepsilon)\} \tag{1}$$

wherein, x_1, x_2, \dots, x_M are different point and x_1, x_2, \dots, x_M belong to X . The box dimension is defined

$$D_b = \lim_{\varepsilon \rightarrow 0} \frac{\ln N(A, \varepsilon)}{\ln(1/\varepsilon)} \tag{2}$$

For signal x_i , ε is the time interval of the signal sampling process. In the calculation of box dimension, ε represents the minimum length of the box and the length growth rate of the box. For a box with a length of $k\varepsilon$ to cover the signal, the number of boxes required is:

$$s_1 = \max\{x_{k(i-1)+1}, x_{k(i-1)+2}, \dots, x_{k(i-1)+k+1}\} \tag{3}$$

$$s_2 = \min\{x_{k(i-1)+1}, x_{k(i-1)+2}, \dots, x_{k(i-1)+k+1}\} \tag{4}$$

$$s(k\varepsilon) = \sum_{i=1}^{N_0/k} |s_1 - s_2| \tag{5}$$

In formula (5), $i = 1, 2, \dots, N_0/k$, $k = 1, 2, \dots, K$, N_0 is signal truncation length, and $K < N_0$, $s(k\varepsilon)$ is the signal amplitude range, then $N_{k\varepsilon}$ is expressed as:

$$N_{k\varepsilon} = s(k\varepsilon)/k\varepsilon + 1 \tag{6}$$

For the fitting curve of $\lg k\varepsilon \sim \lg N_{k\varepsilon}$, calculating the box dimension is chosen to select a better linear segment. After the logarithm, formula (7) is available

$$\lg N_{k\varepsilon} = -d_B \lg k\varepsilon + b \quad (7)$$

In formula (7), $k_1 \leq k \leq k_2$, k_1 is the starting points of the number of boxes. k_2 is the ending points of the number of boxes. The Least-Mean-Square algorithm can be used to calculate the straight slope of the segment. The box dimension of the signal is calculated:

$$D = -\frac{(k_2 - k_1 + 1) \sum_{k=k_1}^{k_2} (\lg k) \cdot \lg N_{k\varepsilon} - \sum_{k=k_1}^{k_2} (\lg k) \cdot \lg N_{k\varepsilon}}{(k_2 - k_1 + 1) \sum_{k=k_1}^{k_2} \lg^2 k - (\sum_{k=k_1}^{k_2} \lg k)^2} \quad (8)$$

2.2 Simulation Experiment

In this paper, we identify four different digital modulation signal types. The modulation signal types include 2FSK, BPSK, 16QAM and MSK. The signal carrier frequency is $f_c = 4$ MHz, sampling frequency is $f_s = 4 \times f_c = 16$ MHz, signal length is $N_s = 2048$, digital signal symbol rate is $R_s = 1000$ Sps, 2FSK signal $f_1 = 1$ MHz, $f_2 = 2$ MHz. The baseband signal is a random code, the modulated signal is shaped by a rectangular pulse, and the noise is white Gaussian noise. For each type signal, 100 Monte Carlo experiments were performed at SNR = 0 dB and SNR = 10 dB to generate a sample signal to calculate the mean and variance of the box dimension. Table 1 is the mean and variance of the fractal dimension of the four digital signals. Table Remarks: (mean, variance).

Table 1. The fractal dimension mean and variance of signals

SNR	2FSK		BPSK		MSK		16QAM	
	μ	σ	μ	σ	μ	σ	μ	σ
0 dB	1.510	0.015	1.549	0.015	1.554	0.016	1.601	0.016
10 dB	1.481	0.015	1.607	0.016	1.611	0.016	1.645	0.016

The mean of the signal feature represents the central position of the feature of the digital signal in the feature space and the class separation between the features of the digital signal. The variance of signal characteristics represents the intra class aggregation of the characteristics of digital signals. As can be seen from Table 1, the fractal box dimension between digital signals has less distance between classes, but the variance of its characteristic value is small. It shows that digital signal fractal dimension within the degree of polymerization is good. For digital signals, there is a strong similarity between the time domain waveforms of signals. So it is not very good to extract the signal fractal feature from the fractal box dimension of the signal time domain waveform. Figure 1 is the digital signal box dimension feature curve with signal to noise ratio (SNR).

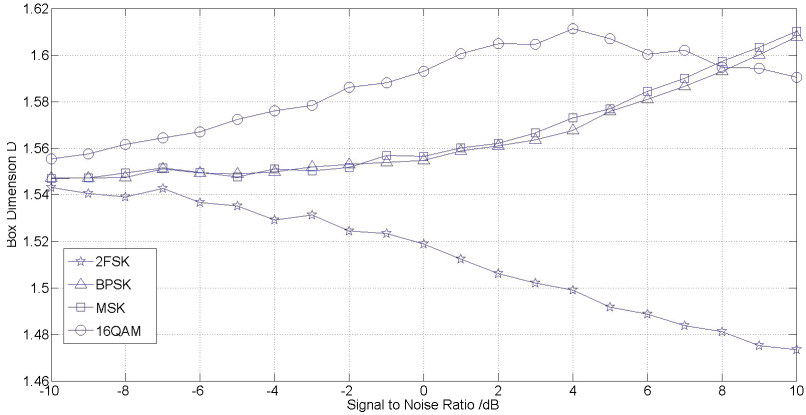


Fig. 1. Digital signal box dimension feature curve with SNR

The signal fractal dimension is a description of the signal dimension. The lower the complexity of the signal, the smaller the box dimension. It can be seen from Fig. 1 that the inter-class distance of fractal box dimension of different signals is small and the characteristic stability is poor. The box dimension curves of different signals have cross phenomena. The fractal box dimension has a good ability to distinguish between 2FSK signals at higher signal-to-noise ratio. But for other digital signals, the inter-class distance between signals is small. It is not conducive to classify and identify digital signals. So, this paper presents a two-dimensional box dimension feature extraction method based on time-frequency image.

3 Research on Two-Dimensional Fractal Dimension Features Based on Time-Frequency Images

Signal time-frequency conversion is to distribute signal energy in signal time-frequency plane. It is a conversion of one-dimensional to two-dimensional. The time-frequency analysis of the signal can effectively reflect the spectral distribution of the signal over time. Compared to the signal’s time domain waveform, the time-frequency transformation of the signal can fine describe the local subtle characteristics of the signal. From the simulation results in the previous section, we can see that the difference between fractal dimension features of digital signal time-domain waveforms is not sufficient to be effectively used for the classification of signals and the interclass distance between fractal dimension features is small [11]. The time-frequency diagram of the signal describes the change of signal over time from the two transform fields, which are time domain and frequency domain. Compared to time domain waveform fractal dimension, signal time-frequency characteristics can be better used for signal classification and identification. Therefore, this paper presents a two-dimensional box dimension feature extraction algorithm based on time-frequency image.

3.1 Research on Algorithm of Time-Frequency Image Box Dimension

The improved fractal dimension algorithm is based on signal time-frequency transform, through the time-frequency image of the signal related processing and extracting two-dimensional box dimension image features of signal. This is a conversion of one-to-two-dimensional. Figure 2 shows the concrete flow chart of the improved algorithm.

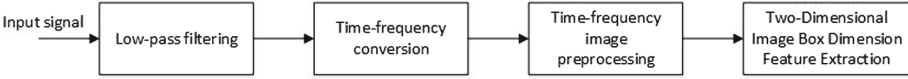


Fig. 2. Flow chart of fractal feature extraction of two-dimensional box dimension based on time-frequency image

The signal time-frequency transform is the distribution of the signal energy in the time-frequency domain. Therefore, we can convert gray value of the image, according to the signal in the time-frequency domain of the energy value, which is the main work of the time-frequency image preprocessing part.

(1) Normalization

In the time-frequency image preprocessing of digital signals, the larger the energy value of the time-frequency distribution, the larger the gray value in the corresponding image. The difference between the time-frequency transformations of different digital signals results in the dynamic range of the gray value of the time-frequency image is not the same. The size of the gray value have a great impact on the extraction of signal characteristics. Therefore, in order to reduce the balance between the different signals, we need to normalize the gray value of the image preprocessing [5]. \bar{x} and σ^2 are the mean and variance of pixel value [12].

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad (9)$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \quad (10)$$

The gray value of the normalized pixel is:

$$\hat{x} = \frac{x_i - \bar{x}}{\sigma} \quad (11)$$

(2) 2D image box dimension feature extraction

For a digital signal time-frequency grayscale, we set the image size is $M \times M$, and divide the $M \times M$ pixel image into $s \times s$ sub-blocks ($1 \leq s \leq M/2$, s is an integer), let

$$r = s/M \quad (12)$$

The time-frequency image gray-scale value of the digital signal is a three-dimensional surface in the spatial coordinates. x and y represent the coordinate position of the pixel, and z represents pixel gray value. $x - y$ plane is divided into $s \times s$ squares, which become a $s \times s \times s'$ box. The s' satisfies formula (13):

$$\frac{M}{s} = \frac{G}{s'} \tag{13}$$

G is the total gray scale. We suppose the $max(i, j)$ of the image grayscale in grid (i, j) fall in the k th box. At the same time, the $min(i, j)$ of the image grayscale in grid (i, j) fall in the l th box. Set (i, j) grid boxes box number is $n_r(i, j)$, the number of boxes require to cover the entire image is N_r , then:

$$n_r(i, j) = l - k + 1 \tag{14}$$

The fractal dimension of the image is:

$$D = \lim_{r \rightarrow 0} \frac{\log N_r}{\log(1/r)} \tag{15}$$

3.2 Simulation Experiment

In order to verify the feasibility of the algorithm, this paper simulates four kinds of common digital signals, in which simulation conditions are consistent with Sect. 2.2. Each signal is subjected to 100 Monte Carlo experiments at each SNR condition. The signal conducts low-pass filtering, SPWVD time-frequency conversion grayscale transformation and normalization, at last extracting the fractal dimension features of time-frequency images according to the algorithm in this paper. Table 2 is the mean and variance when the time-frequency image fractal features is under the SNR = 0 dB and SNR = 10 dB simulation conditions, and the table notes (mean, variance). Figure 3 is the curve in which signal time-frequency image fractal dimension feature changes with SNR.

Table 2. The fractal dimension mean and variance of signals

SNR	2FSK		BPSK		MSK		16QAM	
	μ	σ	μ	σ	μ	σ	μ	σ
0 dB	1.233	0.013	1.237	0.019	1.233	0.035	1.273	0.019
10 dB	1.230	0.009	1.279	0.012	1.293	0.009	1.300	0.015

It can be seen from Tables 1 and 2 that signal time-frequency image fractal box dimensionality variance is smaller than signal one dimensional fractal box dimension characteristic variance. So it proves that the intra-class polymerization degree of fractal box dimension feature of digital signal time-frequency image is relatively good. From Fig. 3, we can see that 16QAM signal and the other

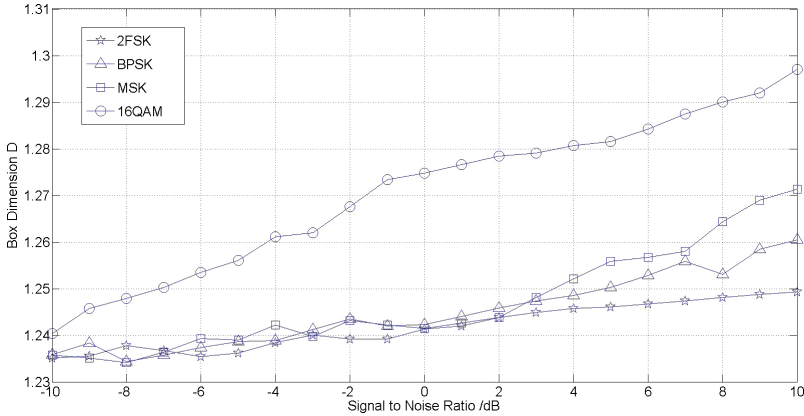


Fig. 3. Signal time-frequency image fractal dimension with SNR

three kinds of signal dimension feature has a clear interclass distance. Compared with the one-dimensional box dimension characteristic curve of digital signal in Fig. 1, the characteristic curve of time-frequency fractal dimension of digital signal is gentle with the change of SNR. For the signal characteristics, it is shown that the time-frequency fractal dimension of the signal is better than the one-dimensional box. Therefore it is more suitable for the signal classification and identification. Comprehensive comparison, the signal time-frequency image feature fractal dimension proposed in this paper is better than signal dimension box dimension features. And it is suitable for classification and recognition of multi-class signal.

4 Conclusion

In this paper, the feature extraction algorithm of digital signal is simulated and improved. For the digital signal fractal box dimension feature distance is small and the box dimension curve has a cross term, it is not suitable for a variety of digital signal classification and recognition. This paper presents a two-dimensional box dimension feature extraction algorithm based on time-frequency image. Simulation results show that the improved algorithm proposed in this paper has a good ability to distinguish 16QAM signals.

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References

1. Zhao, Y., Xing, W.: Building a local geomagnetic reference map by the multifractal Kriging based step-by-step interpolation correction method. *Appl. Sci. Technol.* **06**, 1–5 (2015)
2. Wang, G., Zhang, S., Yan, C., et al.: Probabilistic neural networks and fractal method applied to mineral potential mapping in Luanchuan. In: 2010 Sixth International Conference on Digital Object Identifier, pp. 1003–1007 (2010)
3. Ivanovici, M., Richard, N.: Fractal dimension of color fractal images. *IEEE Trans. Image Process.* **20**(20), 227–235 (2011)
4. Tang, Z., Yang, X., Li, J.: Study on fractal feature of modulated radio signal. *J. Phys.* **60**(5), 0564701 (2011)
5. Zhao, C., Ma, S., Yang, W.: Research on spectrum awareness based on fractal box dimension. *J. Electron. Inf. Technol.* **02**, 475–478 (2011)
6. Chi, F., Hu, Y., Wang, Z.: Feature extraction of turbine blades based on the fractal theory. *Appl. Sci. Technol.* **04**, 64–69 (2015)
7. Bai, H.: Research on radar emitter signal recognition technology based on time-frequency analysis. People's Liberation Army Information Engineering University (2012)
8. Mokni, R., Kherallah, M.: Novel palmprint biometric system combining several fractal methods for texture information extraction. In: 2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC), pp. 002267–002272 (2016)
9. Zehani, S., Mimi, M., Taleb-Ahmed, A., Toumi, A.: Anisotropy analysis of textures using wavelets transform and fractal dimension. In: 2016 2nd International Conference on Advanced Technologies for Signal and Image Processing (ATSIP), pp. 341–347 (2016)
10. Jarvenpaa, E., Jarvenpaa, M., Wu, M.: Random affine code tree fractals: hausdorff and affinity dimensions and pressure. In: *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 100, pp. 297–300 (2017)
11. Latorre, E., Layunta, E., Grasa, L.: Intestinal serotonin transporter inhibition by Toll-like receptor 2 activation. A feedback modulation. *PLOS One* **11**(12), e0169303 (2016)
12. Tam, S.K.E., Hasan, S., Hughes, S.: Modulation of recognition memory performance by light requires both melanopsin and classical photoreceptors. *Proc. Roy. Soc. B-Biol. Sci.* **283**(1845) (2016). <https://doi.org/10.6084/m9.figshare.4285250.v1>