Joint Asynchronous Time and Localization of an Unknown Node in Wireless Sensor Networks

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Abstract. This paper considers the location of an unknown source node with asynchronous anchor nodes in Wireless Sensor Networks (WSN). In this paper, a joint synchronization and localization framework is considered and examined. Firstly, the proposed algorithm obtains algebraic solutions of the source location and the clock skew by improved Taylor series based on regularization theory. Then the clock offset is estimated. The Cramer-Rao lower bound is derived for the considered problem. Simulations show that the proposed algorithm is robust to the inaccurate initial value and better accuracy than the closed-form methods. This proposed algorithm can be widely applied in practice.

Keywords: Localization · Clock offset · Clock skew · WSN

1 Introduction

There has been much research on localization techniques and asynchronous time in Wireless sensor networks. Different measurement techniques used in source localization include time of arrival (TOA) [1], time difference of arrival (TDOA) and frequency difference of arrival (FDOA) [2] and hybrid algorithm with two or three techniques [1]. In [3,4], the authors proposed a close-form algorithms to solve the synchronization and localization parameters at the same time. [5] only considered the client location and clock offset of access points, ignored the clock skews. [6,7] provided two estimators which jointly estimate the position of the target node as well as the unknown clock-skews and clock offsets.

The main contribution of this paper is driving an estimator for determining the source node location, the anchor nodes skews and offsets. Asynchronous

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TDOA-based source localization using improved Taylor series to obtain the three parameters is studied. The paper introduces the Regularization theory to modify the Taylor series to ensure the iteration convergence. The simulation results show that the proposed approach offer better robustness and accuracy.

2 System Model

The localization system consists of M sensors which have known position and they are separated into N groups according to [8]. Group j, $j = 1, 2, ..., N$, has the same clock skew ω_j and clock offset θ_j. And there is 1 anchor node in group 1 for reference. The localization system is used to determine the position of one source node which has unknown transmission time t_s. The internal time is modeled as a function of the reference as (1). t_i and t are the internal time of the ith $(i = 1, 2, ...M)$ node and the reference time, respectively.

$$t_i = \omega_j t + \theta_j \tag{1}$$

$\mathbf{s}_i = [x_i, y_i]$ denote the location of the ith anchor, where $\mathbf{x} = [x, y]$ denotes the location of the source node. d_i is determined by the Euclidean distance: $d_i = \|\mathbf{x} - \mathbf{s}_i\|$. Then, the packet arrival time vector is represented as follows:

$$t_i = \omega_j (t_s + \frac{1}{c} d_i + n_{ti}) + \theta_j \tag{2}$$

Here $c = 3 \times 10^8$ (m/s). n_{ti} is a random measurement error with zero mean and a standard deviation of σ_t.

The transmit time will be abandoned, TDOA based localization is more appropriate than TOA based localization. The clock skew and offset of group 1 is $\omega_1 = 1$ and $\theta_1 = 0$. Then the TDOA vector is obtained by subtracting the packet arrival times from that of the representative as follows:

$$\frac{t_i}{\omega_j} - t_1 = \frac{d_i}{c} - \frac{d_1}{c} + n_{ti} - n_{t1} + \frac{\theta_j}{\omega_j} \tag{3}$$

3 Proposed Algorithm Based on Taylor Series Method

3.1 Improved Taylor Series Method

To progress, an approximation need be applied to the model. The clock skew of the anchor node can be expressed as $\omega_j = 1 + \delta_j$, where $\delta_j \ll 1$ is relatively small value. For sufficiently small δ_j, we have [4] $\frac{1}{\omega_i} = 1 - \delta_i$. Consider Eq. (3), we take $j = 2$, $i = 2$ and $i = 3$ as example. $c(t_2 - t_3) = \frac{1}{c}(d_2 - d_3) + n_{23} + c(t_2 - t_3)\delta_2$, where $r_{23} = c(t_2 - t_3)$ can be regarded as the TDOA measurement. And n_{23} is the additive noise of distance. Extend the situation to N groups, in vector form,

$$\mathbf{r_A} = \mathbf{d_A} + \mathbf{F}\delta_\mathbf{A} + \mathbf{n_A} \tag{4}$$

where $\mathbf{r_A} = [r_{m_1+2,m_1+1}, r_{m_1+3,m_1+1}, \cdots, r_{m_N,m_N-1}]$ is the TDOA vector. $\mathbf{n_A}$ is the noise vector which is zero-mean Gaussian random variables. $\mathbf{d_A}$ is the true distance vector. $\delta_\mathbf{A} = [\delta_2, \delta_3, \cdots, \delta_N]$ is the clock skew after the approximation in (8). And the $(M-N-1) \times (N-1)$ matrix $\mathbf{F}$ is

$$\mathbf{F} = \begin{bmatrix} r_{32}, r_{42} \ldots, r_{m_2 2}, 0 & . \quad . \quad . & 0, 0, & . \quad . \quad . & 0 \\ 0 \quad \cdots \quad 0, r_{m_2+2,m_2+1}, \ldots, r_{m_3,m_2+1}, 0, & 0, & & . \quad . \quad . & 0 \\ \vdots & & & & \\ 0 \quad \cdots \quad 0, 0 & . \quad . \quad . & 0, r_{m_{N-1}+2,m_{N-1}+1} \cdots, r_{m_N,m_{N-1}+1} & & \end{bmatrix}^T \tag{5}$$

The parameter vector is $\mathbf{z}^T = \left[\mathbf{x}^T, \delta_\mathbf{A}^T\right]$. The target function is:

$$f(\mathbf{r}; \mathbf{p}) = k - \frac{1}{2}(\mathbf{r_A} - \mathbf{d_A} - \mathbf{F}\delta_\mathbf{A})^T \mathbf{Q}_{\mathbf{r_A}}^{-1} (\mathbf{r_A} - \mathbf{d_A} - \mathbf{F}\delta_\mathbf{A}) \tag{6}$$

And the partial differentiations in (5) are formulated as follows:

$$\frac{\partial f}{\partial \mathbf{x}} = \frac{\mathbf{x} - \mathbf{s}_i}{\|\mathbf{x} - \mathbf{s}_i\|} - \frac{\mathbf{x} - \mathbf{s}_{m_j+1}}{\left\|\mathbf{x} - \mathbf{s}_{m_j+1}\right\|} \quad \frac{\partial f}{\partial \delta_\mathbf{A}} = \mathbf{F} \tag{7}$$

Taylor series method requires initial guess. Because the measurements are independent, we assume that the $\mathbf{Q}_{\mathbf{r_A}} = 2c^2\sigma_t^2\mathbf{I}$, it is a diagonal matrix. Then the kth iterative change is

$$\Delta\mathbf{z}_k = (\mathbf{B}_k{}^T \mathbf{B}_k)^{-1} \mathbf{B}_k{}^T \mathbf{p}_k \tag{8}$$

where

$$\mathbf{B}_k = \left[\frac{\partial f}{\partial \mathbf{x}}|_{\mathbf{x}_k}, \mathbf{F}\right] \quad \mathbf{p}_k = \mathbf{r_A} - \mathbf{d_A}|_{\mathbf{x}_k} - \mathbf{F}\delta_\mathbf{A}|_{\delta_{\mathbf{A}_k}} \tag{9}$$

And after every iteration, the parameter vector becomes $\mathbf{z}_{k+1} = \mathbf{z}_k + \Delta\mathbf{z}_k$. Taylor series method requires a good initial value, and a bad initial value can cause an ill-posed Jacobian matrix which leads to the iteration divergence. According to (8), We set $\mathbf{A} = \mathbf{B}^T\mathbf{B}$, $\mathbf{b} = \mathbf{B}^T\mathbf{p}$. Jacobian matrix is symmetric matrix. So the SVD of $\mathbf{A}$ is given by $\mathbf{A} = \mathbf{U}\sum\mathbf{U}^T = \sum_{i=1}^{n} \mathbf{u}_i \sigma_i \mathbf{u}_i^T$. Then the Iterative change is $\Delta\mathbf{z} = \mathbf{A}^{-1}\mathbf{b} = \sum_{i=1}^{n} \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{u}_i$.

From the equation, $\Delta\mathbf{z}$ is affected by smaller singular value obviously. As a result, $\Delta\mathbf{z}$ will be sign changed and rand. Then based on Regularization theory, the problem turns into (10) to modify the iterative change.

$$\min \quad \|\mathbf{A}\Delta\mathbf{z} - \mathbf{b}\|_2^2 + \lambda \|\Delta\mathbf{z}\|_2^2 \tag{10}$$

The DSVD method is used to modify the change as:

$$\Delta\mathbf{z}_{dsvd} = (A + \lambda I)^{-1} b = \sum_{i=1}^{n} \frac{\sigma_i}{\sigma_i + \lambda} \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{u}_i \tag{11}$$

From the modification, the cond of matrix becomes lower, and the sign change and rand situation will be decreased. The λ is a regularization parameter controlling the tradeoff between the first and second terms. The L-curve criterion [8] is adopted to determine λ.

The procedure is as follows: the unknown parameter is $\mathbf{z}^T = \left[\mathbf{x}^T, \delta_{\mathbf{A}}^T\right]$. First we assume $\delta_{\mathbf{A}} = 0$, the initial guess $\mathbf{x}_k$. The tradeoff is $\|\Delta\mathbf{z}\| < \varepsilon$. Do $\mathbf{z}_{k+1} = \mathbf{z}_k + \Delta\mathbf{z}_{dsvdk}$ until the tradeoff is satisfied. Then we can get $\hat{\mathbf{z}} = [\mathbf{x}, \delta_{\mathbf{A}}]$.

3.2 Estimation of Clock Offset

Since the d_1 and d_i are not known, we shall replace it by the reconstructed value using Euclidean distance, with $\hat{\mathbf{x}}$ substituted by $\mathbf{x}$ by final result. Let us denote the resulted vector be $\mathbf{R}(\mathbf{x}, \delta_{\mathbf{A}})$. Then the solution of θ is

$$\theta = (\mathbf{H}^T\mathbf{Q}_r^{-1}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{Q}_r^{-1}(\mathbf{R} - \mathbf{R}(\mathbf{x}, \delta_{\mathbf{A}})) \tag{12}$$

where

$$\begin{aligned} \mathbf{H} &= \left[\frac{1}{\omega_2}, \frac{1}{\omega_2} \cdots \frac{1}{\omega_N}\right]^T \\ \mathbf{R} &= [t_2 - t_1, t_3 - t_1, ..., t_M - t_1]^T \\ \mathbf{R}(\mathbf{x}, \delta_{\mathbf{A}}) &= \left[\frac{d_2 - d_1}{c} + t_2\delta_2, ..., \frac{d_M - d_1}{c} + t_M\delta_N\right] \end{aligned} \tag{13}$$

4 Simulation Results and Discussion

As tabulated in Table 1, $M = 8$, $N = 3$. They are used to locate a source whose actual position is [0, 0]m. The time skew is randomly drawn from [0.95 1.05], and the time offset is randomly drawn from [1 10] ns. The RMSEs of the location, clock skew and clock offset estimation are defined as $\sqrt{(\hat{x} - x)^2 - (\hat{y} - y)^2}$, $\sum_{j=2}^{N} \sqrt{(\omega_j - \omega)^2}/(N-1)$ and $\sum_{j=2}^{N} \sqrt{(\theta_j - \theta)^2}/(N-1)$, respectively. Two estimators given in [3,6] are selected for comparisons.

Table 1. Sensor positions [m]

	Set 1	Set 2				Set 3		
x_i	−80	0	60	20	100	50	−65	−20
y_i	50	−100	−10	90	100	100	90	80

The first figure depicts the comparison of proposed algorithm and traditional Taylor series method with different initial guess. The measurement noise is set $\sigma_d^2 = 1\,\text{m}^2$. With good initial guess, the proposed algorithm needs more iterations. But with the bad initial guess, the traditional Taylor method diverges to unexpected result. The amount of calculation of proposed algorithm is larger than traditional Taylor method because of the revise of ill-posed matrix. But the proposed algorithm is robust and accurate.

The rest figures illustrate the estimation accuracy of location root-mean-square error (RMSE), clock skews and offsets with the log function of distance variance deviation changes from −30 to 15. The proposed algorithm is more accurate than LS and the separate approach. When the measurement error is small, the performance gap between Proposed algorithm and CRLB is not significant. The clock offsets rely on the previous estimations. If the localization and clock skews are biased, the clock offsets accuracy will be affected seriously (Fig. 1).

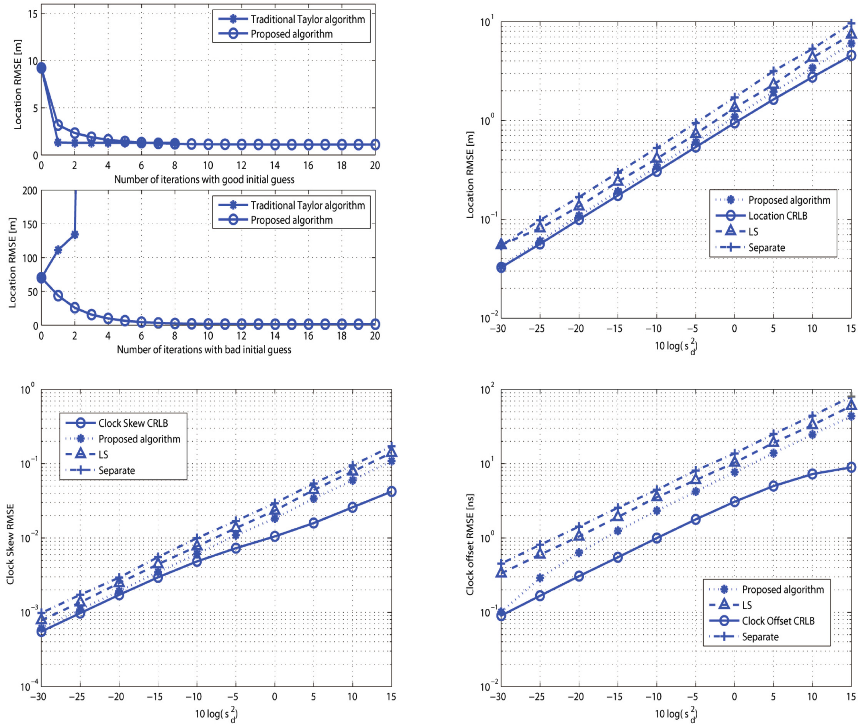

Fig. 1. Simulation of robustness and accuracy of the proposed algorithm

5 Cramer-Rao Lower Bound

$$\text{FIM} = \begin{bmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \mathbf{X}_{13} \\ \mathbf{X}_{12}^T & \mathbf{X}_{22} & \mathbf{X}_{23} \\ \mathbf{X}_{13}^T & \mathbf{X}_{23}^T & \mathbf{X}_{33} \end{bmatrix} \tag{14}$$

We have used the ∇ symbol to denote partial derivative:

$$\nabla_{\mathbf{a},\mathbf{b}} = \frac{\partial \mathbf{a}}{\partial \mathbf{b}^T} \tag{15}$$

$$
\begin{aligned}
\mathbf{X}_{11} &= -E\left[\frac{\partial \ln p}{\partial \mathbf{x}^0 \partial \mathbf{x}^{0T}}\right] = \left(\frac{\partial \zeta}{\partial \mathbf{x}^0}\right)^T Q_r^{-1} \left(\frac{\partial \zeta}{\partial \mathbf{x}^0}\right) \\
\mathbf{X}_{12} &= \nabla_{\zeta,\mathbf{x}^0}^T Q_r^{-1} \nabla_{\zeta,\omega^0} \\
\mathbf{X}_{13} &= \nabla_{\zeta,\mathbf{x}^0}^T Q_r^{-1} \nabla_{\zeta,\theta^0} \\
\mathbf{X}_{22} &= \nabla_{\zeta,\omega^0}^T Q_r^{-1} \nabla_{\zeta,\omega^0} + Q_\omega^{-1} \\
\mathbf{X}_{23} &= \nabla_{\zeta,\omega^0}^T Q_r^{-1} \nabla_{\zeta,\theta^0} \\
\mathbf{X}_{33} &= \nabla_{\zeta,\theta^0}^T Q_r^{-1} \nabla_{\zeta,\theta^0} + Q_\theta^{-1}
\end{aligned}
\tag{16}
$$

where the Q_r, Q_ω and Q_θ are the covariance matrix of measurement error, anchor node clock skew and offsets error. The partial derivatives are obtained using (1), (3) and (7),

$$
\frac{\partial \zeta}{\partial \mathbf{x}^0} = \left[\rho_2^0 - \rho_1^0, \rho_3^0 - \rho_1^0, ..., \rho_M^0 - \rho_1^0\right]^T \quad \rho_i^0 = \frac{\left(\mathbf{x}^0 - \mathbf{s}_i^0\right)}{\|\mathbf{x}^0 - \mathbf{s}_i^0\|} \tag{17}
$$

$$
\nabla_{\zeta,\omega^0} = \frac{\mathbf{d}}{{\omega^0}^2} \otimes \mathbf{1}_M - \frac{c\theta}{{\omega^0}^2} \otimes \mathbf{1}_M \quad \nabla_{\zeta,\theta^0} = \frac{c}{\omega^0} \otimes \mathbf{1}_M \tag{18}
$$

The CRLB for location estimation is given by

$$
\mathrm{CRLB}(\mathbf{x}^0) = \mathbf{X}_{11}^{-1} + \mathbf{X}_{11}^{-1}\mathbf{S}_1 \left(\mathbf{S}_2 - {\mathbf{S}_1}^T \mathbf{X}_{11}^{-1} \mathbf{S}_1\right) {\mathbf{S}_1}^T \mathbf{X}_{11}^{-1} \tag{19}
$$

where

$$
\mathbf{S}_1 = \left[\mathbf{X}_{12}\ \mathbf{X}_{13}\right] \quad \mathbf{S}_2 = \begin{bmatrix} \mathbf{X}_{22} & \mathbf{X}_{23} \\ \mathbf{X}_{23}^T & \mathbf{X}_{33} \end{bmatrix} \tag{20}
$$

By arranging the matrix blocks in (20) properly, the CRLB for the clock skews and offsets can be determined using the same method.

6 Conclusion

This paper investigates the problem of source localization in the presence of sensor clock skews and offsets. A sequential estimation method is developed that obtains the source position and clock skews first, the sensor node clock offset finally. Simulations show that the proposed algorithm can converge with bad initial guess. Compared with the close-formed approaches, the performance of proposed algorithm is closer to the CRLB.

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