A Low-Complexity Power Allocation Method in Ultra-dense Network

Xin Su[1], Bei Liu[2], Jie Zeng[1(✉)], Jing Wang[1], and Xibin Xu[1]

[1] Tsinghua National Laboratory for Information Science and Technology, Research Institute of Information Technology, Tsinghua University, Beijing, China
zengjie@tsinghua.edu.cn

[2] Broadband Wireless Access Laboratory, Chongqing University of Posts and Telecommunications, Chongqing, China

Abstract. This paper considers the downlink power allocation in the ultra-dense network. To acquire the maximum sum rate of all the users, we first make the appropriate approximate hypothesis on the interference, and then adopt the Lagrangian Multiplier method and Karush-Kuhn-Tucker condition to obtain the expression of the optimum power allocation. Finally, the iteratively searching water filling algorithm is used to allocate power for each access node, when the total power is limited. Due to the consideration of the computation complexity of the iteratively searching algorithm, we applied the low-complexity water filling algorithm into the power allocation to reduce the iteration times. The simulation results have shown that the performance of the both two water filling algorithms are close, and can improve the sum rate of the users in the ultra-dense network, and the low-complexity water filling algorithm can converge to the optimum solution more quickly.

Keywords: Ultra-dense network · Water filling algorithm · Power allocation

1 Introduction

The fifth generation (5G) have been expected to provide the larger capacity to meet the rapidly increasing data traffic demands [1]. Ultra-dense network (UDN) shortens the distance of the user and the access node, and can substantially increase the area capacity, by densely deploying the access nodes [1]. However, because of the huge density of the access nodes, the interference in UDN is more complicated than traditional networks [2]. Thus, some better resource allocation methods and interference management algorithms should be studied.

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Many academics have been committed to the studies of UDN. The authors of [3] analyses the relationship of the access node density and the network spectrum efficiency (SE), and concluded that SE can't always increase with the increase of the access node density. *Zhou et al.* [4] introduced the interference management methods from 1G to 5G, and proposed three potential technologies, which are the Coordinated Multi-Point transmission (CoMP) technique, the more advanced interface technique, and the interference alignment technique, to mitigate the more serious inter-cell interference in UDN. *Soret et al.* [5] considers the dominant interference and gives the time-domain and frequency-domain interference coordination to resist the interference in dense networks. *Wang et al.* [6] proposes a dynamical cell muting scheme considering the proportional fair (PF) scheduling of all the users. What's more, the resource allocation based on game theory is expected to obtain the performance improvement in UDN. [7] proposes a power allocation method based on non-cooperative game theory to suppress interference in the downlink dense network. [8] proposes a spectrum allocation combined with the CoMP to mitigate the interference in UDN. What's more, [9] proposes a multi-dimensional bisection search method which is based on the water filling algorithm to maximize the system capacity, but it is too high-complexity to be applied in UDN. In a word, some new interference management methods should be studied.

This paper considers the downlink transmission in UDN, and aims to get the optimal power allocation to maximize the sum rate of all the users in the area. Water filling is a classical algorithm which is always used in Orthogonal Frequency Division Multiplexing (OFDM) systems to allocate the power among the subcarriers [10]. However, the number of the access nodes and the number of the users are very large in UDN, and the inter-cell interference is more complicated. Luckily, we can make a hypothesis on the interference in UDN [9, 11]. Having this hypothesis in mind, we can transform the objective problem into the convex one. Then the water filling algorithm can be applied to search the optimal solution. In addition, considering the high computation complexity of the iteratively searching water filling (ISWF) algorithm, we further proposed the low-complexity water filling (LWF) algorithm, it can reach the nearly sum rate as ISWF with fewer iterations.

The remainder of the paper is organized as follows. Section 2 shows the system model and formulates the problem. Section 3 introduces two power allocation methods, which are the ISWF algorithm and the proposed LWF algorithm. Then the simulation is shown and the complexity is analyzed in Sect. 4. Finally, Sect. 5 concludes the paper.

2 System Model

This part shows the system model, as shown in Fig. 1. Assume the number of the users and the number of the access nodes are K and M, $M \gg K$. And the access nodes and the users are equipped with the single antenna. Assume that each users choose the serving access nodes according to the reference signal receiving power (RSRP); and each access node serves only one user and each user is served by only one access node. Then in order to save energy, turn off the access nodes those serve no users, forming K transmitter-receiver pairs. Simply, define g_{kk} as the channel gain between the user k and its serving access node. d_{mk} and h_{mk} denote the distance and the channel coefficient between the access node m and the user k, then the corresponding channel gain can be dented as $g_{mk} = \frac{h_{mk}}{d_{mk}^{\alpha}}$, where α denotes the pass loss exponent. The receiving signal of user k can be denoted as

$$y_k = \sqrt{p_k g_{kk}} s_k + \sum_{i=1, i \neq k}^{K} \sqrt{p_i g_{ik}} s_i + n_k \tag{1}$$

where $s_i (i = 1, 2, \cdots, K)$ denotes the signal of the serving access node transmitting to user i. $p_i (i = 1, 2, \cdots K)$ denotes the allocated power of the serving access node, and $n_k \sim \mathcal{CN}\left(0, \sigma^2\right)$, which denotes the additional white Gaussian noise.

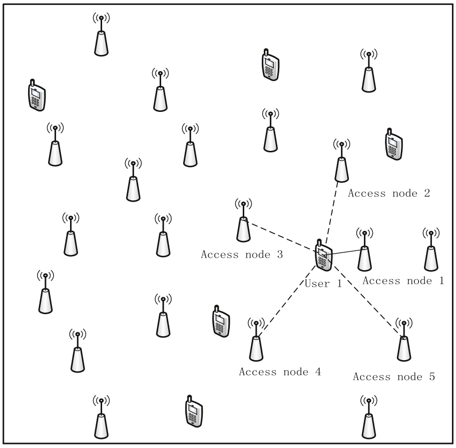

Fig. 1. UDN scenario: There are M access nodes and K users ($M \gg K$) randomly distributed in the specific area. The solid line in the figure represents the desired signal transmission, and the dotted lines represent the interference signal transmission.

We can easily find that the three parts on the right side of the Eq. (1) respectively denote the wanted signal, the interference and the noise. So the receiving signal to interference plus noise ratio (SINR) can be expressed as follows:

$$\text{SINR} = \frac{g_{kk} p_k}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2} \tag{2}$$

The rate of user k could be written as follows [12]:

$$R_k = B_k \log_2\left(1 + \mathrm{SINR}_k\right) = B_k \log_2 \frac{\sum_{i=1}^{K} g_{ik} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2} \tag{3}$$

where B_k denotes the bandwidth allocated to the serving access node of user k. Assume $B_k = B\,(k = 1, 2, \cdots K)$. Thus we can denote the sum rate of all the users as follows:

$$R = B \sum_{k=1}^{K} \log_2 \frac{\sum_{i=1}^{K} g_{ik} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2} \tag{4}$$

2.1 Problem Formulation

With the aim of the maximization of the sum rate of all the users with the restricted power, the following objective function is formed. Since the bandwidth of every access node is identical, we can express the optimization function for simplicity as:

$$\begin{aligned} &\max \sum_{k=1}^{K} \log_2 \frac{\sum_{i=1}^{K} g_{ik} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2} \\ &st. \quad \sum_{k=1}^{K} p_k \leq P_{\max} \end{aligned} \tag{5}$$

Where the total power can be denoted as $P_{\max}$. Visibly, the interference make the optimization function non-convex. Fortunately, we can assume the interferences as constants to transform the function to a convex one [10, 11]. With this in mind, we adopt the Lagrangian Multiplier scheme to search the optimum solution [11]. First, the Lagrangian function is expressed as:

$$L(p_k, \lambda) = \sum_{k=1}^{K} \log_2 \frac{\sum_{i=1}^{K} g_{ik} p_i + \sigma_k^2}{\sum_{i \neq k}^{K} g_{ik} p_i + \sigma_k^2} + \lambda\left(P_{\max} - \sum_{k=1}^{K} p_k\right) \tag{6}$$

where λ is the Lagrangian multiplier, which is for the power constraint. Then take the partial derivative with respect to p_k:

$$\frac{\partial L}{\partial p_k} = \frac{1}{\ln 2}\frac{g_{kk}}{g_{kk}p_k + \sum_{i=1,i\neq k}^{K} g_{ik}p_i + \sigma_k^2} - \sum_{j=1,j\neq k}^{K} \frac{\gamma_j p_j g_{ik}}{\sum_{i=1,i\neq j}^{K} g_{ij}p_i + \sigma_k^2} - \lambda \tag{7}$$

where $\gamma_k = \frac{g_{kk}}{\sum_{i=1,i\neq k}^{K} g_{ik}p_i + \sigma_k^2}$.

Finally, we can easily acquire the illustration of p_k with the Karush–Kuhn–Tucker (KKT) condition, as follows:

$$p_k = \left[\frac{1}{\lambda \ln 2 + \sum_{j=1,j\neq k}^{K} \frac{\gamma_j p_j g_{jk}}{\sum_{j=1,j\neq k}^{K} g_{ij}p_i + \sigma_j^2}} - \frac{1}{\gamma_k}\right]^+ \tag{8}$$

3 Water Filling Based Power Allocation

3.1 ISWF Algorithm

Equation (8) indicates that p_k is relative to γ_k. Having this in mind, the water filling algorithm can be applied. First, define the water-filling level as follows:

$$\beta = \frac{1}{\lambda \ln 2 + \sum_{j=1,j\neq k}^{K} \frac{\gamma_j p_j g_{jk}}{\sum_{i=1,i\neq j}^{K} g_{ij}p_i + \sigma_j^2}} \tag{9}$$

Then rewrite Eq. (8) as:

$$p_k = \left[\beta - \frac{1}{\gamma_k}\right]^+ \tag{10}$$

First, we use the ISWF algorithm to solve the problem. Its main idea is to find the optimal water-filling level by updating the value of β iteratively. In detail, the initial value of β is denoted as follows:

$$\beta = \frac{1}{K}\left[P_{\max} + \sum_{k=1}^{K}\frac{1}{\gamma_k}\right] \tag{11}$$

Then, iteratively update the value of β according to the expression below:

$$\beta \leftarrow \beta + \mu\frac{1}{N_{on}}\left(P_{\max} - \sum_{k=1}^{K}p_k\right) \tag{12}$$

where $0 < \mu < 1$ denotes the water-filling level modification step size. N_{on} denotes the number of the access nodes whose allocated power is not zero. Update β until its value converges to the optimum water filling level. Finally, we can get the optimal power allocation according to Eq. (10).

3.2 Proposed LWF Algorithm

Since the ISWF algorithm is very high-complexity, we propose the LWF algorithm and illustrate it in this part. In the first step, give the power of each access node as the following equation:

$$p_k = \frac{1}{K}\left(P_{\max} + \sum_{i=1}^{K}\frac{1}{\gamma_i}\right) - \frac{1}{\gamma_k} \tag{13}$$

Because different users have different channel gains, the access nodes whose have channel quality are bad may be allocated with negative powers. Then they are divided into the positive set and the negative set: $A = \{p_k | p_k > 0\}$ and $B = \{p_k | p_k < 0\}$. Then calculate the average value,

$$\Delta = \frac{\sum_{p_i \in B} p_i}{|A|} \tag{14}$$

Set the negative power in B to zero, adjust the positive power in A as follows

$$p'_k = p_k(p_k \in A) + \Delta \tag{15}$$

Repeat (14) and (15) until no negative powers exist and we can get the optimum power of each access node.

4 Simulation Results

4.1 Performance Comparison

This section compares the sum rates of above-mentioned methods by Monte Carlo simulations. In the simulation, the number of access nodes and the number of the users are respectively set to 100 and 10. They are randomly positioned in the specific area. And the channels are the randomly generated unit variance Rayleigh fading channels in the simulations. In addition, the pass loss exponent is assumed as 3.75.

As we all know, the average power allocation is the fairest and the simplest method, and is presented as a contrast. Figure 2 shows the sum rate performance of the ISWF algorithm, the proposed LWF algorithm and the average power allocation. From the figure, both the ISWF based power allocation and the proposed LWF based power allocation can achieve the greatly improve the sum rate of the users. Furthermore, sum rates of the two water filling based power allocation methods are very close.

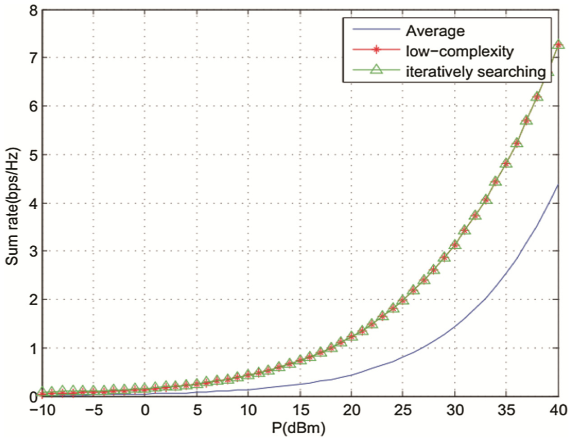

Fig. 2. The sum rate performance of the average power allocation algorithm, the ISWF algorithm, LWF algorithm.

4.2 Complexity Analysis

This part gives the complexity comparison of the above-mentioned algorithms. In one iteration, the ISWF algorithm needs $2K$ times of add operations and $K+2$ times of multiply operations. While the proposed LWF algorithm only needs $2K$ times of add operations and one multiply operation. What's more, the proposed LWF algorithm needs much less iterations to converge than the ISWF algorithm.

Table 1 shows the average iteration times of the two algorithms when the Monte Carlo simulation time is 10000. From the table, the ISWF algorithm needs around 24 iterations to converge, but the LWF algorithm only needs around 4 iteration to converge to the optimal power allocation. In summary, the complexity of the LWF algorithm is much lower.

Table 1. The average iteration times of the two algorithms.

Pmax (dBm)	−10	0	10	20	30	40
ISWF	24.5626	24.5587	24.5577	24.4310	23.9252	22.7437
LWF	4.2231	4.2110	4.1837	4.1347	3.8550	3.1835

In a summary, compared with the ISWF algorithm, the proposed LWF algorithm can reach nearly the same sum rate performance as the ISWF algorithms with much lower complexity.

5 Conclusions and Discussion

This paper creatively proposed an algorithm to obtain the optimal power allocation method, which aimed to maximize the sum rate in UDN. We first simply regarded the interference as content to make the objective problem convex. Then, the ISWF algorithm was applied, and it can significantly improve the sum rate. However, it needs many iterations, which makes it impractical to be applied in UDN. So we further proposed the LWF algorithm, which needs fewer iterations. It is proved that the proposed LWF algorithm reached nearly the same performance in terms of the sum rate to the ISWF algorithm. Both significantly improved the sum rate a lot, compared with the average power allocation. Furthermore, the proposed LWF algorithm needs less iterations to converge, which leads to a substantial complexity decrease. In a word, the proposed LWF based power allocation can be better applied to UDN.

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