

# Wideband Spectrum Sensing by Multi-step Sample Autocorrelation Detection

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**Abstract.** Wideband spectrum sensing capability by using multi-step energy detection has been enabled in GNU Radio software radio platform. To improve the detection performance, we propose a novel approach of wideband spectrum sensing by multi-step sample autocorrelation detection. We first describe the principle of signal sample autocorrelation detection, then we present our proposed multi-step sample autocorrelation detection procedure for wideband spectrum sensing. The proposed procedure is simulated by using MATLAB, and the simulation results demonstrate that our proposal can achieve required detection performance by setting proper decision threshold.

**Keywords:** Spectrum sensing · Energy detection  
Sample autocorrelation detection · GNU Radio

## 1 Introduction

The scarcity of suitable radio spectrum resource becomes ever more pressing with increasing demand on wireless applications and services. The existing command-and-control spectrum management that relies on static allocation results in low spectrum utilization [1]. To address the spectrum scarcity problem, cognitive radio that can sense idle spectrum and perform dynamic spectrum access has been considered to be a promising technology. The existing spectrum sensing techniques include energy detection [2, 3]; matched-filtering detection [4]; cyclo-stationary detection [5] and eigenvalue detection [6]. Among them, energy detection is a signal source detection algorithm which has the advantage of simplicity. However, the performance of energy detection is vulnerable to noise uncertainty [7]. To overcome the shortcoming of energy detection, the authors in [8] proposed a covariance absolute value (CAV) algorithm, where the diagonal and off-diagonal elements in the statistical covariance matrix of the received signal are compared to determine the presence of primary user (PU). In [9], the authors discussed the CAV detection results in Rayleigh fading channel under low signal-noise-ratio (SNR). In practical GNU Radio software radio platform, the spectrum sensing by using multi-step energy detection in frequency domain can be supported [10–12].

To further improve the detection performance of spectrum sensing, we propose a wideband spectrum sensing by multi-step sample autocorrelation detection considering that the sample autocorrelation detection is robust to the noise uncertainty. We first

describe the principle of signal sample autocorrelation detection, then we present our proposed multi-step sample autocorrelation detection procedure. By simulating the procedure using MATLAB, we obtain the detection performance (e.g., detection probability and false alarm probability) in one-step narrowband spectrum sensing, and we discuss the factors influencing the detection performance in one-step spectrum sensing, i.e., the decision threshold, SNR, sensing time, number of available samples, and the smoothing factor. The simulation results demonstrate that our proposal can achieve required detection performance by setting proper detection threshold. Due to the signals can be detected without the knowledge of the noise power, the sample autocorrelation detection is robust to the noise uncertainty.

The remainder of the paper is organized as follows: The principle of signal sample autocorrelation detection is presented in Sect. 2. Section 3 describes our proposed multi-step frequency domain sample autocorrelation detection procedure. Section 4 presents the simulations and discussions. Section 5 draws the conclusions.

## 2 The Principle of Signal Sample Autocorrelation Detection

A signal detector is to find the test statistic of the received signal and compare it with a predefined threshold for deciding whether the signal exists or not. The signal detection can be formulated as a binary hypotheses testing problem as follows.

$$H_0 : y(n) = \eta(n), \quad n = 1, 2, 3, \dots, N \quad (1)$$

$$H_1 : y(n) = h(x(n)) + \eta(n), \quad n = 1, 2, 3, \dots, N \quad (2)$$

where,  $H_0$  and  $H_1$  represent the hypothesis “primary signal absent” and the hypothesis “primary signal present”, respectively;  $x(n)$  denotes  $n$ th sample of the primary signal to be detected;  $\eta(n)$  is  $n$ th sample of noise process;  $y(n)$  denotes  $n$ th sample of the received signal;  $N$  is the total number of samples collected during the sensing time  $T$ ;  $h(\cdot)$  denotes the channel fading process.

### 2.1 Theoretical Analysis for Sample Autocorrelation Detection

When sample autocorrelation detection is performed in time-domain, the signal  $x(t)$  is transmitted through a radio channel and passed through a bandpass filter (BPF) with bandwidth  $W$ ; then the filtered signals is sampled and the sample autocorrelation matrix is calculated. Next, the ratio of its diagonal and off-diagonal elements is treated as the test statistic and is compared to the predefined threshold.

From the discrete sampling sequences of  $y(t)$ , we choose a suitable smoothing factor  $L$  and define the following vectors,

$$\hat{y}(n) = [y(n) \quad [Y(n-1)] \quad \dots \quad y(n-L+1)]^T \quad (3)$$

Where  $n = 0, 1, \dots, N-1$ ,  $[\cdot]^T$  stands for matrix transpose.

The autocorrelation matrix of the received signal is defined as follows,

$$R_y = E[\hat{y}(n)\hat{y}^T(n)] \quad (4)$$

In this paper, we calculate and estimate the sample autocorrelation matrix by limited number of  $y(n)$ . In practice, we can only approximate such a matrix when using limited signal samples. Let us define the sample autocorrelation of the received signals as

$$\lambda(l) = \frac{1}{N_s} \sum_{m=0}^{N_s-1} y(m)y(m-l), \quad l = 0, 1, \dots, L-1 \quad (5)$$

Where  $N_s$  is the number of available samples. In fact, signal autocorrelation matrix  $R_y$  can be approximated by the sample autocorrelation matrix  $R_y(N_s)$  and we defined as

$$R_y \approx R_y(N_s) = \begin{pmatrix} \lambda(0) & \lambda(1) & \dots & \lambda(L-1) \\ \lambda(1) & \lambda(0) & \dots & \lambda(L-2) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda(L-1) & \lambda(L-2) & \dots & \lambda(0) \end{pmatrix} \quad (6)$$

Note that the sample covariance matrix is symmetric and Toeplitz. Obviously, if there is no signal and  $N_s$  is big enough, the off-diagonal elements of  $R_y(N_s)$  are all zeros. If there is signal and the signal samples are correlated,  $R_y(N_s)$  will be almost surely not a diagonal matrix. Let  $r_{ij}$  be the elements of the matrix  $R_y(N_s)$  and we define

$$T_{all} = \frac{1}{L} \sum_{i=1}^L \sum_{j=1}^L |r_{ij}| \quad (7)$$

$$T_{diag} = \frac{1}{L} \sum_{i=1}^L |r_{ii}| \quad (8)$$

Then, the test statistic of the detector is given by

$$Ratio = T_{all}/T_{diag} \quad (9)$$

and it is compared with a predefined threshold  $\gamma$  for hypothesis testing. Therefore, this method can be described as follows: if  $Ratio > \gamma$ , signal is decided to be present; otherwise, signal is decided to be absent. The signal sample autocorrelation detection does not need the information about PU signal, noise and channel, it is robust to noise uncertainty compared with the energy detection.

## 2.2 Threshold Determination

For signal detection, selecting an appropriate detection threshold is the key to improve the detection performance of the algorithm. The authors in [13] derived the threshold

based on the minimum error probability criterion and pointed out that reducing the total error probability can improve the detection performance. In this paper, we select an appropriate threshold based on the false alarm probability and the detection probability.

The detection threshold can generally be determined based on the Newman-Pearson criterion, to obtain the relationship between the false alarm probability and detection threshold.

In [8], the relation between the detection threshold and false alarm probability under the hypothesis  $H_0$  is given as follows,

$$\gamma = \frac{1 + (L - 1) \sqrt{\frac{2}{\pi \cdot N_s}}}{1 - Q^{-1}(P_{fa}) \cdot \sqrt{2/N_s}} \quad (10)$$

Where the function  $Q(\cdot)$  is as follows  $Q(a) = \int_a^\infty \sqrt{\frac{1}{2\pi}} \cdot e^{-\frac{x^2}{2}} dx$ , and  $Q(a) = 1 - Q(-a)$ ,  $Q^{-1}(\cdot)$  represents its inverse function.

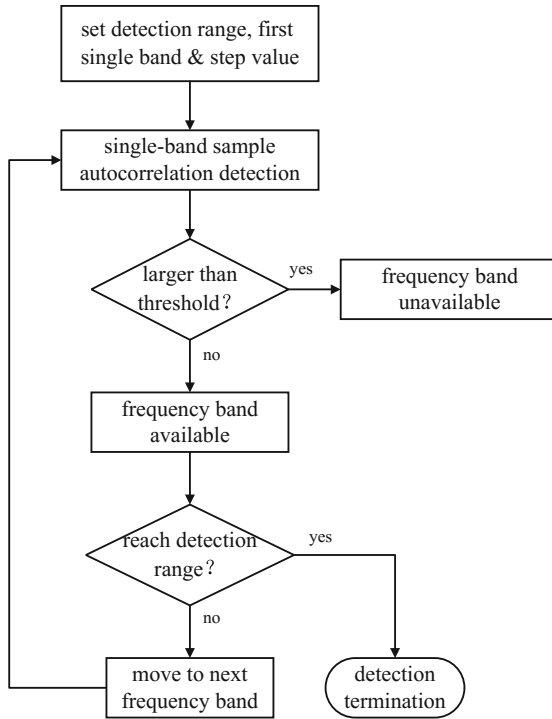
The threshold is set in order to achieve a specified probability of false alarm according to the Newman-Pearson criterion. However, the prerequisite for (10) is that the transformed signal is not filtered by a band-pass filter. In our simulation, the received signal is filtered by a narrow band-pass filter, (10) is no longer applicable to our problem. We proposed a novel approach to the determination of the threshold through the experimental and simulation method by considering both the requirements of detection probability and false alarm probability.

In the IEEE 802.22 draft standard [14], where  $P_{fa} = 0.01-0.1$ ,  $P_d = 0.9-1.0$ . We obtain an appropriate threshold in a certain detection performance ( $P_d > 0.95$  and  $P_{fa} < 0.05$ ). In Sect. 4, we will give a detailed description of how to obtain the appropriate threshold through the experimental and simulation approach.

### 3 Wideband Spectrum Sensing Procedure by Multi-step Sample Autocorrelation Detection

Wideband spectrum detection by using multi-step energy detection in frequency domain has been enable in the GNU Radio package. When the spectrum to be detected is larger than the maximum detection bandwidth in the default settings of USRP, it cannot sense the whole spectrum in one step. For instance, when the A/D sampling rate (`adc_rate`) is set to 64 Mbytes/s and the extraction rate (`decim`) is 16, the maximum detection bandwidth of USRP is `adc_rate/decim = 4 MHz`.

Since energy detection is affected by noise power, we present an approach of wideband spectrum sensing by multi-step sample autocorrelation detection as shown in Fig. 1.



**Fig. 1.** The procedure of wideband spectrum sensing by multi-step sample autocorrelation detection.

In the multi-step sample autocorrelation detection, the equipment first sets up the spectrum range to be sensed, first single band, and step value; then the RF board of USRP changes the central frequency step-by-step, and single-band sample autocorrelation detection is performed for a narrow range of frequency in one step. In each step, it compares the calculated test statistic based on sample autocorrelation with a predefined threshold and determines whether the primary users exists or not. Such a procedure is continued until the whole spectrum range is reached.

It is worth pointing out the frequency step value has influence on the detection bandwidth resolution and the detection time. The smaller the step value, the higher the frequency band resolution is, but the detection time is longer and the complexity of the detection is increased. On the contrary, when using a larger step value, the detection speed is faster, but the frequency band resolution is lower.

#### 4 Simulations and Discussions

We simulate the multi-step sample autocorrelation detection procedure for wideband spectrum sensing by using MATLAB.

Firstly, a wideband signal is generated by modulating a baseband signal. The baseband signal has sinc waveform with 2 MHz bandwidth and BPSK modulation. The symbol duration of the baseband signal is set to 0.5  $\mu$ s. Then, the baseband signal is modulated onto two carriers at 5 MHz and 15 MHz respectively. The signal's occupied spectrum is within 4–6 MHz and 14–16 MHz with such parameters. The whole spectrum range to be sensed is assumed to be 20 MHz. The sampling frequency is set to be 40 MHz. The one-step size for spectrum sensing is set to be 200 kHz. The received signal is obtained by adding white Gaussian noise to the modulated signal. At different SNRs, we change the signal power while keeping the noise power to be 1 at each sampling point. The one-step spectrum sensing at 200 kHz narrowband is implemented by a 200 kHz bandpass filter with 500-order Kaiser window. The multi-step wideband spectrum sensing is realized by adjusting the central frequency of the 200 kHz bandpass filter step by step until the whole 20 MHz spectrum is sensed. In each step, the calculated test statistic in (9) is compared with a threshold for signal detection. At each 200 kHz narrowband, the number of sensed BPSK symbols is set to 2000 (i.e.,  $N = 2000$ ); the number of available samples is set to be 20000 (i.e.,  $N_s = 20000$ ).

Figure 2 shows the detection probability  $P_d$  versus the test statistic when the smoothing factor  $L$  is 15 to 20. At  $L = 15$ , the required decision threshold  $\gamma$  is 9.0 and 9.5 to satisfy  $P_d \geq 0.95$  at SNR = -5 dB and 5 dB respectively; At  $L = 20$ , the required decision threshold  $\gamma$  is 10.5 and 11.5 to satisfy  $P_d \geq 0.95$  at SNR = -5 dB and 5 dB respectively.

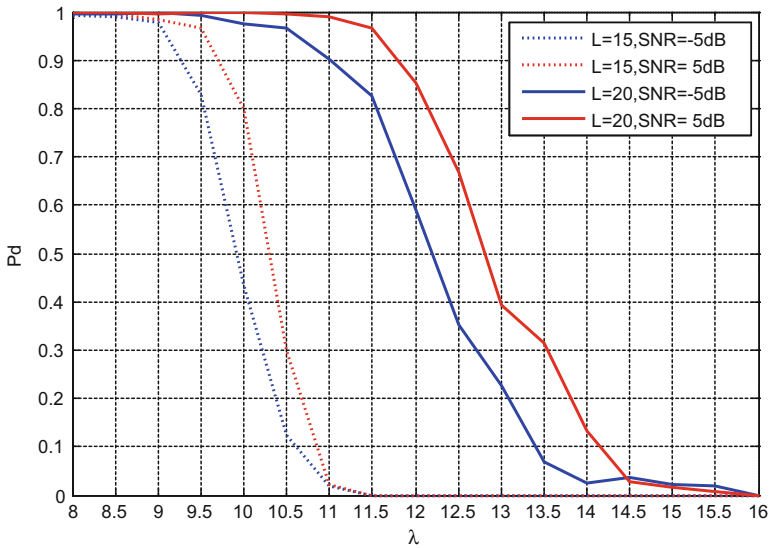
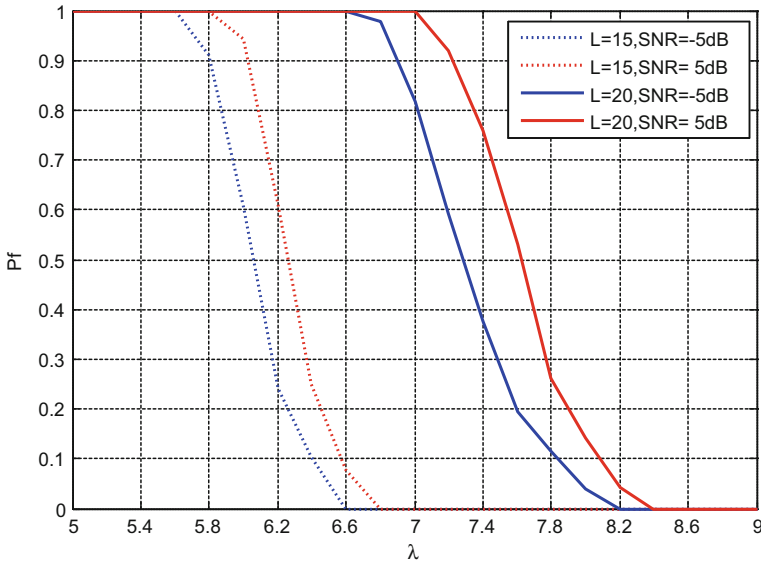


Fig. 2. Detection probability versus test statistic with different  $L$

Figure 3 shows the false-alarm probability  $P_f$  versus the test statistic when  $L$  is 15 and 20. It can be seen that, at  $L = 15$ ,  $\gamma$  increases from 6.4 to 6.7 when the SNR = -5 dB

and 5 dB to satisfy  $P_f \leq 0.05$ ; At  $L = 20$ ,  $\gamma$  increases from 8.0 to 8.6 when the SNR = -5 dB and 5 dB.



**Fig. 3.** False-alarm probability versus test statistic with different  $L$

From Figs. 2 and 3, it also can be seen that the decision threshold  $\gamma$  influences the performance of false-alarm probability and detection probability. In addition, at the same smoothing factor, the decision threshold to satisfy a certain false-alarm probability and detection probability is loosely affected by the SNR. Hence the sample autocorrelation detection is robust to the noise uncertainty. Meanwhile, the decision threshold increases with the increases of the smoothing factor  $L$  to achieve the same detection probability.

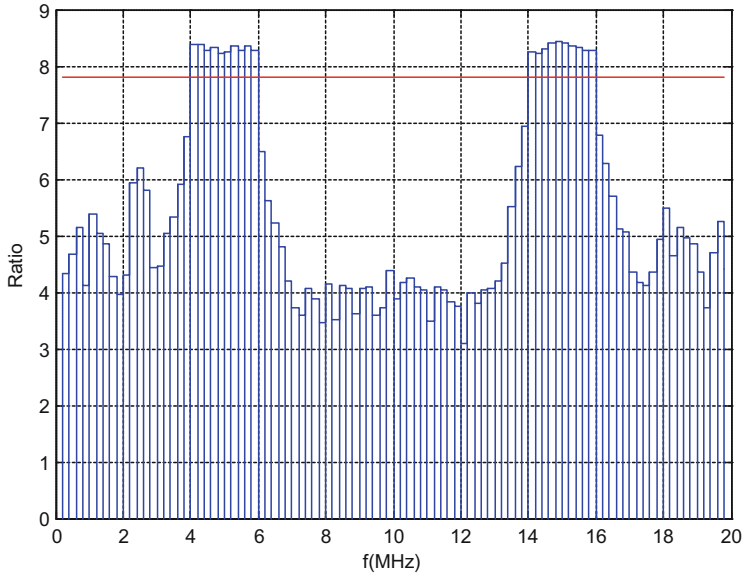
The decision threshold for the sample autocorrelation detection in one-step 200 kHz band can be determined from the simulation results of Figs. 2 and 3.

When  $L$  is 15 and SNR is -5 dB, the decision threshold  $\gamma$  should be less than 8.5 to satisfy  $P_d \geq 0.95$ . Furthermore,  $\gamma$  should be greater than 6.6 to satisfy  $P_f \leq 0.05$ . Therefore, a value between 6.6 and 8.5 can be chosen as the decision threshold in one-step spectrum sensing for the case that  $L = 15$  and SNR = -5 dB.

When  $L$  is 20 and SNR is -5 dB,  $\gamma$  should be less than 10.5 to satisfy  $P_d \geq 0.95$ . Furthermore,  $\gamma$  should be greater than 8.2 to satisfy  $P_f \leq 0.05$ . Therefore, a value between 8.2 and 10.5 can be chosen as the decision threshold in one-step spectrum sensing for the case that  $L = 20$  and SNR = -5 dB.

When  $L$  is 20 and SNR is 5 dB,  $\gamma$  should be less than 10.6 to satisfy  $P_d \geq 0.95$ . Furthermore,  $\gamma$  should be greater than 8.4 to satisfy  $P_f \leq 0.05$ . Therefore, a value between 8.4 and 10.6 can be chosen as the decision threshold in one-step spectrum sensing for the case that  $L = 20$  and SNR = 5 dB.

Figure 4 shows the test statistic of the detector in 100 steps for 20 MHz spectrum sensing by multi-step sample autocorrelation detection for the case that  $L = 15$  and  $\text{SNR} = -5$  dB.  $\gamma$  is chosen to be 7.8 in one-step spectrum sensing for such a case. It can be seen from Fig. 4 that the test statistic of the detector is continuously larger than 7.8 from the 21th to the 30th 200 kHz steps, and from the 71th to the 80th 200 kHz steps. Therefore, the occupied spectrum can be decided to be from 4.0 MHz to 6.0 MHz and from 14.0 MHz to 16.0 MHz, and the idle spectrum is the spectrum range except 4.0 MHz to 6.0 MHz and 14.0 MHz to 16.0 MHz in the 20 MHz.



**Fig. 4.** Detected test statistic in 100 steps for 20 MHz spectrum sensing ( $\text{SNR} = -5$  dB,  $L = 15$ )

Figure 5 shows the test statistic of the detector in 100 steps for 20 MHz spectrum sensing by multi-step sample autocorrelation detection for the case that  $\text{SNR} = -5$  dB and  $L = 20$ .  $\gamma$  is chosen to be 9.0 in one-step spectrum sensing for such a case. It can be seen that from Fig. 5 the test statistic is continuously larger than 9.0 from the 21th to the 30th 200 kHz steps, and from the 71th to the 80th 200 kHz steps. Therefore, the idle spectrum is the spectrum range except 4.0 MHz to 6.0 MHz and 14.0 MHz to 6.0 MHz in the 20 MHz.

Figure 6 shows the test statistic of the detector in 100 steps for 20 MHz spectrum sensing by multi-step sample autocorrelation detection for the case that  $L = 20$  and  $\text{SNR} = 5$  dB.  $\gamma$  is chosen to be 9.0 in one-step spectrum sensing for such a case. It can be seen that from Fig. 5 the test statistic is continuously larger than 9.0 from the 21th to the 30th 200 kHz steps, and from the 71th to the 80th 200 kHz steps. Therefore, the idle spectrum is the spectrum range except 4.0 MHz to 6.0 MHz and 14.0 MHz to 6.0 MHz in the 20 MHz.



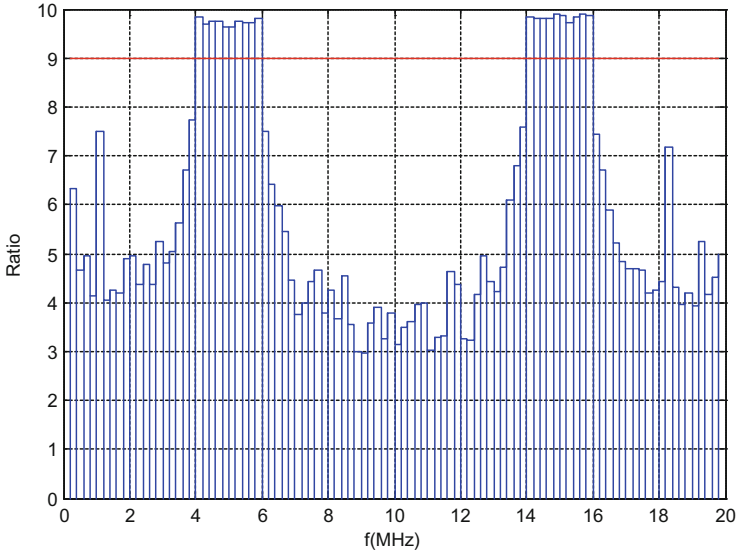


Fig. 5. Detected test statistic in 100 steps for 20 MHz spectrum sensing ( $SNR = -5$  dB,  $L = 20$ )

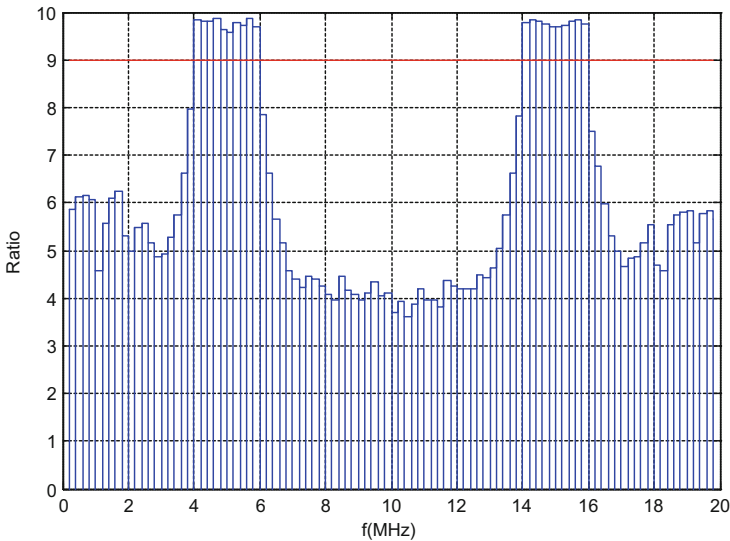


Fig. 6. Detected test statistic in 100 steps for 20 MHz spectrum sensing ( $SNR = 5$  dB,  $L = 20$ )

From (5) – the equation of the sample autocorrelation, it can be seen that the sample autocorrelation matrix is mainly determined by two factors,  $N_s$  and  $L$ . At the same  $N_s$ , the detection probability is increasing. Obviously with the increase of the smoothing factor, the complexity of the algorithm increases in a certain extent. In practice, we can choose a relatively small smoothing factor if it can meet the detection performance.

To investigate the impact of the number of available samples  $N_s$  on the detection probability, we simulate  $P_d$  versus the test statistic when  $N_s$  is 20000 and 40000, respectively. Figure 7 shows that the  $\gamma$  increases from 10.6 to 11.5 with  $N_s$  increases from 20000 to 40000 to satisfy  $P_d \geq 0.95$ .

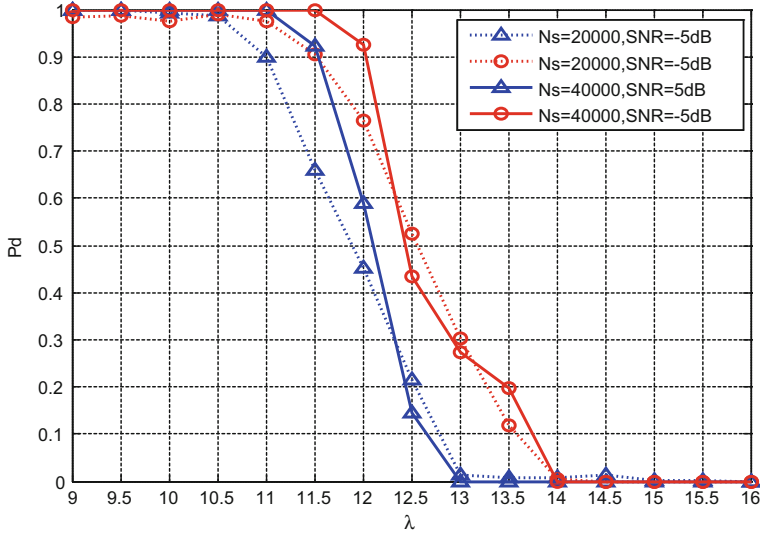


Fig. 7. Detection probability versus  $t$  with different  $N_s$  ( $L = 15$ )

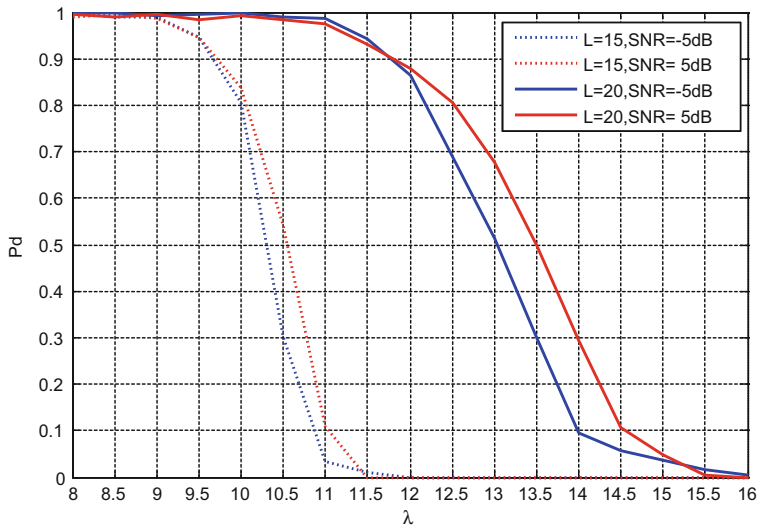


Fig. 8. Detection probability versus test statistic with different  $L$

Next, we evaluate the sensing performance with different sensing time. The number samples at each 200 kHz narrowband is set to 3000 (i.e.,  $N = 3000$ ) instead of 2000. Figure 8 shows  $P_d$  versus  $\gamma$  for  $N = 3000$  when  $L$  is 15 and 20, respectively.  $\gamma$  increases from 9.5 to 11.5 when  $L$  increases from 15 to 20 to satisfy  $P_d \geq 0.95$ ; Comparing Fig. 2 for the case  $N = 2000$ ,  $\gamma$  needs to be increased to satisfy  $P_d \geq 0.95$ . Figure 9 shows  $P_f$  versus  $\gamma$  for  $N = 3000$  when  $L$  is 15 and 20, respectively. It can be seen that, to satisfy  $p_f \leq 0.05$  at the same SNR,  $\gamma$  increases from 7.4 to 9.3. Comparing Fig. 3 for the case  $N = 2000$ ,  $\gamma$  needs to be increased to satisfy  $p_f \leq 0.05$ .

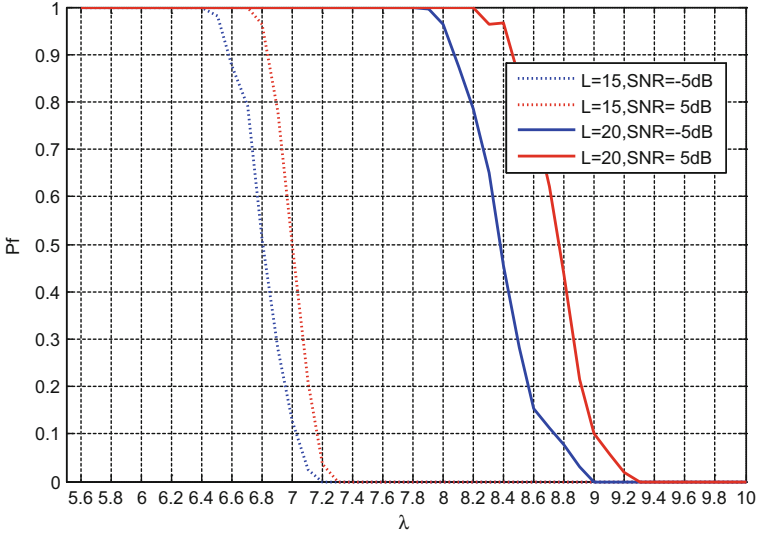


Fig. 9. False-alarm probability versus test statistic with different  $L$

## 5 Conclusions

When a sensing device cannot sense the spectrum range in one step because the whole spectrum to be sensed is wider than the one-step detection bandwidth, the software radio platform (like GNU Radio) employs multi-step energy detection in frequency domain to fulfill the sensing task. Considering that the sample autocorrelation detection is robust to the noise uncertainty, we proposed a novel approach of wideband spectrum sensing by employing the sample autocorrelation detection instead of energy detection in the multiple sensing steps. In the one-step narrowband spectrum sensing, the test statistic of the received signal is calculated based on the sample autocorrelation and it is compared with a predefined threshold to decide whether the signal exists or not. By simulating our proposed procedure, we obtained the achieved detection probability and false alarm probability of spectrum sensing in one-step spectrum sensing. From the simulation results, we examined the factors (SNR, sensing time, the number of available samples and the smoothing factor) that influence the decision threshold. In the performance evaluation, the decision threshold is determined considering both the requirements

of detection probability and false alarm probability. Due to the fact that the sample autocorrelation detection can work without the knowledge of noise power, our proposal provides a more robust approach for wideband spectrum sensing than the multi-step energy detection.

**Acknowledgments.** This paper was supported by the National Natural Science Foundation of China (Grant No. 61561017 and Grant No. 61261024), National Science & Technology Pillar Program (Grant No. 2014BAD10B04), and Hainan Province Major Science & Technology Project (Grant No. ZDKJ2016015).

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