Increase MIMO Systems Performances by Concatenating Short Polar Codes to Spatial Time Block Codes

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Abstract. Polar codes, proposed by Erdal Arikan, have attracted a lot of interest in the field of channel coding for their capacity-achieving trait as well as their low encoding and decoding complexity in order O (NlogN) under successive cancellation (SC) decoder. However, there remains one significant drawback, that is, the error correction performance of short and moderate length polar codes is unsatisfactory, especially when compared with low-density parity check (LDPC) codes and turbo codes. In this paper, we propose a concatenation scheme performance, which employs a short polar encoder following to Spatial Time Block Codes (STBC), and we develop an efficient detector for Multiple Input Multiple Output (MIMO) antennas, which adaptively combines Minimum Mean Square Error Successive Interference Canceller together (MMSE-SIC). We also compared to Maximum Likelihood in the literature and finally present a simulation results in binary input Additive White Gaussian Noise (BI-AWGN) with binary phase shift keying (BPSK) modulation, and we observe that, our proposed concatenation scheme significantly outperforms the Maximum Likelihood performance in the high Signal-to-Noise-Ratio (SNR).

Keywords: Polar codes · STBC · BPSK · MIMO · MMSE-SIC · BER · FER

1 Introduction

Polar codes, proposed by Arikan [1] are a big breakthrough in coding theory, which proven to be capacity achieving for any given Binary Discrete Memoryless Channel (BDMC) W based on the phenomenon called channel polarization. Moreover, they can be implemented with a simple encoder and a simple SC decoder, both with low complexity of the order of O(NlogN), where N is the code block length. Due to those excellent proprieties, polar codes have arisen a lot of research interest among researchers especially the combination with MIMO antennas for data transmission [2]. Unfortunately, the performance of long length polar codes is generally used.

STBC provides full spatial diversity in the collocated MIMO systems, but it doesn't have the coding gain over fading channels. In the documentation many approach of concatenate STBC techniques to other codes have been proposed [3, 4]. In [5, 6], a

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concatenation scheme of good encoding and decoding named Polar codes with STBC called Polar-STBC of long length have been discussed and achieved sufficient gain due to this concatenation. In [7] authors propose the antennas detection and reduce the complexity of the receiver by offering Maximum Likelihood detection algorithm. In [8] a linear filter detection MMSE-SIC using a small polar code, which allowed reducing the complexity while maintaining the BER, performance is presented. In [6] long polar codes concatenate to STBC gives good BER performance when ML is applied to the detector.

In this paper, we analysis the BER and FER performances of short polar codes concatenated to STBC when MMSE-SIC detector is set to the output. We compare the result with a Maximum Likelihood Detector (MLD) as described in [6]. We are working on short codes because their hardware implementation is more easily.

The rest of the paper is organized as follows. Section 2 gives a brief review of Polar codes. The system model Polar-STBC and the Soft Output detector are presented in Sect. 3. Section 4 gives firstly numerical simulations of the Polar-STBC and STBC only systems using MMSE-SIC at the receiver. While Sect. 5 conclude the document.

2 Polar Codes

The polar encoding can be represented in [5] as,

$$x_1^N = u_1^N G_N \tag{1}$$

Where u_1^N is the source code, x_1^N is the encoded code, and we call G_N the generator matrix of polar codes of code length N. From the basic facts of channel polarization, we know that, some of the polarized channels are used to transmit the information bits, whereas the remaining is used to transmit the frozen bits. Alternatively, we can denote the polar encoding with another expression, namely,

$$x_1^N = u_A G_N(A) \oplus u_A^c G_N(A^c) \tag{2}$$

Where u_A and u_A^c denote the part of the source code which contains information bits respectively for an arbitrary set $A \subset \{1, 2, \ldots, N\}$, and A^c denotes the complementary set of A. Finally $G_N(A)$ and $G_N(A^c)$ denotes the sub matrix of G_N generated by the row with indices in A and A^c respectively.

The construction of polar codes is based on channel polarization [5, 8].

3 System Model

In our system model, we consider a transmission scheme using MIMO communication system with $\boldsymbol{L_T}$ antennas at the transmitter and $\boldsymbol{L_R}$ transmitter at the receiver. The channel is assumed to be in flat fading, Rayleigh channel, with Additive White Gaussian Noise Channel (AWGN). We propose double encoding, a small polar coding following to STBC, after their concatenation. The items are sent to the MIMO systems. To cooperative diversity system, Rayleigh channel and AWGN are also use. We used the same encoding offer to the first section (Fig. 1).

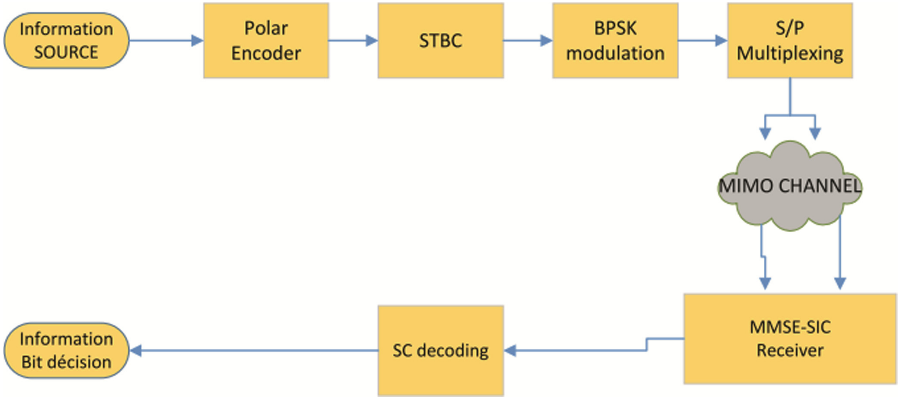

Fig. 1. The system detailed diagram block

After polar encoding, these polar code words are STBC encoded and fed to the L_t transmitting antennas by using

$$\begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix} \tag{3}$$

In STBC, the information bits to be transmitted are first divided into two parts, one part for selecting transmit antenna pair, and the other part for BSPKS modulation, to get two modulation symbols x_1, x_2. The row of matrix corresponds to the transmit time slot and the column of the matrix to the transmit antenna. In the first time slot, x_1, x_2 are respectively transmitted by two active transmit antennas, and $-x_2^*, x_1^*$ are transmitted by the same transmit antenna pair in second time slot.

For $N_R = 4$, there are two different codebooks $\mathcal{L}_1$, $\mathcal{L}_2$, which can be denoted as

$$\mathcal{L}_1 = \left\{ \begin{bmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & x_1 & x_2 \\ 0 & 0 & -x_2^* & x_1^* \end{bmatrix} \right\}$$
$$\mathcal{L}_2 = \left\{ \begin{bmatrix} 0 & x_2 & x_2 & 0 \\ 0 & -x_2^* & x_1^* & 0 \end{bmatrix} \begin{bmatrix} x_2 & 0 & 0 & x_1 \\ x_1^* & 0 & 0 & -x_2^* \end{bmatrix} \right\} e^{j\theta}$$

Where each codebook has two different codewords $\mathcal{L}_{i,j}$ $j = 1, 2$, and the codewords in the same codebook do not have overlapping non-zero column, θ is a rotation angle, which can be optimized for a given modulation format to ensure maximum diversity and coding gain. It is assumed the $2 \times N_r$ codeword $\mathcal{L}$ is transmitted over a $N_T x N_R$ Rayleigh flat fading MIMO channel H, which remains constant in two consecutive symbols intervals.

lthe received signal can be written as:

$$y = \sqrt{\rho}\mathrm{H}\mathbf{x} + \mathbf{n} \tag{4}$$

where ρ is the average SNR at the each antenna, and n is the $2 \times N_r$ noise matrix. The entries of both **H** and n are independent an identically distributed (i.i.d) and $y = [y_1, \ldots, y_{L_R}]^T \in \mathbb{C}^{L_R x1}$ is the received signal vector, $x = [x_1, \ldots, x_{L_R}]^T \in \mathbb{C}^{L_R x1}$ is the transmitted symbol vector.

In the noisy channel here the BER is mainly expressed as a function of the normalized by E_b/N_0, (Energy per bit to the noise power spectral density ratio). Here in BPSK modulation and AWGN channel, the BER as the function of the E_b/N_0 is given by:

$$BER = \frac{1}{2}\mathrm{erf}(\sqrt{E_b/N_0}) \tag{5}$$

After these two encoding, our contribution is to set the detector MMSE-SIC to the output, the proposed algorithm is presented to the previous section. Simulation results of Bit Error Rate (BER) result versus Signal to Noise Ratio (SNR) shown that the MMSE-SIC is a good detector for short polar code. To support this theory, we compare our result to Maximum Likelihood Detector (MLD) as described by Zhao et al. in [6].

3.1 Review of MMSE-SIC

MMSE-SIC has been one of the most popular suboptimal detector for MIMO systems, which outperforms considerably Zero Forcing (ZF) and MMSE at the expense of a mild increase in the computational complexity [10]. See top of this page for detailed algorithm, in which $\bar{H}[i] \in \mathbb{C}^{(L_R-i+1)X(L_R-i+1)}$ represents the channel of the residual data stream at the i^{th} step and $\mathcal{F}$ denote the set of transmit antennas with the largest Signal to Noise Ratio (SNR). Specially at the i^{th} step, there are $(i-1)$ data stream to be cancelled in the previous steps.

The MMSE-SIC principle is to detect signal in one iteration by nulling out other co-Channel Interference. The idea is to use MMSE detector in order to exploit this combining weight matrix [10]. If the signal is detected, it is immediately fed back to the linear combining process and its contribution is cancelled from the received signal in the next detection iteration and found the next minimum MSE as shown [8–10].

Algorithm 1.MMSE-SIC

Initialization:

$$\begin{cases} \mathcal{F} = \emptyset \\ \boldsymbol{H}[1] = \boldsymbol{H} \end{cases}$$

Main Program

For i =1,…, L_T,

$P[i] = (\widehat{\mathbf{H}}^{\downarrow}[i]\widehat{\mathbf{H}}[i] + \sigma_n^2 IL_T - i + 1)^{-1}$

$SNR_{i,l} = \frac{1}{\sigma_n^2 (P[i])_{l,l}}$ -1

$l_i = \mathrm{argmax}_{\, l \in [1, L_T], l \notin A} SNR_{i,l}$

$\mathcal{F}[i] = l_i$

$\hat{b}_{l_i} = \mathrm{Dec}\left\{\left(P[i]\widehat{\mathbf{H}}^{\downarrow}[i]y\right)_{l_i}\right\}$

$y \leftarrow (y - \mathbf{H}_{l_i}\hat{x}_{l_i})$

$\widehat{\mathbf{H}}[i+1] \leftarrow \widehat{\mathbf{H}}[i]$ Without the l_i^{th} column

end

MMSE-SIC detects the data stream of the maximum SNR and subtracts a replica signal of the detected symbol from the received signal. This process is repeated until the last transmitted data is cancelled.

This algorithm allows us to determine the first BER and probably FER performances.

3.2 Noise Improving in MMSE-SIC

In this subsection, an analysis about the noise enhancement at each step of MMSE-SIC is proved [11]. We noted $C_n[i] \in \mathbb{C}^{(L_R - i+1)X(L_R - i+1)}$ as the covariance matrix of the noise compenent at the ith step. Then, we have

$$R_n[i] = \sigma_n^2 P[i] H^{\downarrow}[i]\hat{H}[i]P[i], \tag{6}$$

$$R_n[i] = \sigma_n^2 V^{\downarrow}[i] D^{-1}[i] V[i], \tag{7}$$

Where

$$D^{-1}[i] = Diag(\overline{\lambda}_1[i], \overline{\lambda}_2[i], \ldots, \overline{\lambda}_1 L_T - i + 1[i]) \tag{8}$$

$$\overline{\lambda}_l[i] = \frac{(\lambda_l[i] + \sigma_n^2)^2}{\lambda_l[i]} \tag{9}$$

With $\overline{\lambda}_l[i]$ satisfying

$$\overline{\lambda}_l[i] \geq \overline{\lambda}_l[i] \geq \ldots \geq \overline{\lambda}_1 L_T - i + 1[i] \tag{10}$$

And $V_n[i] \in \mathbb{C}^{(L_R - i+1)X(L_R - i+1)}$ denotes the unitary matrix that diagonalizes $H^{\downarrow}[i]\hat{H}[i]$ and $\overline{\lambda}_l[i]$ represents an eigenvalue of $H^{\downarrow}[i]\hat{H}[i]$. The number of negligeable $\overline{\lambda}_l[i]$ represents also the number of noise enhancement directions.

Simulation results of Bit Error Rate (BER) versus Signal to Noise Ratio (SNR) shown that the MMSE-SIC is a good detector for short polar code. To support this theory, we compute the FER performances and finally we compare our result to Maximum Likelihood Detector (MLD) as described by Zhao et al. in [6].

3.3 Maximum Likelihood (ML) Detector

The maximum likelihood detector provides BER performances of all MIMO detectors, but at exceedingly high complexity [6, 7]. For Q ary modulation and L_R transmit antenna system, the number of symbol set combination is Q^{L_R}, leading to exponential complexity. ML detection performs a search of the closet symbol combination sent to receiver therefore, its solution is given by:

$$\hat{y} = argmin_{y \in S^{L_T}} \|x - Hy\|^2 \quad (11)$$

4 Simulation Results

In this first section, we present the BER results for the proposed MMSE-SIC receiver using Polar-STBC.

In Fig. 2, BER performances is analysis at 6×10^{-2} for Polar-STBC using $N_t = 2$ and $N_r = 2$ vs STBC only using $N_t = 2$ and $N_r = 2$ and $N_t = 2$ and $N_r = 3$.

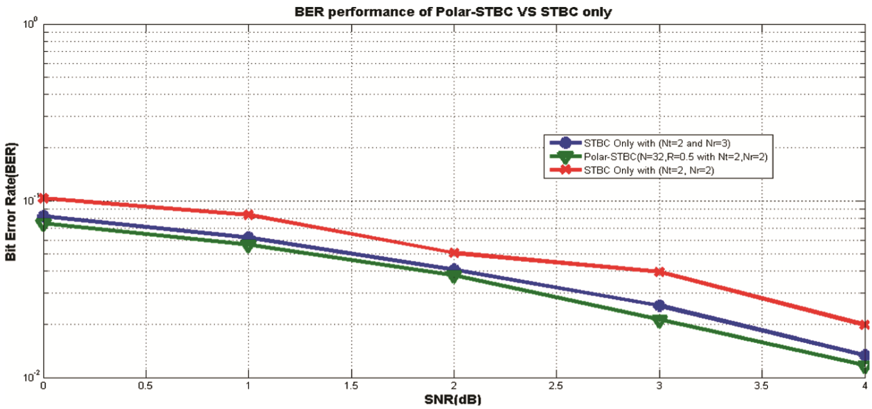

Fig. 2. BER performance between small polar-STBC and STBC only

We noted a big improvement when the BER is 6×10^{-2}, the SNR for Polar-STBC $2 * 2$ MIMO antennas is about 1 dB improvement over the STBC only using $N_t = 2$ and $N_r = 2$ and better than STBC only using $N_t = 2$ and $N_r = 3$ around 0.3 dB improvement.

At the other hand we introduces the frame error rate performance versus SNR per receiver antenna (e.g. $SNR = 2 * \frac{E_s}{N_0}$) in Fig. 3.

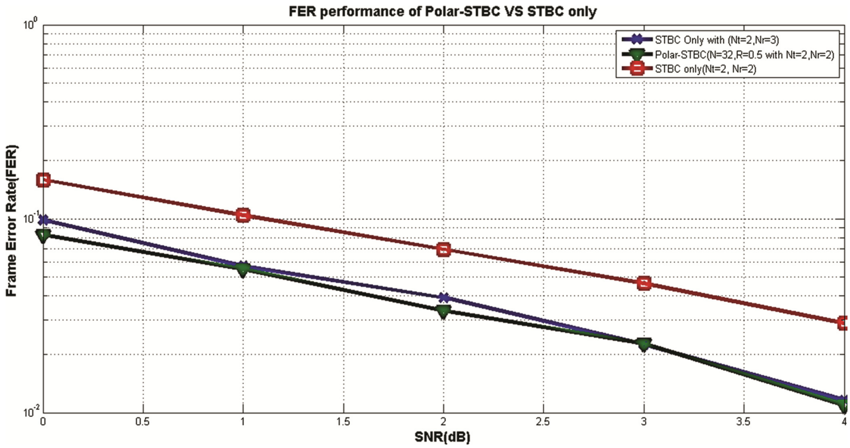

Fig. 3. FER performance between small polar-STBC and STBC only

The FER allows to judge efficiency of coding, more its weak, better it is. The FER represents the number of erroneous messages after detection on the number of transmitted messages.

Its shown in Fig. 3 that the FER at 6×10^{-2}, a slight improvement of Polar-STBC using $N_t = 2$ and $N_r = 2$ versus OSTBC only using $N_t = 2$ and $N_r = 2$ about 1.7 dB, but also outperform STBC only using $N_t = 2$ and $N_r = 3$ MIMO around 0.4 dB. These results illustrate that the STBC used in MIMO provide transmit diversity communication over fading channel, but also the coding gain is improved by using polar channel coding.

After simulation result, we compared to ML detection paper [6]. For instance, we added number of transmitters $N_t = 4$ and number receiver antennas $N_r = 4$.We observe that our proposed scheme concatenation significantly outperforms the polar-STBC with ML all over if the number of antenna is the same.

We noted that at BER $= 10^{-2}$ the proposed outperform to 1 dB compared to polar-STBC (N = 4096) with STBC (4 × 4 antennas) using ML detector which proves that our proposal is also a good candidate for MIMO antennas (Fig. 4).

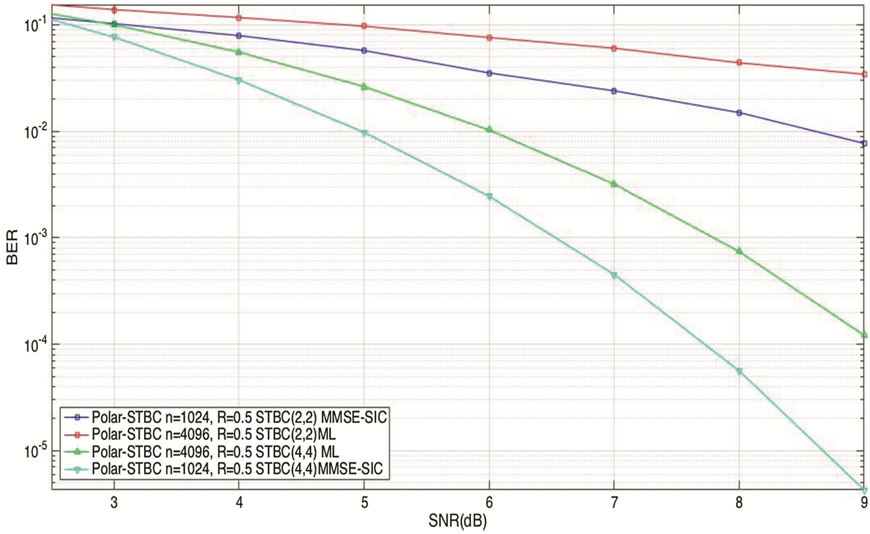

Fig. 4. BER performance concatenation Polar-STBC with detector ML and MMSE-SIC

5 Conclusion

In this paper, we presented a combination of STBC to short polar length codes when MMSE-SIC is applied to the detector. This soft output is compared to ML by improve the number of transmit and receiver's antennas. The proposed scheme permits to achieve near optimal BER performance over highly correlated performance MIMO channels at much reduced complexity. This proposition opens many perspectives such as architecture implementation for small Polar-STBC codes.

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