Low-Complexity Equalization of Continuous Phase Modulation Using Message Passing

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Abstract. In this paper, based on factor graphs and Laurent decompositions, we propose an iterative receiver of CPM signals over interference (ISI) channels. We adopt the Gaussian message passing to simplify the message passing in factor graphs. Compared with the conventional receivers with the minimum mean squared error (MMSE) frequency domain equalization (FDE) and the BCJR demodulator, the proposed algorithm has advantages in terms of complexity. And the proposed algorithm can achieve better performance with the convolutional code.

Keywords: Continuous phase modulation · Iterative receivers
Factor graphs · ISI

1 Introduction

Continuous phase modulation (CPM) is a nonlinear modulation in which the phase is a continuous function of time [1]. CPM is attractive for its good spectral efficiency and constant envelope. However, the optimal receiver of CPM using maximum-likelihood sequence detection (MLSD) is consists of a filter bank followed by a Viterbi decoder. As a consequence, the optimal receiver of M-ary CPM signals requires a bank of $2M^L$ matched filters and a trellis diagram with pM^{L-1} states, where L is the memory of the CPM pulse and p is the number of phase states.

Considering the practical applications of CPM, several methods have been proposed in the literature to reduce the complexity of the receiver, such as Rimoldi decomposition [2], Walsh decomposition [3] and Laurent decomposition (LD) [4]. In [4], by Laurent decomposition (LD), the binary CPM signal is decomposed into a sum of pulse amplitude modulation (PAM) signals. Moreover, Mengali and Morelli extend the LD from binary CPM signals to M-ary CPM signals [5]. The LD can significantly reduce both the number of matched filters and the number of states in trellis diagram, and is more popular used than the other methods. Such as, a reduced-complexity suboptimal detection of CPM signals is proposed based on the extended Laurent representation in [6].

To mitigate the inter-symbol interference (ISI) caused by multipath environments, the equalization is necessary at the receiver to mitigate the ISI for CPM. The optimum receiver of CPM with ISI is a kind of maximum likelihood sequence estimation

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(MLSE) receiver in the time domain. The complexity of this receiver is effected by both the memory of CPM signals and the delay spread of the multipath channel. As the complexity grows exponentially with the length of the spread, the MLSE receiver is unfeasible for the channel with long delay multipath taps. To solve this problem, the frequency domain equalization (FDE) approach is extended to the CPM scenarios. A series of literature concerned on the study of FDE in CPM receivers appears since 2000s. In 2005, Tan and Stuber study the application of linear single-carrier frequency-domain equalization (SC-FDE) to CPM [7]. Their method can provide better BER performance and lower complexity cost than the optimal receivers for multipath channels having long delay components. A year later, Pancaldi and Vitetta combined the frequency-domain equalization and iterative information exchange [8]. In [9], a frequency domain double turbo equalizer of CPM is proposed by combining the soft-input soft-output (SISO) FDE, the SISO CPM demodulator and the SISO decoder.

Iterative receivers based on factor graphs (FG) and the sum-product algorithm (SPA) have been widely used in linear modulation scenarios to solve the problem of the inter-symbol interference. Moreover, various approximate inference algorithms have been proposed to reduce the complexity of message passing in graphical models, such as the Gaussian message passing (GMP) [10], the expectation propagation [11–13]. However, less attention has been devoted to the application of this method to CPM scenario. In [14], the FG and SPA is used in the detection of CPM signals over channel affected by phase noise.

In this paper, we consider the equalization and detection of CPM signals over multipath channels based on factor graphs and the SPA. By using the Laurent decomposition and the Gaussian message passing, we proposed a kind of low-complexity time-domain turbo equalization of CPM signals. It will be shown that the designed receivers have similar performance with respect to the optimal detectors regardless of the code part. Considering the convolutional code, it has better performance than the MMSE-FDE and optimal detectors.

The paper is organized as follows. In Sect. 2, we describe the system model including the signal model of CPM based on the Laurent decomposition and the model of received signals after multipath channel. The system model is used in Sect. 3 to realize the low-complexity turbo equalization. Section 4 shows the simulation results and conclusions are drawn in Sect. 5 finally.

2 Signal Model

In this paper, only single modulation index CPM is considered. In general, a CPM signal can be expressed as [1]:

$$s(t,\vec{a}) = \sqrt{\frac{2E_s}{T}}\exp\{j2\pi h\sum_n a_n q(t-nT)\} \tag{1}$$

where E_s is the energy per symbol, T is the symbol period, $h = r/p$ is the modulation index (r and p are relatively prime integers), $\{a_n\}$ are the transmitted information symbols, $a_n \in \{\pm 1, \pm 3, \ldots, \pm M-1\}$, $n = 0, \ldots, N-1$. The function $q(t)$ is the phase response and has the form as:

$$q(t) = \begin{cases} 0, & \mathit{if}\ t \le 0 \\ \int_0^t g(\tau)d\tau, & \mathit{if}\ 0 < t < LT \\ 1/2, & \mathit{if}\ t \ge LT \end{cases} \tag{2}$$

where the $g(\tau)$ is the frequency pulse defined over a finite time interval $0 \le t \le LT$.

Exploiting the extended Laurent decomposition [4, 5], the CPM signals can be expressed as a sum of linearly modulated signals:

$$s(t, \vec{a}) = \sum_{k=0}^{K-1} \sum_n \alpha_{k,n} p_k(t - nT) \tag{3}$$

in which $K \le Q^{\log_2 M}(M-1)$, $Q = 2^{L-1}$ and $p_k(t)$ is the k th PAM component and the symbols $\{\alpha_{k,n}\}$ are the function of the transmitted information symbols $\{a_n\}$. The Laurent decomposition in (3) can exactly express the CPM signals when $K = Q^{\log_2 M}(\mathrm{M}-1)$. Considering most of the signal power is concentrated on the first $M-1$ PAM components, a value of $K = M-1$ is usually be used to attain a good tradeoff between the complexity of the system and the approximation quality of Laurent decomposition. In this paper, we only consider the first PAM component to further reduce the complexity of the receiver. As a consequence, we obtain an approximation of $s(t, \vec{a})$:

$$s(t, \vec{a}) \approx \sum_n \alpha_{0,n} p_0(t - nT). \tag{4}$$

As shown in [6], the symbol $\alpha_{0,n}$ can be expressed as a function of $\alpha_{0,n-1}$ and a_n:

$$\alpha_{0,n} = \alpha_{0,n-1} e^{j\pi h a_n}. \tag{5}$$

Furthermore, we employ an equivalent representation of information symbols:

$$\overline{a_n} = \frac{a_n + (M-1)}{2} \tag{6}$$

in which $\overline{a_n} \in \{0, 1, \ldots, M-1\}$. Substituting (6) into (5), we can get a new expression about $\alpha_{0,n}$:

$$\alpha_{0,n} = e^{-j\pi h(M-1)(n+1)} e^{j2\pi h \phi_n} \tag{7}$$

$$\phi_n = [\phi_{n-1} + \overline{a_n}]_p \tag{8}$$

where ϕ_n is the accumulation of phase, $\phi_n \in \{0, 1, \ldots, p-1\}$ and $[\cdot]_p$ denotes the "modulo p" operator.

Considering the multipath channels, the received signals can be expressed as:

$$r(t) = \sum_{l=0}^{N_L-1} h_l s(t - \tau_l T) + w(t) \tag{9}$$

where h_l and τ_l ($\{\tau_l\}$ are positive integers in this paper) are the gain and the symbol number of the propagation delay for the lth path and N_L is the number of channel paths. The function $w(t)$ is complex-valued Additive White Gaussian Noise(AWGN) with variance σ_w^2. Exploiting the Laurent decomposition of CPM signals, a discrete-time expression of received signals can be written as:

$$r_n = \sum_{l=0}^{N_L-1} h_l s_{n-\tau_l} + w_n \tag{10}$$

$$r_n \triangleq \int r(t) p_0(t - nT)dt \tag{11}$$

$$s_n \triangleq \int s(t) p_0(t - nT)dt = \sum_{m=0}^{N-1} \int p_0(t - mT) p_0(t - nT) dt \alpha_{0,m} \tag{12}$$

where w_n is assumed to be independent identical distributed Gaussian sequence with variance σ_n^2. As shown in [5], $p_0(t)$ is the first component of pulse amplitude modulation (PAM) signals. The integral value of $\int p_0(t - mT)p_0(t - nT)dt$ varies with the value of $|m - n|$ which is maximum when $|m - n|$ is zero and takes zero when $|m - n|$ is big enough. Based on this property, we can further simplify the expression (12):

$$s_n \approx \sum_{m=n-L_m}^{n+L_m} \int p_0(t - mT) p_0(t - nT) dt \alpha_{0,m} = \sum_{m=n-L_m}^{n+L_m} p_{|m-n|} \alpha_{0,m} \tag{13}$$

where L_m is an integer associated with M and L.

3 The Proposed Algorithm

Based on the signal model mentioned in Sect. 2, we derive the factor graph based receiver of CPM signals in this section. Our goal is to restore the transmitted information bits $\mathbf{b}$ from the received signals $\mathbf{r}$. Generally, we use the Maximum a Posteriori (MAP) strategy to estimate the information sequence:

$$\widehat{b_i} = \arg\max_{b_i} p(b_i|\mathbf{r}) \tag{14}$$

where b_i denotes the i th information bit and $p(b_i|\mathbf{r})$ is the marginal probability mass function of the joint posterior probability distribution $p(\mathbf{b}|\mathbf{r})$. According to the Bayesian rule, $p(\mathbf{b}|\mathbf{r})$ can be expressed as:

$$p(\mathbf{b}|\mathbf{r}) \propto p(\mathbf{r}|\mathbf{s})p(\mathbf{s}|\boldsymbol{\alpha})p(\boldsymbol{\alpha}|\boldsymbol{\phi})p(\boldsymbol{\phi}|\mathbf{a})p(\mathbf{a}|\mathbf{c})p(\mathbf{c}|\mathbf{b})p(\mathbf{b}) \tag{15}$$

where $\propto$ denotes proportionality and $\mathbf{c}$ is the code bits. The element c_n^q denotes the q th information bit of the n th symbol a_n. Using (10) in Sect. 2, the conditional probability $p(\mathbf{r}|\mathbf{s})$ can be factorized into:

$$p(\mathbf{r}|\mathbf{s}) = \prod_n f_n(r_n|\mathbf{s}) = \prod_n \exp\left\{-\frac{\left(r_n - \sum_{l=0}^{N_L-1} h_l s_{n-\tau_l}\right)^2}{\sigma_n^2}\right\} \tag{16}$$

in which $n = 0, 1, \ldots, N-1$ and we assumed the channel is already known to the receiver – in other words, the h_l and τ_l are already known. Similarly, we can get the factorization of the conditional probabilities $p(\mathbf{s}|\boldsymbol{\alpha})$,$p(\boldsymbol{\alpha}|\boldsymbol{\phi})$ and $p(\boldsymbol{\phi}|\mathbf{a})$ using Eqs. (13), (7) and (8)

$$p(\mathbf{s}|\boldsymbol{\alpha}) = \prod_n g_n(s_n, \boldsymbol{\alpha}) = \prod_n \delta\left(s_n - \sum_{m=n-L_m}^{n+L_m} p_{|m-n|}\alpha_{0,m}\right) \tag{17}$$

$$p(\boldsymbol{\alpha}|\boldsymbol{\phi}) = \prod_n I_n\left(\alpha_{0,n}, \phi_n\right) = \prod_n \delta\left(\alpha_{0,n} - e^{-j\pi h(M-1)(n+1)} e^{j2\pi h\phi_n}\right) \tag{18}$$

$$p(\boldsymbol{\phi}|\mathbf{a}) = \prod_n J_n(a_n, \phi_n, \phi_{n-1}) \tag{19}$$

in which $g_n(\cdot)$, $I_n(\cdot)$ and $J_n(\cdot)$ are the indicator functions and $\{a_n\}$ are assumed to be unipolar symbols, $a_n \in \{0, 1, \ldots, M-1\}$. The conditional probability $p(\mathbf{a}|\mathbf{c})$ in (15) can be factorized into

$$p(\mathbf{a}|\mathbf{c}) = \prod_n p(a_n|\mathbf{c_n}) = \prod_n \delta(a_n - \varphi(\mathbf{c_n})) \tag{20}$$

where $\delta(\cdot)$ is the Kronecker delta function and $\varphi(\mathbf{c_n})$ is the mapping function and $\mathbf{c_n}$ is comprised of $\left\{c_n^q, \forall q\right\}$, $q = 0, 1, \ldots, Q$,$Q = \log_2 M - 1$.

According to the factorization (16)–(20), we can get the factor graph representation of the receiver, as depicted in Fig. 1. In Fig. 1, f_n denotes the channel transition function $f_n(r_n|\mathbf{s})$, g_n denotes the Laurent decomposition constraint $g_n(s_n, \boldsymbol{\alpha})$, M_n denotes the mapping constraint $p(a_n|\mathbf{c_n})$,$P(\phi_{-1})$ and $P(\phi_{N-1})$ denotes the initial probabilities of the variables ϕ_{-1} and ϕ_{N-1},$P(\phi_{-1}) = P(\phi_{N-1}) = 1/p$.

Given the factor graph representation, the marginals can be computed exactly by message passing. Before study the detailed message computation, we introduce the representation of messages that follows. The messages passing between the function nodes A and variable nodes B of the ith iteration are denoted as $\mu^i_{A\to B}(B)$ and $\mu^i_{B\to A}(B)$. For example, the messages passing between the nodes $\{f_n\}$ and $\{s_n\}$ are denoted as $\mu^i_{f_n\to s_k}(s_k)$ and $\mu^i_{s_n\to f_t}(s_n)$ respectively, $k = n - \tau_l$, $t = n + \tau_l$, $l = 0, \ldots, N_L - 1$, $n = 0, \ldots, N-1$.

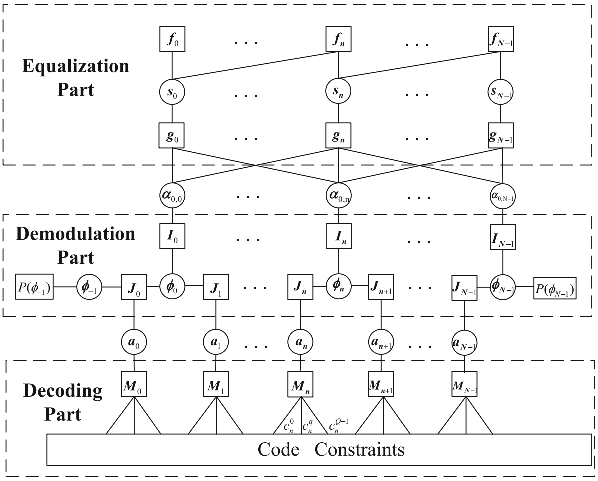

Fig. 1. Factor graph representation.

Firstly, we focus on the equalization part of the factor graph. By applying the updating rules of the SPA in the factor graph, messages $\mu^i_{f_n \to s_k}(s_k)$ and $\mu^i_{s_n \to f_t}(s_n)$ can be calculated by

$$\mu^i_{s_n \to f_t}(s_n) = \prod_{t' \neq t} \mu^{i-1}_{f_{t'} \to s_n}(s_n) \mu^i_{g_n \to s_n}(s_n) \tag{21}$$

$$\mu^i_{f_n \to s_k}(s_k) = \int_{\overrightarrow{s}/s_k} f_n(\overrightarrow{s}) \prod_{k' \neq k} \mu^i_{s_{k'} \to f_n}(s_{k'}). \tag{22}$$

However, a direct computation of Eqs. (21) and (22) is intractable for high-dimensional integral. Hence, we derive the reduced-complexity receiver based on the Gaussian message passing method.

Considering the practical application scenarios, we can assume that s_n is a continuous complex Gaussian random variable. According to the Eq. (10), w_n is a Gaussian variable, and then $(r_n - \sum_{l=0}^{N_L-1} h_l s_{n-\tau_l})$ is also a Gaussian variable. We can reasonably assume that the messages $\mu^i_{f_n \to s_k}(s_k)$ and $\mu^i_{s_n \to f_m}(s_n)$ are approximated as Gaussian density function:

$$\mu^i_{s_n \to f_t}(s_n) = N_C(s_n; x^i_{s_n \to f_t}, v^i_{s_n \to f_t}) \tag{23}$$

$$\mu^i_{f_n \to s_k}(s_k) = N_C(h_l s_k; x^i_{f_n \to s_k}, v^i_{f_n \to s_k}) \tag{24}$$

where $x^i_{s_n \to f_t}$ and $v^i_{s_n \to f_t}$ respectively denote the mean and the variance of the variable s_n with respect to the message $\mu^i_{s_n \to f_t}(s_n)$, $x^i_{f_n \to s_k}$ and $v^i_{f_n \to s_k}$ denote the mean and the variance of the variable $h_l s_k$ with respect to the message $\mu^i_{f_n \to s_k}(s_k)$. According to

Eqs. (13) and (17), considering the linear relationship between variables s_n and $\alpha_{0,n}$, we can assume $\alpha_{0,n}$ is a continuous complex Gaussian random variable too. According to SPA, the messages passing between nodes s_n and g_n can be express as:

$$\mu^i_{s_n \to g_n}(s_n) = N_C(s_n; x^i_{s_n \to g_n}, v^i_{s_n \to g_n}) \tag{25}$$

$$\mu^i_{g_n \to s_n}(s_n) = N_C(s_n; x^i_{g_n \to s_n}, v^i_{g_n \to s_n}) . \tag{26}$$

Similarly, the messages passing between nodes $\alpha_{0,n}$ and g_n can be express as:

$$\mu^i_{\alpha_{0,\mathrm{n}} \to g_j}(\alpha_{0,\mathrm{n}}) = N_C(\alpha_{0,\mathrm{n}}; x^i_{\alpha_{0,\mathrm{n}} \to g_j}, v^i_{\alpha_{0,\mathrm{n}} \to g_j}) \tag{27}$$

$$\mu^i_{g_n \to \alpha_{0,\mathrm{m}}}(\alpha_{0,\mathrm{m}}) = N_C(p_{|m-n|}\alpha_{0,\mathrm{m}}; x^i_{g_n \to \alpha_{0,\mathrm{m}}}, v^i_{g_n \to \alpha_{0,\mathrm{m}}}) \tag{28}$$

where $n = 0, \ldots, N-1$, $n - L_m \le j \le n + L_m$, $n - L_m \le m \le n + L_m$.

According to the properties of Gaussian distribution and the rules of SPA, the means and the variances mentioned above can be exactly calculated as:

$$x^i_{f_n \to s_k} = r_n - \sum_{l' \ne l} h_{l'} x^i_{s_{n-\tau_{l'}} \to f_n},\; v^i_{f_n \to s_k} = \sigma_n^2 + \sum_{l' \ne l} |h_{l'}|^2 v^i_{s_{n-\tau_{l'}} \to f_n} \tag{29}$$

$$v^i_{s_n \to f_t} = \left(\frac{1}{u^i_{g_n \to s_n}} + \sum_{l' \ne l} \frac{|h_{l'}|^2}{v^{i-1}_{f_{n+\tau_{l'}} \to s_n}} \right)^{-1},\; x^i_{s_n \to f_t} = v^i_{s_n \to f_t} \left(\frac{x^i_{g_n \to s_n}}{v^i_{g_n \to s_n}} + \sum_{l' \ne l} \frac{|h_{l'}| x^{i-1}_{f_{n+\tau_{l'}} \to s_n}}{v^{i-1}_{f_{n+\tau_{l'}} \to s_n}} \right) \tag{30}$$

$$v^i_{s_n \to g_n} = \left(\sum_l \frac{|h_l|^2}{v^i_{f_{n+\tau_l} \to s_n}} \right)^{-1},\; x^i_{s_n \to g_n} = v^i_{s_n \to g_n} \sum_l \frac{h_l x^i_{f_{n+\tau_l} \to s_n}}{v^i_{f_{n+\tau_l} \to s_n}} \tag{31}$$

$$x^i_{g_n \to s_n} = \sum_{m=n-L_m}^{n+L_m} p_{|m-n|} x^i_{\alpha_{0,m} \to g_n},\; v^i_{g_n \to s_n} = \sum_{m=n-L_m}^{n+L_m} \left| p_{|m-n|} \right|^2 v^i_{\alpha_{0,m} \to g_n} \tag{32}$$

$$x^i_{g_n \to \alpha_{0,m}} = x^i_{s_n \to g_n} - \sum_{m' \ne m} p_{|m'-n|} y^{i-1}_{\alpha_{0,m'} \to g_n},\; v^i_{g_n \to \alpha_{0,m}} = v^i_{s_n \to g_n} + \sum_{m' \ne m} \left| p_{|m'-n|} \right|^2 v^{i-1}_{\alpha_{0,m'} \to g_n} \tag{33}$$

$$v^i_{\alpha_{0,n} \to g_j} = \left(\frac{1}{v^i_{I_n \to \alpha_{0,n}}} + \sum_{j' \ne j} \frac{\left| p_{|j'-n|} \right|^2}{v^i_{g_{j'} \to \alpha_{0,n}}} \right)^{-1},\; x^i_{\alpha_{0,n} \to g_j} = v^i_{\alpha_{0,n} \to g_j} \left(\frac{x^i_{I_n \to \alpha_{0,n}}}{v^i_{I_n \to \alpha_{0,n}}} + \sum_{j' \ne j} \frac{p_{|j'-n|} x^i_{g_{j'} \to \alpha_{0,n}}}{v^i_{g_{j'} \to \alpha_{0,n}}} \right) \tag{34}$$

where $p_{|m-n|} \triangleq \int p_0(t - mT) p_0(t - nT) dt$.

Next we study the demodulation part of the factor graph in Fig. 1. By applying the rules of SPA continually, the message $\mu^i_{\alpha_{0,n}\to I_n}(\alpha_{0,n})$ can be computed:

$$\mu^i_{\alpha_{0,n}\to I_n}(\alpha_{0,n}) = \prod_j \mu^i_{g_j\to\alpha_{0,n}}(\alpha_{0,n}). \tag{35}$$

Considering the equations in (34), we can further compute the message $\mu^i_{\alpha_{0,n}\to I_n}(\alpha_{0,n})$ as:

$$v^i_{\alpha_{0,n}\to I_n} = \left(\sum_j \frac{\left|p_{|j-n|}\right|^2}{v^i_{g_j\to\alpha_{0,n}}}\right)^{-1},\ x^i_{\alpha_{0,n}\to I_n} = v^i_{\alpha_{0,n}\to I_n}\left(\sum_j \frac{p_{|j-n|}x^i_{g_j\to\alpha_{0,n}}}{v^i_{g_j\to\alpha_{0,n}}}\right) \tag{36}$$

where $x^i_{\alpha_{0,n}\to I_n}$ and $v^i_{\alpha_{0,n}\to I_n}$ are the mean and the variance of the message $\mu^i_{\alpha_{0,n}\to I_n}(\alpha_{0,n})$.

Before computing the message $\mu^i_{\phi_n\to I_n}(\phi_n)$, we focus on the value of the variable $\alpha_{0,n}$. According to the Eq. (7), $\alpha_{0,n}$ is actually a discrete random variable with p values. The probability distribution of $\alpha_{0,n}$ can be calculated by

$$P_\downarrow(\alpha_{0,n}) = \frac{N_C(\alpha_{0,n}; x^i_{\alpha_{0,n}\to I_n}, v^i_{\alpha_{0,n}\to I_n})}{\sum_{\alpha_{0,n}} N_C(\alpha_{0,n}; x^i_{\alpha_{0,n}\to I_n}, v^i_{\alpha_{0,n}\to I_n})}. \tag{37}$$

By applying the updating rules of the SPA, messages with reference to the demodulation part can be recursively computed as:

$$\mu^i_{I_n\to\phi_n}(\phi_n) = \sum_{\alpha_{0,n}} I_n(\alpha_{0,n},\phi_n)\mu^i_{\alpha_{0,n}\to I_n}(\alpha_{0,n}) \tag{38}$$

$$\mu^i_{\phi_n\to I_n}(\phi_n) = \mu^i_{J_n\to\phi_n}(\phi_n)\mu^i_{J_{n+1}\to\phi_n}(\phi_n) \tag{39}$$

$$\mu^i_{I_n\to\alpha_{0,n}}(\alpha_{0,n}) = \sum_{\phi_n} I_n(\alpha_{0,n},\phi_n)\mu^i_{\phi_n\to I_n}(\phi_n) \tag{40}$$

$$\mu^i_{J_n\to\phi_n}(\phi_n) = \sum_{a_n}\sum_{\phi_{n-1}} J_n(a_n,\phi_n,\phi_{n-1})\mu^{i-1}_{a_n\to J_n}(a_n)\mu^i_{\phi_{n-1}\to J_n}(\phi_{n-1}) \tag{41}$$

$$\mu^i_{\phi_n\to J_{n+1}}(\phi_n) = \mu^i_{J_n\to\phi_n}(\phi_n)\mu^i_{I_n\to\phi_n}(\phi_n) \tag{42}$$

$$\mu^i_{J_n\to\phi_{n-1}}(\phi_{n-1}) = \sum_{a_n}\sum_{\phi_n} J_n(a_n,\phi_n,\phi_{n-1})\mu^{i-1}_{a_n\to J_n}(a_n)\mu^i_{\phi_n\to J_n}(\phi_n) \tag{43}$$

$$\mu^i_{\phi_n\to J_n}(\phi_n) = \mu^i_{J_{n+1}\to\phi_n}(\phi_n)\mu^i_{I_n\to\phi_n}(\phi_n) \tag{44}$$

$$\mu^i_{J_n\to a_n}(a_n) = \sum_{\phi_n}\sum_{\phi_{n-1}} J_n(a_n,\phi_n,\phi_{n-1})\mu^i_{\phi_n\to J_n}(\phi_n)\mu^i_{\phi_{n-1}\to J_n}(\phi_{n-1}) \tag{45}$$

In Eqs. (41) and (44), the messages $\mu^i_{\phi_{n-1}\to J_n}(\phi_{n-1})$ and $\mu^i_{J_{n+1}\to\phi_n}(\phi_n)$ have the following initial conditions:

$$\mu^i_{\phi_{-1}\to J_0}(\phi_{-1}) = \mu^i_{J_N\to\phi_{N-1}}(\phi_{N-1}) = 1/p. \tag{46}$$

In Eq. (38), the message $\mu^i_{I_n\to\phi_n}(\phi_n)$ is a discrete probability distribution function about the variable ϕ_n and can be computed just like the means in Eq. (37). As for the message $\mu^i_{I_n\to\alpha_{0,n}}(\alpha_{0,n})$ in Eq. (40), it can be approximated as Gaussian density function and can be computed using the message $\mu^i_{\phi_n\to I_n}(\phi_n)$:

$$\mu^i_{I_n\to\alpha_{0,n}}(\alpha_{0,n}) = N_C(\alpha_{0,n}; x^i_{I_n\to\alpha_{0,n}}, v^i_{I_n\to\alpha_{0,n}}) \tag{47}$$

$$P_\uparrow(\alpha_{0,n}) = P(\phi_n) = \mu^i_{\phi_n\to I_n}(\phi_n) \tag{48}$$

$$x^i_{I_n\to\alpha_{0,n}} = E_{P_\uparrow(\alpha_{0,n})}[\alpha_{0,n}], v^i_{I_n\to\alpha_{0,n}} = E_{P_\uparrow(\alpha_{0,n})}[|\alpha_{0,n}|^2] - \left|x^i_{I_n\to\alpha_{0,n}}\right|^2. \tag{49}$$

Finally, we focus on the decoder part of the factor graph in Fig. 1. A soft-input-soft-output (SISO) decoder is used to implement the turbo iteration with the demodulation part. According to the rules of SPA, the message $\mu^i_{a_n\to M_n}(a_n)$ can be updated by

$$\mu^i_{a_n\to M_n}(a_n) = \mu^i_{J_n\to a_n}(a_n). \tag{50}$$

Using the message $\mu^i_{a_n\to M_n}(a_n)$ in Eq. (50) and the a priori logarithm likelihood ratios (LLRs) $\{\lambda^{i-1}_a(c^q_n), \forall q\}$ fed back from the SISO decoder at previous turbo iteration, the extrinsic LLRs $\{\lambda^i_e(c^q_n), \forall q\}$ which are the input of the SISO decoder can be obtained as follows:

$$\lambda^i_e(c^q_n) = \ln\frac{\sum_{a_n\in A^1_q}\mu^i_{a_n\to M_n}(a_n)}{\sum_{a_n\in A^0_q}\mu^i_{a_n\to M_n}(a_n)} - \lambda^{i-1}_a(c^q_n). \tag{51}$$

Once the extrinsic LLRs $\{\lambda^i_e(c^q_n), \forall q\}$ are available, the decoder performs decoding and feeds back the a priori logarithm likelihood ratios (LLRs) $\{\lambda^i_a(c^q_n), \forall q\}$ which can be used to compute the message $\mu^i_{M_n\to a_n}(a_n)$.

$$\beta(a_n) = \prod_q \frac{\exp\{c^q_n\lambda^i_a(c^q_n)\}}{1+\exp\{c^q_n\lambda^i_a(c^q_n)\}} \tag{52}$$

$$\mu^i_{M_n\to a_n}(a_n) = \frac{\beta(a_n)}{\sum\limits_{a_n}\beta(a_n)}. \tag{53}$$

In Eq. (53), $\mu^i_{M_n \to a_n}(a_n)$ is a discrete probability distribution function about the discrete random variable a_n. By applying the updating rules of SPA, the message $\mu^i_{a_n \to J_n}(a_n)$ can be expressed as:

$$\mu^i_{a_n \to J_n}(a_n) = \mu^i_{M_n \to a_n}(a_n). \tag{54}$$

In summary, the messages passing in one turbo iteration are exactly derived as the equations in this section shown. Because of the factor graph in Fig. 1 is loopy, we consider a message passing form the bottom to the top and then back again as one turbo iteration. At the beginning of the first turbo iteration, we set $\mu^0_{\phi_n \to I_n}(\phi_n) = 1/p$, $\forall n$, $\lambda^0_a(c^q_n) = 0$, $\forall q$, $\forall n$.

4 Simulation Results

In this section, we present the simulation results for the proposed algorithm. The performance of the proposed algorithm is assessed in terms of bit error rate (BER) versus E_b/N_0. For each considered channel, we compare the proposed algorithm with the conditional receiver of the CPM signals which uses the minimum mean squared error (MMSE) frequency domain equalization (FDE) and the optimal BCJR demodulator. In both receivers, we consider the $1/2 - rate$ convolutional code with generators $G_1 = 91$ and $G_2 = 121$ (octal notation) and the encoding length is 1024.

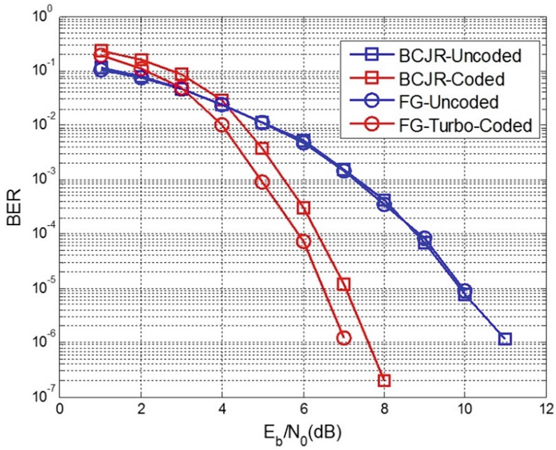

Fig. 2. 1RC modulation with h = 1/2 and M = 2 for AWGN channel.

In Fig. 2, we consider a relatively simple scenario, AWGN channel, to verify the performance of the algorithm proposed in Sect. 3. A binary CPM signal with frequency pulse of duration $L = 1$ symbol interval and the modulation index $h = 1/2$ is considered. When using the proposed algorithm, we set the parameters as follows $N_L = 1$ and $L_m = 1$. As shown in Fig. 2, we compare the BER performance of the proposed algorithm and the BCJR method with and without the convolutional code respectively. The performance of the two algorithms is similar in the scenario without the convolutional code. When we consider the convolutional code in both systems, the performance of the proposed algorithm using the turbo iteration is better than the BCJR demodulation.

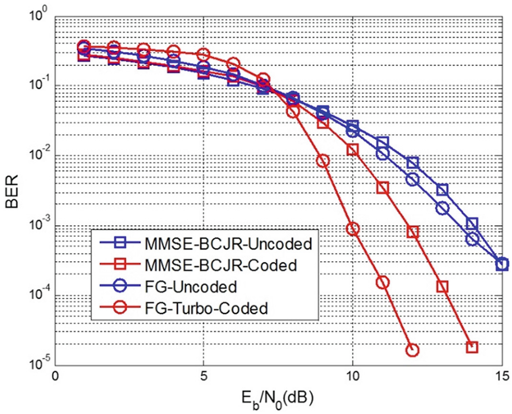

Fig. 3. 1RC modulation with h = 1/4 and M = 4 for Channel I

In Fig. 3, we consider an ISI channel (channel I) characterized by $N_L = 2$ and

$$\boldsymbol{h} = [0.8165\ \ 0.5773],\ \boldsymbol{\tau} = [0\ \ 20] \tag{55}$$

in which the vector $\boldsymbol{h}$ denotes the gain of each path of the channel and the vector $\boldsymbol{\tau}$ denotes the symbol number of the propagation delay for each path of the channel. A quaternary CPM signal with frequency pulse of duration $L = 1$ symbol interval and the modulation index $h = 1/4$ is considered. We set the parameters of the proposed algorithm $N_L = 2$ and $L_m = 2$. The length of FFT in the MMSE-FDE method is 1024. In this scenario, the receiver with the MMSE-FDE and the BCJR demodulator and the receiver using the proposed algorithm are considered. Similar to Fig. 2, we compare the BER performance of the two methods with and without the convolutional code respectively. As shown in Fig. 3, the performance of the two methods is similar without the convolutional code. When take the convolutional code into account, the performance of the proposed algorithm is better than the MMSE-FDE-BCJR receiver.

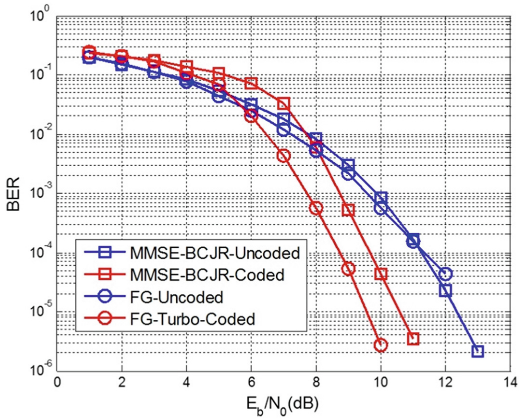

Fig. 4. 1RC modulation with h = 1/4 and M = 4 for Channel II.

In Fig. 4, we consider an ISI channel (channel II) characterized by $N_L = 5$ and

$$\boldsymbol{h} = [0.7\ 0.2\ 0.05\ 0.03\ 0.02], \boldsymbol{\tau} = [0\ 5\ \ 10\ \ 15\ \ 20] \tag{56}$$

where the vector $\boldsymbol{h}$ denotes the gain of each path of the channel and the vector $\boldsymbol{\tau}$ denotes the symbol number of the propagation delay for each path of the channel. A quaternary CPM signal with frequency pulse of duration $L = 1$ symbol interval and the modulation index $h = 1/4$ is considered. The parameters of the proposed algorithm are set as $N_L = 5$, $L_m = 2$. As shown in Fig. 4, the performance of the two methods is similar without the convolutional code. And the performance of the proposed algorithm is better than the MMSE-BCJR receiver with the convolutional code.

In terms of computational complexity, we point out that the decode parts in both receivers is the same. They differ in the equalization part and the demodulation part. For the equalization part, the computational complexity of the MMSE-FDE is $O(N \log_2 N)$ and that of the proposed algorithm is $O(NN_L)$ which is associated with the number of the path of the channel. For the demodulation part, the computational complexity of the BCJR algorithm is $O(pM^L N)$ while that of the proposed algorithm is $O(p(N+1))$.

5 Conclusion

Based on the Laurent decomposition and the Gaussian message passing, we presented a low-complexity turbo iterative receiver of CPM signals. According the simulations of different scenarios as shown in Sect. 4, the performance of the proposed algorithm is similar to the receiver with the MMSE-FDE and the BCJR demodulator in uncoded systems. When take the coding module into account, the proposed algorithm can achieve better performance. Moreover, the proposed algorithm has lower computational complexity than the receiver with the MMSE-FDE and the BCJR demodulator.

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