Study on Rolling Bearing On-Line Health Status Estimation Approach Based on Vibration Signals

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Abstract. As the rolling bearing vibration signal is of nonlinear and nonstationary characteristics, the condition-indicating information distributed in the rolling bearing vibration signal is complicated, and using traditional time domain and frequency domain approaches cannot easily make an accurate estimation for the rolling bearing health status. In this paper, a simple and efficient fault diagnostic approach was proposed to accommodate to the requirements of both real-time monitoring and accurate estimation of fault type as well as severity. Firstly, a four-dimensional feature extraction algorithm using entropy and Holder coefficient theories was developed to extract the characteristic vector from the vibration signals, and secondly a gray relation algorithm was employed for achieving bearing fault pattern recognition intelligently. The experimental study have illustrated the proposed approach can efficiently and effectively improve the fault diagnostic performance compared with the existing artificial intelligent methods, and can be suitable for on-line health status estimation.

Keywords: Rotating machines · Rolling bearing · Vibration signals Health status

1 Introduction

Rolling bearing is one of main components in various types of rotational machinery (e.g., gas turbine engine, steam turbine engine and electrical machine), and its failure is usually the foremost cause to the failure and breakdown of rotating machines, which results in enormous property loss [1–3]. So as to maintain rotating machine operating reliably, it is important to propose an effective and efficient bearing health status estimation approach. Vibration-based bearing fault diagnosis approaches have attracted broad attention in the near past among all kinds of bearing fault diagnosis methods [4], due to that bearing vibration signals carry abundant machine health status information, which make it possible to extract health status feature vectors using signal processing technology based on vibration signals [5].

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Recently, great deal signal processing approaches have been applied for rolling element bearing off-line heath status estimation. As the result of the nonlinear factors, such as stiffness, friction and clearance, rolling bearing vibration signals always bear nonlinear and nonstationary performance at different operating conditions [6]. what's more, bearing vibration signals involve not only the working information related to the bearing itself, but also plentiful information related to other rotating parts of the machine, which in comparison with the former is usually taken as the background noise [7]. The slight bearing health status information may easily be submersed by the background noise due to that background noise is often relatively large. Therefore, the common time domain or frequency domain signal processing approaches may not easily obtain an accurate estimation result about the bearing health status, even the advanced signal processing approaches, e.g., Hilbert transform, fractional fourier transform and wavelet transform [8]. AS the nonlinear dynamics theory develops, many nonlinear analysis techniques have been used to recognize the complicated bearing nonlinear dynamic behavior [9]. The most common manner to extract and refine the health status information from the vibration signals is to combine a few of advanced signal processing approaches (e.g., Hilbert transform [10], wavelet package transform [11], higher order spectra [12] and empirical mode decomposition), to further extract the fault frequency with the aid of empirical judgement by the experts. Recently, the procedure of bearing fault diagnosis is gradually taken as a process of fault pattern recognition with the aid of artificial intelligence (AI) approaches, and its reliability and real-time performance is essentially determined by the effectiveness of the fault feature extraction algorithm and pattern recognition algorithm [13]. In recent years, some entropy based feature extraction algorithms (e.g., hierarchical entropy [14, 15], fuzzy entropy [16], sample entropy [17], approximate entropy [18] and hierarchical fuzzy entropy) were proposed used for extracting fault feature vectors based on the bearing vibration signals. We exploit a four-dimensional feature extraction algorithm using entropy and Holder coefficient theories, which are fit for processing complicated nonstationary and nonlinear problem, for extracting fault feature vectors based on the bearing vibration signals in the paper. When the fault feature extraction is ready, a fault pattern recognition method is required to implement the fault diagnosis automatically [13]. Nowadays, different fault pattern recognition approaches have been proposed for mechanical fault diagnosis, and one of the most common approaches is support vector machines [22] and artificial neural networks [19–21]. However, a large number of samples are needed for the training of ANNs, which may be hard to obtain in the practical applications. The support vector machines are based on statistical learning theory, and have better generalization than artificial neural networks under a smaller number of samples [23]. However, the selection of optimal parameters of SVMs has big effect on the performance of a SVM classifier [23, 24], and thus a multi-class concept [24] or an optimization algorithm has been used to improve the performance of support vector machines. In this paper, so as to balance the issue of accuracy versus real-time performance, a gray relation algorithm was employed to fulfill an intelligent fault pattern recognition based on the extracted four-dimensional feature vector.

The rest of the paper is organized as follows. Firstly, the methodology of the proposed approach is introduced in Sect. 2, and secondly the experimental validation of the proposed approach is illustrated in Sect. 3, and at last the conclusions are presented in Sect. 4.

2 Methodology

In this paper, a simple and efficient fault diagnostic approach was proposed to accommodate to the requirements of both real-time monitoring and accurate estimation of different fault types and in addition different severities for rolling bearing. Firstly, a four-dimensional feature extraction algorithm using entropy and Holder coefficient theories was developed to extract characteristic vectors from the vibration signals, and secondly a gray relation algorithm was used to achieve fault pattern recognition based on the extracted four-dimensional feature vectors.

2.1 Feature Extraction

Entropy is an important concept in the information theory, which is a measure for information uncertainty [25]. Suppose the event set is X, and the probability set for the event set is an n-dimensional probability vector $P = (p_1, p_2, \ldots, p_n)$, and satisfy:

$$0 \leq p_i \leq 1 \tag{1}$$

and

$$\sum_{i=1}^{n} p_i = 1 \tag{2}$$

Then the entropy E is defined as follows:

$$E(P) = E(p_1, p_2, \ldots, p_n) = -\sum_{i=1}^{n} p_i \log p_i \tag{3}$$

Therefore, entropy E can be taken as an entropy function for the n-dimensional probability vector $P = (p_1, p_2, \ldots, p_n)$.

Shannon entropy theory points out that, if there are many possible outcomes for an event and the probability for each possible outcome is $p_i (i = 1, 2, \ldots, n)$, whose sum is equal to 1, then the information obtained from a possible outcome can be expressed by $I_i = \log_a(1/p_i)$, and the information entropy defined for the time series is as follows:

$$S = -\mathrm{k} \sum_{i=1}^{N} p_i \log_e p_i \tag{4}$$

when $k = 1$, S stands for a Shannon entropy E_1 and can be used to depict the uncertainty degree of signals.

Based on the definition of Shannon entropy E_1, the definition of Exponential entropy E_2 is also introduced for the feature extraction purpose. Suppose the probability for each possible outcome is p_i, and its information content can be defined as:

$$\Delta I(p_i) = e^{1-p_i} \tag{5}$$

According to the basic entropy definition, the exponential entropy E_2 can be defined as:

$$E_2 = \sum_{i=1}^{n} p_i e^{1-p_i} \tag{6}$$

From Eqs. (5) and (6), it can be seen that compared with conventional information content $\Delta I(p_i) = \log(1/p_i)$, the definition of exponential entropy has the same meaning. The defining domain of $\Delta I(p_i)$ is $[0, 1]$, and $\Delta I(p_i)$ is a monotonic reduction function with the value domain of $[1, e]$. The exponential entropy E_2 is maximal only if the probability of all events is equal.

Holder coefficient can be used to measure the similar degree of two sequences, which may extract signals' features. It is evolved from Holder inequality and the definition of Holder inequality can be described as follows [26, 27]:

For any vector $X = [x_1, x_2, \ldots, x_n]^T$ and $Y = [y_1, y_2, \ldots, y_n]^T$, they satisfy:

$$\sum_{i=1}^{n} |x_i \cdot y_i| \leq \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p} \cdot \left(\sum_{i=1}^{n} |y_i|^q \right)^{1/q} \tag{7}$$

where $\frac{1}{p} + \frac{1}{q} = 1$ and $p, q > 1$.

Based on the Holder inequality, for two discrete signals $\{f_1(i) \geq 0, i = 1, 2, \ldots, n\}$ and $\{f_2(i) \geq 0, i = 1, 2, \ldots, n\}$, if $p, q > 1$, and $\frac{1}{p} + \frac{1}{q} = 1$, then Holder coefficient of these two discrete signals can be calculated as follows:

$$H_c = \frac{\sum f_1(i) f_2(i)}{\left(\sum f_1^p(i)\right)^{1/p} \cdot \left(\sum f_2^q(i)\right)^{1/q}} \tag{8}$$

where $0 \leq H_c \leq 1$.

Holder coefficient characterizes the similar degree of two discrete signals, if and only if $f_1^p(i) = k f_2^q(i)$, $i = 1, 2, \ldots, n$, in which n denotes the length of the discrete signal and k is a real number, H_c will be the biggest value. In this case, the similar degree of two signals is biggest, which indicates that these two signals belong to the same type of signals; if and only if $\sum_{i=1}^{n} f_1(i) f_2(i) = 0$, H_c get the minimum value, and in this case, the similarity of two signals is smallest, which indicates the signals are irrelevant, and belong to different types of signals.

Rectangular sequence $s_1(i)$ and triangular sequence $s_2(i)$ are selected as reference sequences, and then the Holder coefficient value of the vibration signal to be identified with the two reference signal sequences is obtained as follows:

$$H_1 = \frac{\sum f(i)s_1(i)}{\left(\sum f^p(i)\right)^{1/p} \cdot \left(\sum s_1^q(i)\right)^{1/q}} \tag{9}$$

where the rectangular sequence $s_1(i)$ is as follows:

$$s_1(i) = \begin{cases} s, & 1 \le i \le N \\ 0, & else \end{cases} \tag{10}$$

Similarly, H_2 is obtained as follows:

$$H_2 = \frac{\sum f(i)s_2(i)}{\left(\sum f^p(i)\right)^{1/p} \cdot \left(\sum s_2^q(i)\right)^{1/q}} \tag{11}$$

where the triangular sequence $s_2(i)$ is as follows:

$$s_2(i) = \begin{cases} 2i/n, & 1 \le i \le n/2 \\ 2 - 2i/n, & n/2 \le i \le n \end{cases} \tag{12}$$

2.2 Pattern Recognition

As the basis of gray system theory, the gray relation algorithm is to calculate the gray relation coefficient and relation degree between each comparative feature vector and reference feature vectors based on the basic theory of space mathematics [28].

Suppose the feature vectors (i.e., the four-dimensional feature vector extracted based on entropy and Holder coefficient theories) extracted based on vibration signals, to be identified are as follows:

$$B_1 = \begin{bmatrix} b_1(1) \\ b_1(2) \\ b_1(3) \\ b_1(4) \end{bmatrix}, B_2 = \begin{bmatrix} b_2(1) \\ b_2(2) \\ b_2(3) \\ b_2(4) \end{bmatrix}, \cdots, B_i = \begin{bmatrix} b_i(1) \\ b_i(2) \\ b_i(3) \\ b_i(4) \end{bmatrix}, \cdots \tag{13}$$

where B_i (i = 1, 2, …) is a certain fault pattern to be recognized (i.e., fault types and in addition severities).

Suppose the knowledge base between the health status patterns (i.e., fault types and in addition severities) and fault signatures (i.e., the feature vectors) from a part of samples is as follows:

$$C_1 = \begin{bmatrix} c_1(1) \\ c_1(2) \\ c_1(3) \\ c_1(4) \end{bmatrix}, C_2 = \begin{bmatrix} c_2(1) \\ c_2(2) \\ c_2(3) \\ c_2(4) \end{bmatrix} \cdots, C_j = \begin{bmatrix} c_j(1) \\ c_j(2) \\ c_j(3) \\ c_j(4) \end{bmatrix}, \cdots \tag{14}$$

where C_j (j = 1, 2, …) is a known health status pattern (i.e., fault types and in addition severities); c_j (j = 1, 2, …) is a certain feature parameter.

For $\rho \in (0,1)$:

$$\xi\big(b_i(k), c_j(k)\big) = \frac{\min\limits_j \min\limits_k \left|b_i(k) - c_j(k)\right| + \rho \cdot \max\limits_j \max\limits_k \left|b_i(k) - c_j(k)\right|}{\left|b_i(k) - c_j(k)\right| + \rho \cdot \max\limits_j \max\limits_k \left|b_i(k) - c_j(k)\right|} \tag{15}$$

$$\xi(B_i, C_j) = \frac{1}{4}\sum_{k=1}^{4} \xi\big(b_i(k), c_j(k)\big), j = 1, 2, \cdots \tag{16}$$

where ρ is a distinguishing coefficient; $\xi\big(b_i(k), c_j(k)\big)$ is the gray relation coefficient of k_{th} feature parameter for B_i and C_j; $\xi(B_i, C_j)$ is the gray relation degree for B_i and C_j. Thereafter B_i is categorized to a certain health status pattern where the maximal $\xi(B_i, C_j)$ (j = 1, 2, …,) is calculated.

2.3 Proposed Approach

Totally, the proposed approach for rolling bearing health status estimation is as follows:

(1) The vibration signals from the object rolling element bearing in the rotating machine are sampled under different working conditions, including normal operation condition and various fault types and severities, for the establishment of the sample knowledge base.
(2) Through a four-dimensional feature extraction algorithm using entropy and Holder coefficient theories, the health status feature vectors are extracted from the sample knowledge base.
(3) The sample knowledge base for GRA is established based on the fault symptom (i.e., the extracted feature vector) and the fault pattern (i.e., the known fault types and severity).
(4) The health status feature vectors extracted based on bearing vibration signals to be identified are input into GRA, and the diagnostic results (i.e., fault types and severity) are output.

3 Experimental Validation

All the rolling element bearing vibration signals for analysis are from Case Western Reserve University Bearing Data Center in the paper [29]. The test bearing is a deep groove rolling bearing of 6205-2RS JEM SKF. The related rolling element bearing experimental device consists of a torque meter, a power meter and a three-phase induction motor, and the load power and speed are measured over the sensor, shown in Fig. 1. Over controlling the power meter, the desired torque load can be obtained. The motor drive end rotor is supported over a test bearing, where a single point of failure is set through discharge machining. The fault diameter (i.e., fault severities) includes 28 mils, 21 mils, 14 mils and 7 mils, and the fault types includes outer race fault, the inner race fault and the ball fault. An accelerometer is installed on the motor drive end housing with a bandwidth of up to 5000 Hz, and the vibration data for the test bearing in different operating conditions is collected by a recorder, where the sampling frequency is 12 kHz.

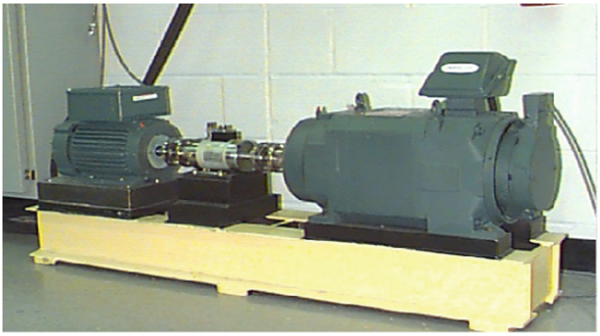

Fig. 1. Experimental setup

The bearing vibration data used for analysis was obtained under the load of 0 horsepower and the motor speed of 1797 r/min. Totally 11 types of vibration signals considering different fault categories and severities are analyzed, seen in Table 1. Each

Table 1. Description of experimental data set

Health status condition	Fault diameter (mils)	The number of base samples	The number of testing samples	Label of classification
Normal	0	10	40	1
Inner race fault	7	10	40	2
	14	10	40	3
	21	10	40	4
	28	10	40	5
Ball fault	7	10	40	6
	14	10	40	7
	28	10	40	8
Outer race fault	7	10	40	9
	14	10	40	10
	21	10	40	11

data sample from vibration signals is made up of 2048 time series points. For those 550 data samples, 110 data samples are chosen randomly for establishment of knowledge base, with the rest 440 data samples as testing data samples (Figs. 2 and 3).

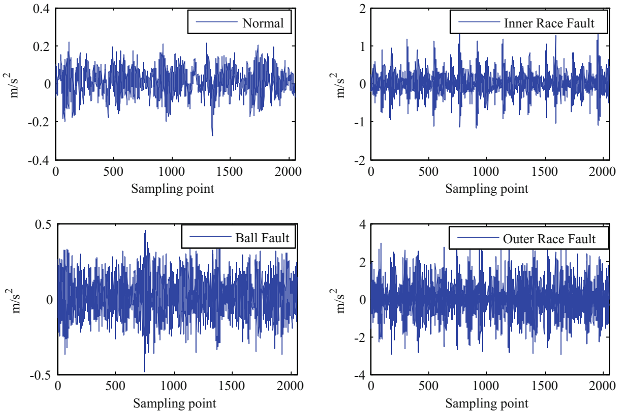

Fig. 2. Rolling bearing normal operating condition and fault conditions with fault diameter 7mils

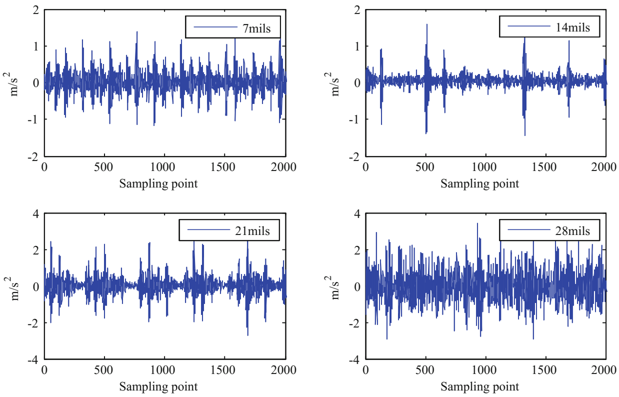

Fig. 3. Bearing inner race fault conditions with various severities

The health status feature vectors extracted from rolling bearing normal operating condition and different fault conditions with 7mils fault diameter over the four-dimensional feature extraction algorithm using entropy and Holder coefficient theories were shown in Figs. 4 and 5 respectively. And the health status feature vectors extracted from rolling bearing inner race fault condition with different severities over the four-dimensional feature extraction algorithm using entropy and Holder coefficient theory were shown in Figs. 6 and 7 respectively.

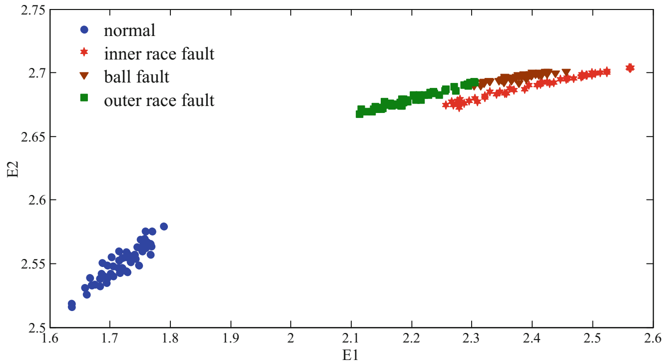

Fig. 4. Entropy features of a random chosen sample from bearing normal operating condition and different fault conditions with fault diameter 7mils

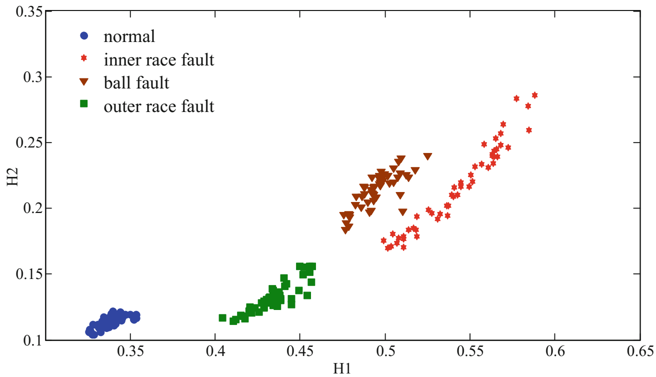

Fig. 5. Holder coefficient features of a random chosen sample from bearing normal operating condition and different fault conditions with fault diameter 7mils

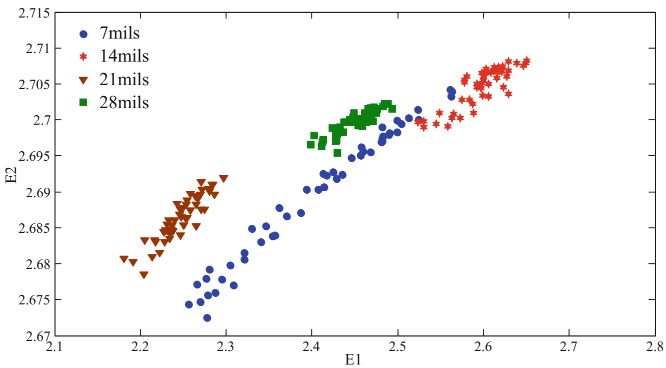

Fig. 6. Entropy features of a random chosen sample from rolling bearing inner race fault condition with various severities

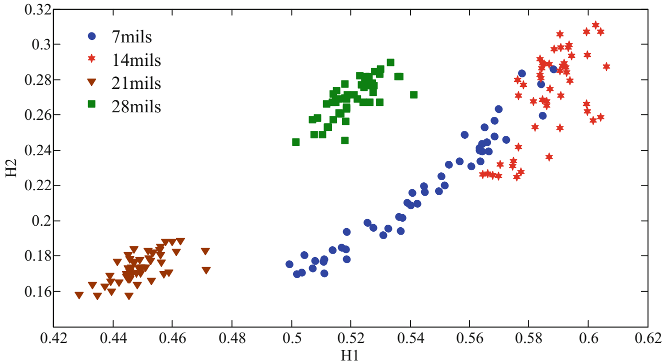

Fig. 7. Holder coefficient features of a random chosen sample from rolling bearing inner race fault condition with various severities

From Figs. 4, 5, 6 and 7, it can clearly be seen that the fault feature vectors extracted based on the rolling bearing vibration signals with different fault types and in addition different severities through the four-dimensional feature extraction algorithm based on entropy and Holder coefficient theory show apparent differences. The sample knowledge base for GRA is established based on the fault symptom (i.e., the extracted feature vector) and the fault pattern (i.e., the known fault types and severity). The fault feature vectors extracted based on the testing rolling bearing vibration signals to be identified are input into GRA, and the diagnostic results (i.e., fault types and severity) are output, shown in Table 2.

Table 2. The diagnostic results by GRA compared with results from references [30, 31]

Label of classification	The number of testing samples	The number of misclassified samples			Testing accuracy (%)		
		[30]	[31]	Proposed	[30]	[31]	Proposed
1	40	0	0	0	100	100	100
2	40	0	0	0	100	100	100
3	40	0	4	2	100	90	95
4	40	3	0	0	92.5	100	100
5	40	0	0	0	100	100	100
6	40	2	4	3	95	90	92.5
7	40	3	0	0	92.5	100	100
8	40	3	4	4	92.5	90	90
9	40	0	0	0	100	100	100
10	40	0	0	3	100	100	92.5
11	40	4	4	0	90	90	100
In total	440	15	16	12	96.59	96.3636	96.9697

The diagnostic results from Table 2, the fault pattern recognition success rate for detecting bearing faulty conditions can reach 100%, and the total fault pattern recognition success rate can reach almost 97%, which shows a certain improvement in diagnostic accuracy compared with the methods from references [30, 31]. The time cost by these methods for one Test Case is shown Table 3 by using a laptop computer with a 2.0 GHz dual processor.

Table 3. The time consumption comparison of these approaches

	[30]	[31]	Proposed
Time consumption/s	0.056695	0.011198	0.002160

From Table 3, the experimental results have demonstrated the proposed approach can be suitable for on-line health status estimation.

4 Conclusion

In this paper, a simple and efficient fault diagnostic approach was proposed to accommodate to the requirements of both real-time monitoring and accurate estimation of fault type as well as severity. The experimental results have many meaningful conclusions as follows:

(1) The proposed approach can accurately and effectively identify the different types of rolling bearing failure and the severity of the fault.
(2) The diagnostic results by the proposed approach show that the fault pattern recognition success rate for detecting bearing faulty conditions can reach 100%, and the total fault pattern recognition success rate can reach almost 97%.
(3) The proposed approach can improve the fault diagnostic performance compared with the existing artificial intelligent methods, and can be suitable for on-line health status estimation.

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