Power Allocation for NOMA System via Dual Sub-gradient Descent

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Abstract. Non-orthogonal multiple access (NOMA) has attracted great attention as a promising downlink multiple access technique for the next generation cellular networks (5G) due to its superior spectral efficiency. Power allocation of multi-user scenario in NOMA is a challenging issue and most of existing works focus on two-user scenario. In this work, we develop a dual sub-gradient descent algorithm based on Lagrange dual function to optimize multi-user power allocation for the multiple-input single-output (MISO) downlink NOMA system. The objective function is a non-convex optimization problem and we can solve it with a log-convex method and an approximation based approach. Numerical results demonstrate that the proposing scheme is able to achieve higher capacity performance for a NOMA transmission system compared with the traditional orthogonal multiple access (OMA) with a few iterations.

Keywords: Non-orthogonal multiple access · Power allocation
Log-convex · Dual sub-gradient descent

1 Introduction

The power domain based NOMA utilizes superposition coding (SC) [1] at transmitter and successive interference cancellation (SIC) [2] at the receiver to achieve a higher spectral efficiency. Compared with the contemporary orthogonal multiple access approaches, non-orthogonal communication techniques have more advantages, such as larger system capacity, higher spectral efficiency and reduced latency, making NOMA an attractive option for implementation in future wireless standards.

There are two key techniques contained in NOMA: (1) NOMA utilizes SIC to process the received signals on the user equipment where users are sorted based on their effective channel gains. At the receiving terminal, the stronger user can eliminate interference imposed by the weaker user through SIC operation. (2) NOMA is a multiplexing scheme which is applied in power domain. User de-multiplexing is

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ensured by large power difference between paired users. Theoretically, pairing the user having the worst channel gain with the user having the best channel gain in each cluster can obtain a better channel capacity. In a word, employing SC and SIC in NOMA can achieve a better system performance [3].

NOMA applying non-orthogonal transmission technique between the sub-channels, thus there is no apparent near-far effect and multiple access interference (MAI) compared with the traditional CDMA and OFDMA in 3G. However, it would be rather complex than other receivers as a non-orthogonal transmission receiver and the power-domain multiplexing is still on the researching stage, NOMA is facing some challenges in technical implementation, and there is still much of work to do.

Power allocation in NOMA has been studied in many existing works and has been extended to various systems and different schemes. In [4], a power allocation approach named Fair-NOMA is introduced. The key idea of Fair-NOMA is that the capacity of two mobile users has the opportunity to always achieve that of the OMA system. According to the Shannon capacity equation, if the capacity of NOMA is greater than or equal to OMA, a reasonable power allocation coefficient can be derived. In both single-input single-output (SISO) and multiple-input multiple-output (MIMO) scenarios, two optimal power allocation solutions with closed form based on the Karush-Kuhn-Tucker (KKT) condition have been studied in [5]. For MIMO-NOMA with layered transmission, Choi [6] explored an approach based on alternating maximization (AM) algorithm and showed that the sum rate optimization problem is concave in allocated powers to multiple layers of users. Energy efficient (EE) resource allocation problem has not been well studied for NOMA system until [7], where the author proposed a low-complexity suboptimal matching scheme for sub-channel assignment (SOMSA) algorithm to maximize the system energy efficiency and numerical simulation results in this work have shown that NOMA has much better sum rate and superior EE performance compared with OMA.

In [8] Ding utilized an approximation of the original non-convex optimization problem with a minorization-maximization algorithm (MMA) in downlink MISO-NOMA system. In each step of the MMA, the author utilized a second-order cone program to get a subset of the feasibility set of the original problem and the algorithm is numerically shown to converge within a few iterations. Finally, a linear multiuser superposition transmission (MUST) scheme is studied in [9], in which a Monte Carlo simulations based approach is devised to maximizing the total mutual information with a reasonable power allocation scheme.

Compared to the previous power allocation schemes in NOMA system, our main contributions can be summarized as follows:

(i) The prior studies of power allocation about NOMA have focused on two-user scenarios. In this paper, we will investigate multi-users power allocation problems in the downlink of a MISO-NOMA system. Furthermore, constraints are also included to guarantee the capacity of the weaker users fulfilling their target data rates and it should be noted that this power allocation scheme is not quality-of-service (QoS) guaranteed for the strong user because the minimum rate requirement of the strong user is always accessibility in our algorithm.

(ii) Using the log-convex concept, and combined with the Lagrange dual function, we develop a dual sub-gradient descent algorithm that solves the NOMA sum capacity maximization problems. We also show that the proposed algorithm is convergent in a few iterations.

(iii) We present an approximation to the original optimization problem to reduce the complexity of the proposed method. To provide more insight, we perform this approximation specifically designed for the characteristics of log-convex. Finally, numerical examples are presented to show the validity of the proposed algorithm.

The rest of the paper is organized as follows: The system model for downlink MISO-NOMA is outlined in Sect. 2. The based approach for the optimal power allocation to maximize the sum capacity is investigated in Sect. 3. Simulation results are presented in Sect. 4. Finally, Sect. 5 concludes the paper with some remarks and discussed the future works to be considered.

2 System Model

In this section, we present a multiple-input single-output (MISO) downlink NOMA transmission system. We consider that the base station (BS) in a cellular system is equipped with N antennas and K single antenna users. The channel gain between the k-th user and the BS is denoted by h_k. It is assumed that $h_k = d_k^{-\theta} g_k$, with d_k being the distance from the BS to the k-th user, where $g_k \sim CN(0, 1)$ and θ is the pass loss exponent. Furthermore, we assume that the distances from the users to the BS are fixed and users are equally spaced in the cellular system. We allocate powers for each user based on their channel state information (CSI) and user who have better channel gain will obtain lower power. Thus, the stronger users can detect the weaker users according to SIC at the receiving terminals. Generally, the distances are sorted as $d_1 \geq \ldots \geq d_k$. The channel gains of each user are sorted as $|h_1|^2 \leq \ldots \leq |h_k|^2$ and the powers of them are then allocated as $p_1 \geq \ldots \geq p_k$, accordingly. Based on NOMA transmission protocol, the BS will send $\sum_{k=1}^{K} \sqrt{p_k s_k}$ to each user, where the message for the k-th user is s_k, and p_k is the transmission power for user k. Therefore, the received signal at the k-th user is given as:

$$y_k = h_k \sum_{k=1}^{K} \sqrt{p_k} s_k + n_k, \tag{1}$$

Here n_k denotes the additive noise and $n_k \sim CN(0, \sigma^2)$. The k-th user signal to interference plus noise ratio (SINR) is

$$SINR_k = \frac{|h_k|^2 p_k}{|h_k|^2 \sum_{i=k+1}^{K} p_i + \sigma^2}, \tag{2}$$

The SINR of the K-th user is

$$SINR_K = \frac{|h_K|^2 p_K}{\sigma^2}. \tag{3}$$

The target rate for each user will be set as to achieve an acceptable QoS requirement. Therefore, for user k, we can define

$$\log_2(1 + SINR_k) \geq R_{target}. \tag{4}$$

This constraint provides a guarantee that all users can meet their QoS requirements.

3 Problem Formulation

In this section, a dual sub-gradient descent algorithm based on Lagrange dual function is used to maximize the sum capacity of NOMA system. Firstly, we express the optimization function as:

$$\sum_{k=1}^{K} R_k = \sum_{k=1}^{K} \log_2(1 + SINR_k) \tag{5}$$

To solve the optimization problem (5), a mathematical model is developed according to standardized Lagrange dual function as follows:

$$\begin{aligned} &\max \sum_{k=1}^{K} R_k \\ &s.t. \begin{cases} C1: R_k \geq R_{target} \\ C2: 0 \leq p_k \leq P,\ k = 1, 2..., K \\ C3: \sum_{k=1}^{K} p_k = P \end{cases} \end{aligned} \tag{6}$$

In this optimization problem, constraint C1 represents the fact that the capacity of each user must meet their corresponding QoS requirement, and constraint C2 reflects the NOMA principle that the power allocation of the weak user must be greater than that of the strong user. By defining A_k as the upper bound of interference plus noise, we can define that

$$|h_k|^2 \sum\nolimits_{i=k+1}^{K} p_i + \sigma^2 = A_k. \tag{7}$$

In NOMA, power allocation is of importance to enhance the achievable capacity of each user and it is a non-convex optimization problem. Inspired from the results presented in [10, 11], we devise a new power allocation algorithm named dual sub-gradient descent in the NOMA system based on Lagrange dual function. The core idea of Lagrange function is to embed constraint conditions into the objective function, adding weighted sum of the constraint conditions to obtain an augmented objective function. The Lagrange dual function is the minimum value of Lagrange function.

Even though the Lagrange function has no lower bound on x, the value of dual function is $-\infty$ [12]. The dual function is a kind of affine function on x, so even if the original question is non-convex, the dual function is still a concave function. Thus, on account of using log-convex algorithm, it is obvious that maximizing the sum capacity in (5) is equivalent to maximizing the objective function in (8).

$$\max \sum_{k=1}^{K} \ln\left(|h_k|^2 p_k A_k^{-1}\right) \tag{8}$$

Through logarithm transformation, we can define $p_k = e^{x_k}$, $A_k = e^{y_k}$ and substitute (8) into (6), then the mathematical model in (6) can be rewritten as:

$$\begin{aligned} &\max \sum_{k=1}^{K} \ln\left(e^{x_k - y_k + \ln(|h_k|^2)}\right) \\ &s.t. \begin{cases} C1: |h_k|^{-2} e^{y_k - x_k}\left(2^{R_{target}} - 1\right) - 1 \le 0 \\ C2: e^{x_k} P^{-1} - 1 \le 0, k = 1, 2\ldots, K \\ C3: P^{-1} \sum_{k=1}^{K} e^{x_k} - 1 = 0 \end{cases} \end{aligned} \tag{9}$$

Algorithm 1 Power Allocation with Dual Sub-gradient Descent Function

```
Niter:the iteration number
Unum:the total number of users
Initialization
Set λ_0 =0, μ_0 =0, γ_0 =0;
  Set P_1 ≥ P_2 ≥ P_3.
for n = 1 to Niter do
    for k = 1 to Unum do
        Set x_k = P_k and y_k = |h_k|^2 ·sum(P_{k+1} : P_3) + σ^2;
        λ_{n+1} = {λ_n + α(∂L(v)/∂λ)}^+;
        where α is the update step.
        μ_{n+1} = {μ_n + α(∂L(v)/∂μ)}^+;
        γ_{n+1} = {γ_n + α(∂L(v)/∂γ)}^+;
        where {z}^+ = max(0, z).
        x_{n+1} = min{x_{n+1} − α(∂L(v)/∂x), x_{n,max}};
        y_{n+1} = y_{n+1} − α(∂L(v)/∂y);
    end for
end for
    R_k = log_2{1 + |h_k|^2 exp(x_Niter)/exp(y_Niter)}.
  where h_k is the effective channel gain.
end procedure
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Proving the optimization model in (9) is a convex optimization problem, amounts to proving the objective function is being a convex function and all of the constraint inequalities are being the convex set of these optimization variables. Since the left side of these constraint inequalities in (9) are the sum of exponential functions after transformation, we can conclude that these constraint inequalities are the convex sets of optimization variables x and y. Meanwhile, the logarithmic function is a kind of monotone increasing function and the objective function after variable substitution is a convex function on x and y. Therefore, we can prove that the problem in (9) is a convex optimization problem.

To solve the above-mentioned problem, we adopt the convex optimization algorithm to obtain the globally optimal solution of this log-convex problem. Combined with the characteristics of NOMA system, we draw Lagrange multipliers into each communication link and assume that λ, μ, γ denote the Lagrange dual variables of formulae C1, C2 and C3, respectively. The corresponding Lagrange dual function is shown as follows:

$$\begin{aligned} L(x, y, \lambda, \mu, \gamma) = & -\sum_{k=1}^{K} \ln(e^{x_k - y_k + \ln(|h_k|^2)}) \\ & + \lambda_k[|h_k|^{-2} e^{y_k - x_k}(2^{R_{target}} - 1) - 1] \\ & + \mu_k(P^{-1} e^{x_k} - 1) + \gamma_k(P^{-1} \sum_{i=1}^{K} e^{x_k} - 1) \end{aligned} \tag{10}$$

Defining $v = \{x, y, \lambda, \mu, \gamma\}$ to express the optimization variables and Lagrange dual variables, we use grad $\nabla L(v)$ to iterate and update them until the algorithm converges. Detailed steps are given in Algorithm 1.

In Algorithm 1, we fix the total power of users and allocate it to each user according to their CSI by defining $x_k = p_k$, $y_k = |h_k|^2 \sum_{i=k+1}^{K} p_i + \sigma^2$ to initialize x_k and y_k, where $k = 1, 2, \ldots, K$. After the algorithm converges, we can obtain the optimal value of each user's power p_k and interference plus noise A_k through inverse the transformation. Finally, we use the Shannon Formula to compute each user's capacity.

4 Simulation Results

In this section, we investigate the performance of the proposed method to the NOMA power allocation problem. It should be noted that in simulations the user distances are fixed, we adopt the common path-loss model with pass-loss exponent $\theta = 2$ for a fading channel, where the mean value of each user's channel is 0 and whose variance is taken to be unity.

For comparison, we also consider OMA transmission as a reference scheme. In [13], we studied a kind of power allocation scheme in the TDMA system which requires K time slots to support K users, while NOMA can support K users during a single time slot. Thus, the achievable rate of user k in OMA system is given as

$$R_{k,OMA} = a(k)\log_2(1 + \frac{b(k)P|h_k|^2}{a(k)\sigma^2}) \tag{11}$$

where $a(k)$ and $b(k)$ are the time division weighting coefficient and power allocation coefficient for user k, respectively, P is the total power of all users and $|h_k|^2$ is the k-th user effective channel gain. Then we assume that $a(k) = b(k)$, which yields a modified format as

$$R_{k,OMA} = a(k)\log_2(1 + \frac{P|h_k|^2}{\sigma^2}) \tag{12}$$

The sum rate of OMA system is obtained via full search to meet the user rate requirements as well as to maximize the system capacity.

4.1 Convergence Verification

In [5], the KKT algorithm needs $2N_R N_T + 4$ and $2N_R N_T + 8$ iterations to calculate the value of minimum and maximum power coefficients in two users scenario, where N_R and N_T are the numbers of antennas equipped at the BS and mobile users, respectively. In this paper, we set that $N_T = 3$ and $N_R = 1$, thus the iteration number is 24 with KKT algorithm. In the proposed method, the calculation of the Lagrange dual variables λ, μ, γ, results of power allocation and rate for each user using less than 50 iterations for the three users' scenario. The results are shown in Figs. 1, 2, 3, 4 and 5, where the update step $\alpha = 0.1$.

Figures 1, 2 and 3 depict the convergence features of Lagrange dual variables λ, μ, γ. In Fig. 1, the value of lamda3 is always be zero due to which always being smaller than or equal to zero. In Fig. 3, the value of γ in each communication link is the same because γ is the dual variable of formula C3 in (9), which is a constraint of sum transmit power and is unrelated to an individual user power.

Figure 4 provides the characteristics of the convergent power allocation, where UE1 is the weakest user and UE3 is the strongest user, and it is clearly depicting that UE1 gets more power than others. UE 3 is allocated the lowest power, simultaneously. In Fig. 5, it is shown that UE1 has the lowest user rate because it is the weakest user and UE3 has the highest rate. Both of the power allocation and user rate have good convergent property.

In Fig. 6, the solid lines give the shapes of the target function R_k, where Capacity1 represents user one's capacity, and the remaining are the same. The dotted lines express the value of dual function for each user, where Lower Bound 1 represents the lower bound of dual function for user one. Since dual function gives the lowest bound of Lagrange function, which is always smaller than the target function R_k. Therefore, we can conclude that the solutions of our target function are in the feasibility region.

Furthermore, after those Lagrange dual variables λ, μ, γ converging, we substituted them into the original optimal model (9) and proved that all constraints are satisfied, which means that the algorithm we proposed is exact in calculation.

4.2 Comparison with OMA

In this subsection, we provide some simulation results to evaluate the system performance of the proposed power allocation algorithm. All the simulations are conducted by averaging 10^5 channel realizations to guarantee the accuracy of the proposed algorithm.

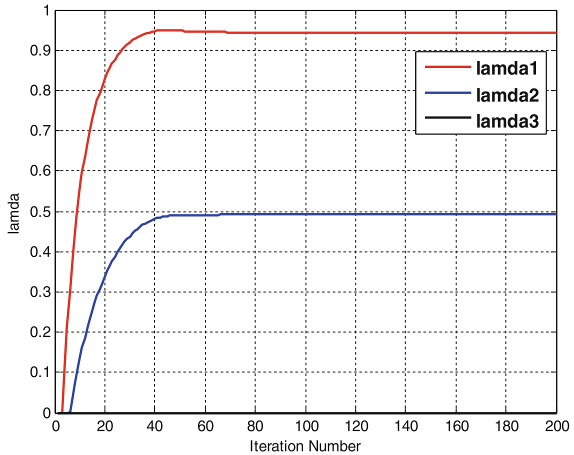

Fig. 1. Convergence behavior of Lagrange dual variable λ

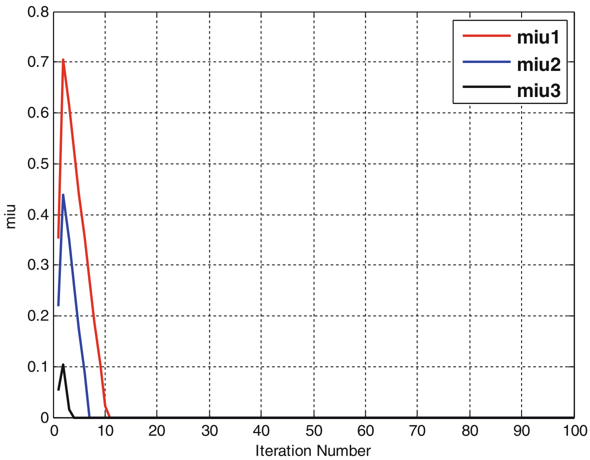

Fig. 2. Convergence behavior of Lagrange dual variable μ

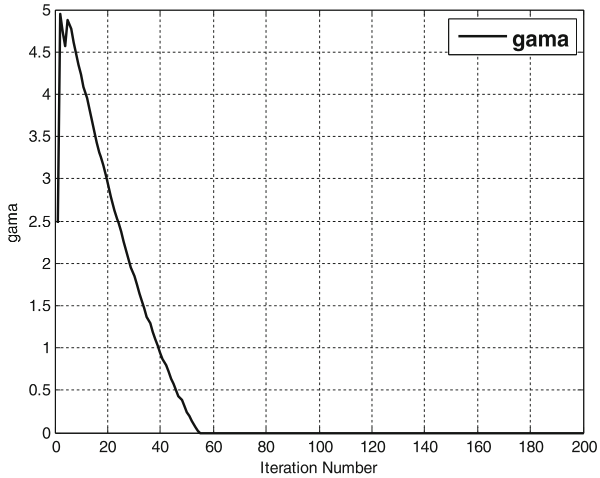

Fig. 3. Convergence behavior of Lagrange dual variable γ

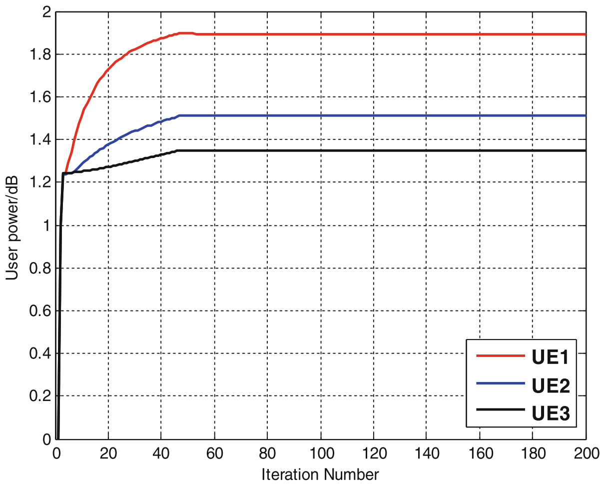

Fig. 4. Convergence behavior of user allocated powers

Figure 7 provides a comparison for the proposed NOMA system power allocation algorithm and traditional OMA communication scheme by depicting the achievable ergodic capacity of one user or the sum capacity of all users versus transmit signal to noise ratio (TX-SNR), where NOMA-UE1 and NOMA-UE3 denote the capacities of strongest user and the weakest user, respectively, achieved by the proposed NOMA. Similarly, OMA-UE1 and OMA-UE3 represent those for the OMA system; where NOMA Sum-Capacity and OMA Sum-Capacity represent the sum capacity in NOMA and OMA system, respectively. Figure 7 demonstrates that the performance of the NOMA system is better than the performance of an OMA system.

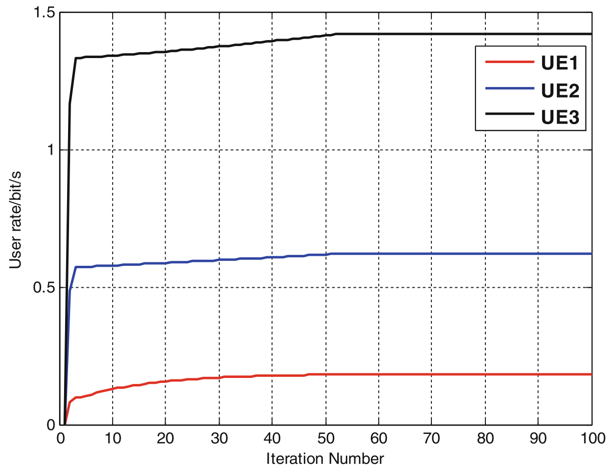

Fig. 5. Convergence behavior of user rates

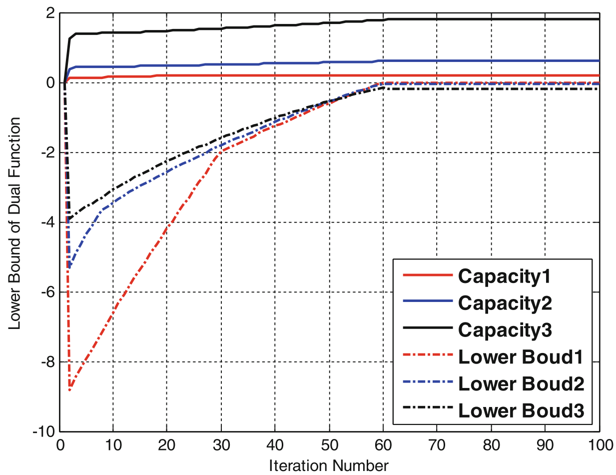

Fig. 6. The lower bound of dual function compared with target function R_k.

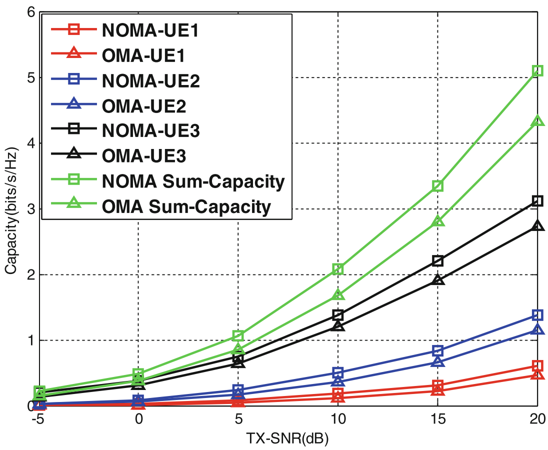

Fig. 7. The ergodic capacity of one user or the sum capacity versus TX-SNR for the proposed NOMA scheme compared with the OMA scheme.

5 Conclusions

This work proposed a dual sub-gradient descent algorithm based on Lagrange dual function and approximation-function based approach to optimize the power allocation for downlink NOMA systems. Numerical results illustrate that the proposed approximation-function based method can significantly speed up the convergence. Simulation results show that the NOMA system based on the proposed power allocation scheme outperforms OMA system by $20\% - 25\%$ in terms of sum capacity. It must be stressed that the extension of this proposed scheme to other difficult non-convex optimization problems is straightforward.

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