Spectrum Sensing Based on Modulated Wideband Converter with CoSaMP Reconstruction Algorithm

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Abstract. Modulated Wideband Converter (MWC) provides a sub-Nyquist sampling approach for wideband spectrum sensing. In previous studies, reconstruction algorithm, OMP (Orthogonal Matching Pursuit) is used in the CTF (continuous-to-finite) block of MWC for frequency support recovery. However, the percentage of correct support recovery is low at low SNR (Signal Noise Ratio) using OMP algorithm. In this paper, the reconstruction algorithm of CoSaMP (Compressive Sampling Matching Pursuit) is investigated to be used in the CTF block instead of OMP. Simulation results demonstrate that such a proposal can achieve higher percentage of correct support recovery than with OMP algorithm.

Keywords: Spectrum sensing · Modulated Wideband Converter (MWC)
Orthogonal Matching Pursuit (OMP)
Compressive Sampling Matching Pursuit (CoSaMP)

1 Introduction

In the field of wireless communications, spectrum resources become increasingly scarce with the emerged new services and growing number of users. Nevertheless, regulatory bodies found that allotted radio frequency spectrum can be inefficiently utilized. As a solution, cognitive radio (CR) technique can exploit the unused frequency regions on an opportunistic basis by using spectrum sensing to identify available spectrum holes [1, 2]. Hence, quick and efficient spectrum sensing is an essential component of CR functionality.

CR typically operates in a wideband environment, and the Nyquist sampling rate can be prohibitively large. When the band positions are unknown, it is a more challenging problem because standard demodulation cannot be used. The modulated wideband converter (MWC) provides a sub-Nyquist sampling approach for wideband spectrum sensing [3, 4]. It can blindly sample multiband analog signals at a low sub-Nyquist rate. The MWC first multiplies the analog signal by a bank of periodic waveforms. Then the product is lowpass filtered and sampled uniformly at a low rate, which is orders of magnitude smaller than the Nyquist rate. The spectral support can be

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recovered by using sparse recovery algorithm of compressed sensing (CS) [5], which is performed in the CTF (continuous-to-finite) block in the MWC architecture [6]. The CTF constructs a frame from the input samples, then it solves a finite dimensional sparse representation problem, from which it identifies the indices of the active spectrum slices, namely those containing signal energy.

In the previous study of MWC, Orthogonal Matching Pursuit (OMP) algorithm is used in the CTF block [7–9]. However, the percentage of correct support recovery is low at low SNR (Signal Noise Ratio). In this paper, CoSaMP (Compressive Sampling Matching Pursuit) algorithm [10] is investigated to be used in the CTF block instead of OMP algorithm. Simulation results demonstrate that such a proposal can achieve higher percentage of correct support recovery than with OMP algorithm.

This paper is organized as follows. In Sect. 2, we describe the MWC architecture for wideband spectrum sensing. Section 3 describes sparse signal recovery algorithms, namely OMP algorithm and CoSaMP algorithm, which are used in the CTF block. Section 4 presents and discusses the simulation results of MWC with OMP algorithm and CoSaMP algorithm. Section 5 concludes this paper.

2 MWC Architecture for Wideband Spectrum Sensing

The MWC architecture for wideband spectrum sensing is shown in Fig. 1. The MWC performs sub-Nyquist sampling to obtain input samples, and the CTF block conducts frequency support recovery to identify spectrum holes.

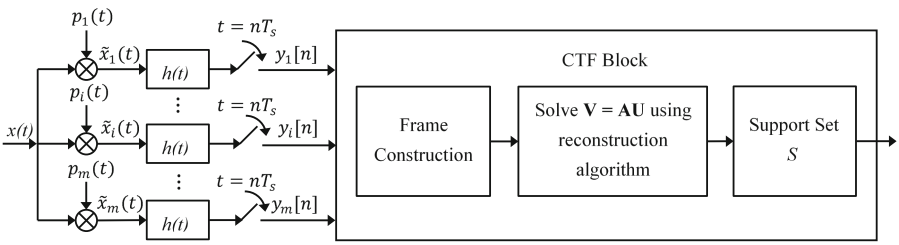

Fig. 1. MWC architecture for wideband spectrum sensing

2.1 MWC

The MWC can treat multiband signals when knowledge of the carrier frequencies is unknown. The only assumption is that the spectrum is concentrated on N frequency intervals with individual widths not exceeding B. The sampling rate is proportional to the effective spectrum occupation NB rather than f_{NYQ} which represents Nyquist rate. Typically, the spectrum is underutilized so that $NB \ll f_{NYQ}$. For the purpose of spectrum sensing, the goal of MWC is to detect the inactive support. Other tasks such as reconstruction and processing of the primary transmissions are not required in the CR settings.

The MWC consists of a front end of m channels. In the ith channel, the input signal $x(t)$ is multiplied by a periodic mixing waveform $p_i(t)$ with period T_p, low-pass filtered by $h(t)$, and then sampled at rate $1/T_s$. The basic MWC configuration has $f_p = 1/T_p \geq B$, $T_p = T_s$, $m \geq 4N$. Such a parameter choice results in a sampling rate, $mf_s \approx 4NB$, which, in general, is far below f_{NYQ}.

The mixing sequence $p_i(t)$ is periodic, and it has a Fourier expansion

$$p_i(t) = \sum_{l=-\infty}^{\infty} c_{il} e^{j2\pi f_p l t} \tag{1}$$

for some coefficients c_{il}.

Denoting by $z_l[n]$ the sequence that would have been obtained if the signal was mixed by a pure sinusoid $e^{j2\pi f_p l t}$ and low-pass filtered. This sequence corresponds to uniform samples at rate f_p of a section of $x(t)$, conceptually obtained by bandpass filtering an f_p-width interval around lf_p and demodulating to the origin. Since the system is linear, modulating by $p_i(t)$ and low-pass filtering is equivalent to summing the weighted combinations of all the sequences $z_l[n]$

$$y_i[n] = \sum_{l=-L}^{L} c_{il} z_l[n] \tag{2}$$

where the sum limits $-L \leq l \leq L$ represent the range of coefficients c_{il}, with nonnegligible amplitudes. It follows that the number of spectrum intervals that are aliased to the origin is $M = 2L + 1$.

Mixing by periodic waveforms aliases the spectrum to baseband, and each frequency interval of width $f_p = 1/T_p$ receives a different weight. The energy of the various spectral intervals is overlayed at baseband. Nonetheless, the fact that only a small portion of the wideband spectrum is occupied, together with the different weights in the different channels, permits the recovery of $x(t)$.

(2) can be rewritten as a linear system

$$\mathbf{y}[\mathrm{n}] = \mathbf{C}\mathbf{z}[\mathrm{n}] \tag{3}$$

where the vector $\mathbf{y}[\mathrm{n}]$ collects the measurements at $t = nTs$. The matrix $\mathbf{C}$ consists of the coefficients c_{il}, and $z_l[n]$ consists of the values of $z_l[n]$ arranged in vector form. From (3) and the definition of $z_l[n]$, it follows that at most $2\ N$ sequences, $z_l[n]$ are active, namely contain signal energy.

2.2 CTF Block

The CTF block constructs a finite-dimensional frame (or basis) from the samples, from which a small-size optimization problem is formulated. The solution of that problem indicates those spectrum slices that contain signal energy. The CTF outputs an index set S of active slices.

The spectrum sensing functionality is to finding the index set

$$S = \{l | z_l[n] \neq 0\} \tag{4}$$

which reveals the spectrum support of $x(t)$ at a resolution of f_p Hz.

Detecting S by inverting $\mathbf{C}$ in (3) is not possible, since the $m \times M$ matrix $\mathbf{C}$ is underdetermined; the MWC uses $m \ll M$ to reduce the sampling rate below Nyquist rate. Underdetermined systems have in general infinitely many solutions. Nonetheless, under the parameter choice, and additional mild conditions on the waveforms $p_i(t)$, a sparse **z**[n] with at most *2 N* nonzero entries is unique and can be recovered in polynomial time by relying on results in the field of compressed sensing.

Solving for a sparse vector solution of an underdetermined system of equations has been widely studied in the literature of compressed sensing. Recovery of **z**[n] using any of the existing sparse recovery techniques is inefficient, since the sparsest solution **z**[n], even if obtained by a polynomial-time CS technique, is computed independently for every n. Instead, the CTF method exploits the fact that the bands occupy continuous spectral intervals. This analog continuity boils down to **z**[n] having a common nonzero location set S over time. To take advantage of this joint sparsity, the CTF builds a frame from the measurements using

$$\mathbf{y}[n] \overset{\text{Frame construct}}{\rightarrow} \mathbf{Q} = \mathbf{y}[n]\mathbf{y}^H[n] \overset{\text{Decompose}}{\rightarrow} \mathbf{Q} = \mathbf{V}\mathbf{V}^H. \tag{5}$$

The active spectrum slices are detected from the sparse solution of the following finite dimensional system

$$\mathbf{V} = \mathbf{C}\mathbf{U}. \tag{6}$$

It is proven in [6] that (6) has a unique solution matrix $\mathbf{U}$ with minimal number of nonidentically zero rows, and that the locations of these rows coincide with the support set S of $x(t)$. The CTF effectively locates the signal energy at a spectral resolution of f_p. From that support set, the CR device can decide the spectrum holes to be used as

$$\text{Spectrum holes } = \bigcup_{l \notin S} [lf_p - \frac{f_p}{2}, lf_p + \frac{f_p}{2}]. \tag{7}$$

In previous studies, OMP algorithm is used in the CTF block for frequency support recovery. However, the percentage of correct support recovery is low at low SNR using OMP algorithm. In this paper, CoSaMP algorithm is investigated to be used in the CTF block instead of OMP algorithm. In the sequel, we demonstrate by simulation that such a proposal can achieve higher percentage of correct support recovery than with OMP algorithm.

3 Sparse Signal Reconstruction Algorithm

The sparse signal reconstruction algorithms, OMP and CoSaMP, are presented as follows.

3.1 OMP Algorithm

MP (Matching Pursuit) is a greedy iterative algorithm for approximately solving the original l_0-norm problem for sparse signal recovery. MP works by finding a basis vector in the dictionary that maximizes the correlation with the residual, and then recomputing the residual and coefficients by projecting the residual on all atoms in the dictionary using existing coefficients. The main difference of OMP from MP is that after every step, all the coefficients extracted are updated by computing the orthogonal projection of the signal onto the set of atoms selected so far. The algorithm maintains an active set of atoms already picked, and adds a new atom at each iteration. The residual is projected on to a linear combination of all atoms in the active set, so that an orthogonal updated residual is obtained. The pseudo code of the OMP algorithm is given as follows.

Input:

(1) $M \times N$ dimensional sensing matrix $A = \Phi\Psi$
(2) $N \times 1$ dimensional measurement vector y
(3) Sparsity of signal K

Output:

(1) Estimation of signal sparse representation coefficient $\overset{\wedge}{\theta}$
(2) $N \times 1$ dimensional residual $r_K = y - A_K\,\hat{\theta}_K$

In the following processes, r_t is the residual, t is the number of iteration, $\varnothing$ is the empty set, Λ_t is the column index set at tth iteration, λ_t is the column index at the tth iteration, a_j is the jth column of the matrix A, A_t is the column set of $M \times t$ dimensional matrix of matrix A, which is chosen according to index Λ_t; θ_t is a $t \times 1$ dimensional column vector; the symbol $\cup$ refers to the union operation of sets; $\langle \cdot, \cdot \rangle$ refers to the inner product of vectors.

(1) Initializing $r_o = y$, $\Lambda_o = \varnothing$, $A_o = \varnothing$, $t = 1$;
(2) Finding the index λ_t, which satisfies $\lambda_t = \arg \max_{j=1,2,\cdots,N} \left|\left\langle r_{t-1}, a_j \right\rangle\right|$;
(3) Making $\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}$, $A_t = A_{t-1} \cup a_\lambda$;
(4) Calculating the least square solution of $y = A_t\,\theta_t$: $\hat{\theta}_t = \arg \min_{\theta_t} \|y - A_t\,\theta_t\| = \left(A_t^T A_t\right)^{-1} A_t^T y$;
(5) Updating the residual $r_t = y - A_t\,\hat{\theta}_t = y - A_t\left(A_t^T A_t\right)^{-1} A_t^T y$;
(6) $t = t + 1$, if $t \le K$, the program will turn back to the second step to continue the iteration, otherwise the iteration will be stopped and the program will run the seventh step;

(7) The $\hat{\theta}$ from reconstruction has nonzero terms in Λ_t, and their values are $\hat{\theta}_t$ which are obtained from the last iteration respectively.

3.2 CoSaMP Algorithm

CoSaMP algorithm is another reconstruction algorithm which is proposed by Needell and Tropp [10]. CoSaMP algorithm chooses multiple atoms during each iteration rather than only one atom in OMP algorithm. In the selection criteria, the atoms in every iteration chosen by OMP algorithm are saved forever, whereas the atoms in every iteration chosen by CoSaMP algorithm may be discarded in next iteration. CoSaMP algorithm is better to approximate a compressible signal from noisy samples than OMP algorithm. The pseudo code of the CoSaMP algorithm is given as follows.

Input:

(1) $M \times N$ dimensional sensing matrix $A = \Phi\Psi$
(2) $N \times 1$ dimensional measurement vector y
(3) Sparsity of signal K

Output:

(1) Estimation of signal sparse representation coefficient $\hat{\theta}$
(2) $N \times 1$ dimensional residual $r_S = y - A_S \hat{\theta}_S$

In the following processes, r_t is the residual, t is the number of iteration, $\varnothing$ is the empty set, J_0 is the column index in each iteration, Λ_t is the column index set at tth iteration, a_j is the jth column of the matrix A, A_t is the column set of matrix A which is chosen according to index Λ_t, θ_t is a $L_t \times 1$ column vector; $abs[\cdot]$ is calculating the module value namely absolute value.

(1) Initializing $r_o = y$, $\Lambda_o = \varnothing$, $A_o = \varnothing$, $t = 1$;
(2) Calculating $u = abs[A^T r_{t-1}]$ (that is calculating $\langle r_{t-1}, a_j \rangle$, $1 \le j \le N$), choosing the $2K$ maximum in u, forming the set J_0 (column ordinal set) with the column ordinals j of A that correspond to these $2K$ maximum;
(3) Making $\Lambda_t = \Lambda_{t-1} \cup J_0$, $A_t = A_{t-1} \cup a_j$ (for all $j \in J_0$);
(4) Calculating the least square solution of $y = A_t \theta_t$: $\hat{\theta}_t = \arg\min_{\theta_t} \|y - A_t \theta_t\| = \left(A_t^T A_t\right)^{-1} A_t^T y$;
(5) Choosing the K largest absolute values of $\hat{\theta}_t$, and they are denoted as $\hat{\theta}_{tK}$. The K columns corresponding to A_t are denoted as A_{tK} and the column ordinals corresponding to A are denoted as Λ_{tK}, then updating the set $\Lambda_t = \Lambda_{tK}$;
(6) Updating the residual $r_t = y - A_{tK} \hat{\theta}_{tK} = y - A_{tK} \left(A_{tK}^T A_{tK}\right)^{-1} A_{tK}^T y$;
(7) $t = t + 1$, if $t \le S$, the program will turn back to the second step to continue the iteration, if $t > S$ or the residual $r_t = 0$, the iteration will be stopped and the program will run the eighth step;
(8) The $\hat{\theta}$ from reconstruction has nonzero terms in Λ_{tK}, and their values are $\hat{\theta}_{tK}$ which are obtained from the last iteration respectively.

4 Simulation and Discussion

In the simulation, the overall spectrum sensing range is 20 MHz. There are 2 active frequency bands ($2N = 4$). On each active frequency band, binary phase shift keying (BPSK) signal with Sinc waveforms is generated as the transmitted signal with bandwidth $B = 2$ MHz, and the signals on two frequency bands are modulated with frequency carriers 5 MHz and 15 MHz respectively. The signal energy is normalized to 1 at each band. The Nyquist rate is $f_{NYQ} = 40$ MHz. The sampling rate fs equals the aliasing rate fp whose value is 2.105264 MHz. The signals are sent over AWGN channel before reaching the MWC sensing device. The number of MWC channels is $m = 20$.

Figure 2 shows the time-domain signal waveform and PSD (Power Spectrum Density) of original signal, signal with noise and reconstructed signal when SNR is 30 dB.

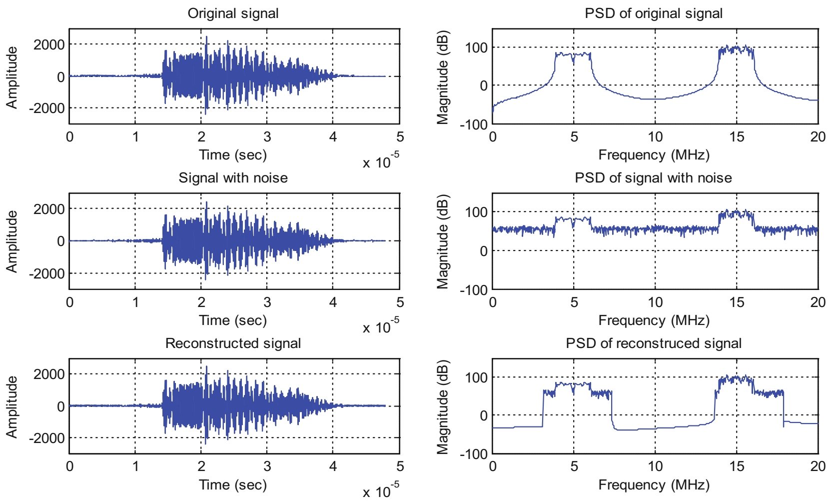

Fig. 2. Signal waveform and PSD of original signal, signal with noise, and reconstructed signal when SNR = 30 dB

Figure 3 shows the time-domain signal waveform and PSD (Power Spectrum Density) of original signal, signal with noise and reconstructed signal when SNR is 5 dB.

Figure 4 shows normalized residual of reconstructed signal versus SNR using OMP and CoSaMP algorithms. The normalized residual is defined as follows

$$residual = y - solution \tag{8}$$

$$normalized\,residual = resnorm/norm(solution) \tag{9}$$

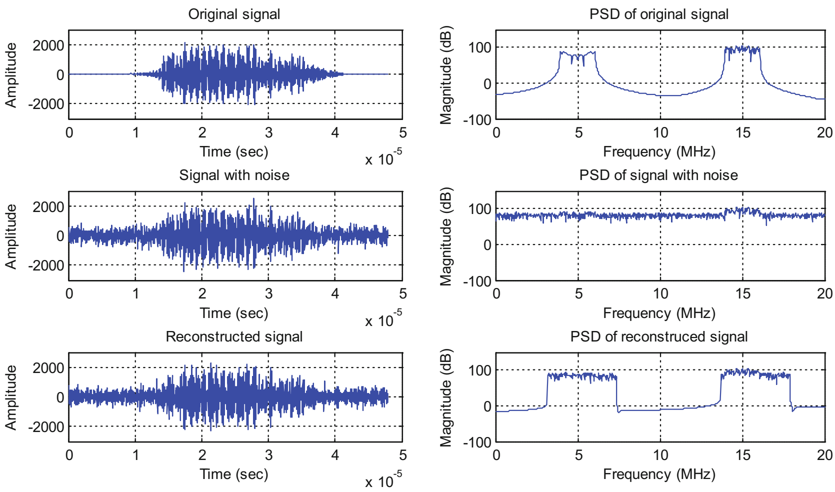

Fig. 3. Signal waveform and PSD of original signal, signal with noise, and reconstructed signal when SNR = 5 dB

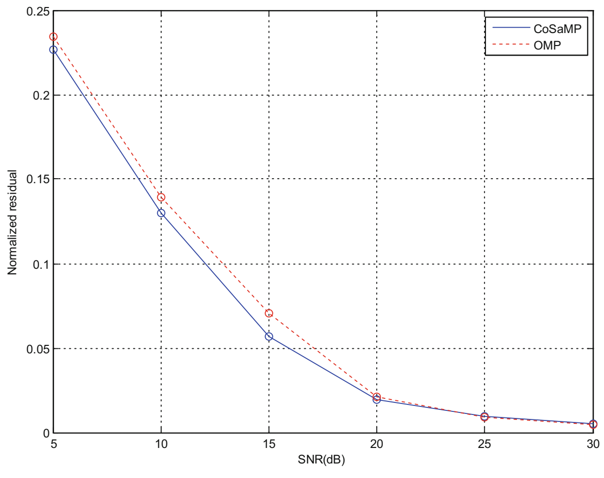

Fig. 4. Normalized residual of reconstructed signal at different SNRs

The *residual* can be calculated by (8), where y is the observed signal; the *solution* is $A_t\,\hat{\theta}_t$ in the fifth step of OMP algorithm or $A_{tK}\,\hat{\theta}_{tK}$ in the sixth step of CoSaMP algorithm. The normalized residual is given in (9), where *resnorm* is the norm of residual.

It can be seen from Fig. 4 that the normalized residual of the reconstructed signal with CoSaMP algorithm is lower than that with OMP algorithm, though there is no significant difference.

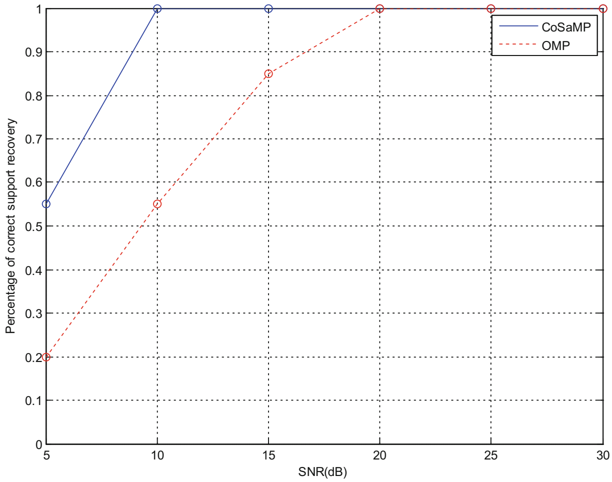

Fig. 5. Percentage of correct support recovery at different SNRs

Figure 5 shows that the percentage of correct support recovery at different SNRs using OMP and CoSaMP algorithms. It can be seen from Fig. 5 that the percentage of correct support recovery with CoSaMP algorithm is much higher than that with OMP algorithm at low SNR. Particularly, when SNR = 5 dB, the percentage of correct support recovery with CoSaMP algorithm is 55%. On the other hand, the percentage of correct support recovery with OMP algorithm is only 20% at the same SNR. Moreover, exact support recovery can be achieved when SNR is larger than 10 dB by using CoSaMP algorithm; OMP algorithm cannot reach such a performance until SNR is larger than 20 dB.

In addition, CoSaMP algorithm $(O(mn))$ has reduced complexity than OMP algorithm $(O(kmn))$ [10]. The notation k refers to the sparsity level, the notation m refers to the number of measurements and the notation n refers to the signal length. Consequently, CoSaMP algorithm can be a better algorithm to be used in the CTF block of the system architecture for multiband spectrum sensing for support recovery than OMP algorithm.

5 Conclusion

In this paper, MWC with CoSaMP reconstruction algorithm for support recovery is proposed for multiband spectrum sensing. Our simulation results demonstrated that the method with CoSaMP algorithm can achieve higher percentage of correct support recovery than OMP algorithm, especially at low SNR regime. We also show that lower normalized residual can be achieved with CoSaMP algorithm than with OMP algorithm. In addition, CoSaMP algorithm has less complexity than OMP algorithm. These benefits make CoSaMP algorithm a better candidate to be used with MWC for wideband spectrum sensing.

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References

1. Mitola III., J.: Cognitive radio for flexible mobile multimedia communications. J. Mob. Netw. Appl. **6**(5), 435–441 (2001)
2. Yücek, T., Arslan, H.: A survey of spectrum sensing algorithms for cognitive radio applications. J. IEEE Commun. Surv. Tutor. **11**(1), 116–130 (2009)
3. Mishali, M., Eldar, Y.C., Dounaevsky, O., et al.: Xampling: analog to digital at sub-nyquist rates. J. IET Circuits Dev. Syst. **5**(1), 8–20 (2011)
4. Mishali, M., Eldar, Y.C.: From theory to practice: sub-nyquist sampling of sparse wideband analog signals. IEEE J. Sel. Top. Signal Process. **4**(2), 375–391 (2015)
5. Donoho, D.L.: Compressed sensing. IEEE Trans. Inform. Theory. **52**(4), 1289–1306 (2006)
6. Mishali, M., Eldar, Y.C.: Blind multi-band signal reconstruction: compressed sensing for analog signals. IEEE Trans. Signal Process. **57**(3), 993–1009 (2009)
7. Pati, Y.C., Rezaiifar, R., Krishnaprasad, P.S.: Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition. In: Proceedings of Conference on Recommendation 27th Asilomar Conference on Signals, Systems and Computers, pp. 40–44 (1993)
8. Yang, Z.Z., Yang, Z., Sun, L.H.: A survey on orthogonal matching pursuit type algorithms for signal compression and reconstruction. J. Signal Process. **29**(4), 486–496 (2013)
9. Tropp, J.A., Gilbert, A.C.: Signal recovery from random measurements via orthogonal matching pursuit. IEEE Trans. Inform. Theory. **53**(12), 4655–4666 (2008)
10. Needell, D., Tropp, J.A.: CoSaMP: iterative signal recovery from incomplete and inaccurate samples. J. Appl. Comput. Harmon. Anal. **26**(3), 301–321 (2009)