Power Allocation for Downlink of Non-orthogonal Multiple Access System via Genetic Algorithm

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Abstract. Non-orthogonal multiple access (NOMA) is a promising technology in future communication systems due to high spectral efficiency. In this paper, we propose an efficient power allocation method based on the genetic algorithm (GA) to solve the non-linear optimization problem for maximizing the achievable sum rate under a total power constraint and the users' quality of service (QoS) in the downlink NOMA systems. Different power allocation coefficients can be obtained with different objective functions and optimization criteria. Simulation results demonstrate that the NOMA systems with power allocation using GA can achieve better performance than the orthogonal multiple access (OMA) systems in terms of the achievable sum rate.

Keywords: Non-orthogonal multiple access (NOMA) · Quality of service (QoS) · Power allocation · Genetic algorithm

1 Introduction

Along with the development of wireless communication technology, spectral scarcity has become a serious problem [1]. Spectrum efficiency has ignited great interest from both academia and industry. The traditional mobile communication systems are faced with drastic changes and enormous challenges, including the explosive growth of mobile data services and massive machine-type communications. The 5th generation of communication systems (5G) will support high data rate communications, massive device connections, ultra-low latency, high reliability, and so on [2]. But the conventional multiple access technique-orthogonal multiple access (OMA) schemes, for instance, frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA), will hardly meet those

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requirements and challenges in 5G. Non-orthogonal multiple access (NOMA) is a promising multiple access technique for 5G communication systems with utilizing superposition coding (SC) at the transmitter and successive interference cancellation (SIC) [3] at the receiver. Compared to the conventional OMA, NOMA can be able to support multi-users to share the same time-frequency resources [4]. In essence, the NOMA systems achieve high spectral efficiency at the cost of increased receiver complexity.

Power allocation in OFDMA has been well studied in [5, 6], however, power allocation in the NOMA systems is still a challenging open problem and important for optimizing the achievable sum rate under a total power constraint in the NOMA downlink systems. Many previous works have already focused on power allocation for the NOMA systems. In [7], a minorization-maximization algorithm (MMA) was applied to maximize the downlink sum rate and the nonconvex optimization problem was converted into a convex optimization problem. In [8], Choi proposed an approach to optimize the sum capacity of multiple-input multiple output NOMA (MIMO-NOMA) systems with layered transmissions which allocated power to multiple layers and used the alternating maximization (AM) algorithm that can be regarded as a two-block Gauss-Seidel method. In [9], the mutual information was chosen as the optimal objective function to optimize power allocation for the maximum achievable rate. In [10], Liu demonstrated that the performance of MIMO-NOMA is better than MIMO-OMA in terms of the sum channel capacity (except for the case in which there is only one user being communicated to).

The existing works about NOMA power allocation under the users' QoS constraints are mostly analyzed for two users. In [11], a bisection search algorithm was proposed along with a low complexity suboptimal algorithm to optimize two users' ergodic capacity of MIMO-NOMA system under the total transmission power constraint and the minimum achievable rate constraint of the weak user. In [12], Wang utilized the Karush–Kuhn–Tucker (KKT) conditions to obtain closed-form solutions for maximizing the channel capacity in terms of two users' power allocation under a total power constraint and the QoS constraints of each user, and moreover extended the solutions to a MIMO scenario. In [13], Oviedo proposed a Fair-NOMA that means the two users are capable of achieving higher capacity in the NOMA systems than the OMA systems. In [14], Choi proposed proportional fairness scheduling (PFS) to obtain two users' optimal power allocation with different criteria in the downlink NOMA systems.

In this paper, we analyze the multi-user NOMA power allocation under a total power constraint and the users' QoS constraints, regardless of the user selection criteria, and utilize the effective methods based on the genetic algorithm (GA) to solve the nonconvex optimal problem. The rest of the paper is organized as follows. The system model is outlined in Sect. 2. Section 3 formulates an optimization problem of power allocation in the NOMA systems. Section 4 introduces the genetic algorithm. The simulation results are presented and discussed in Sect. 5. Finally, the conclusions are given in Sect. 6.

2 System Model

Consider a downlink communication scenario, with a base station $\mathcal{B}$ equipped with a single antenna and N users each equipped with a receive antenna; the base station $\mathcal{B}$ transmits signal data to each user and the total transmitted total power is P; $\alpha_i(i = 1, 2...N)$ are the fractions of the total power allocated to the *i-th* user. The 1st user is the weakest user (the furthest from the base station $\mathcal{B}$), and the *N-th* user is the strongest user (the nearest from the base station $\mathcal{B}$). The channel fading coefficients $h_i(i = 1, 2...N)$ satisfy the Gaussian distribution with zero mean and variance $\sigma^2_{h_n}$. The channels are sorted as $0 \leq |h_1|^2 \leq |h_2|^2 ... \leq |h_N|^2$. The additive white Gaussian noise (AWGN) is assumed to be normalized with zero mean and variance σ^2_n. According to the NOMA principle, the system will allocate more power to the users with weak channel conditions and less power to the users with strong channel conditions. The users' power allocation coefficients are ordered as: $\alpha_1 \geq \alpha_2 \geq ... \geq \alpha_N$. The weak user decodes its signal information, and perceives the signal information from the strong user as interference due to its less power. The strong user utilizes the SIC at the receiver and decodes its own signal information after decoding and removing the reference induced by the weak user.

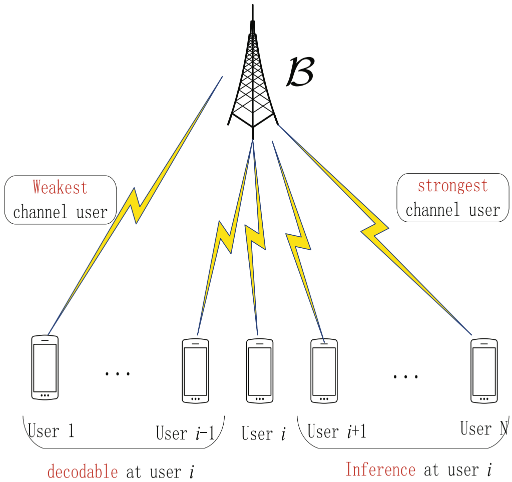

Fig. 1. Multiuser downlink system topology.

The multi-user NOMA scheme is shown as Fig. 1. It is shown that the *i-th* user can decode and remove the *m-th* (when $m < i$) user's signal information and perceive the signal information from the *k-th* (when $k > i$) users as interference since they are negligible to the *i-th* user. In this way, the achievable rate for the *i-th* $(i = 1, 2, \cdots, N - 1)$ user is formulated as follows:

$$R_i = log_2\left(1 + \frac{\alpha_i P|h_i|^2}{P|h_i|^2 \sum_{k=i+1}^{N} \alpha_k + \sigma_n^2}\right) \tag{1}$$

The *N-th* user can decode and remove all the other users' signal information. Thus, the achievable rate of the *N-th* user is formulated as follows:

$$R_N = log_2\left(1 + \frac{\alpha_N P|h_N|^2}{\sigma_n^2}\right) \tag{2}$$

Therefore, the system achievable sum rate is formulated as follows:

$$R_{sum} = \sum_{i=1}^{N-1} log_2(1 + SINR_i) + log_2\left(1 + \frac{\alpha_N P|h_N|^2}{\sigma_n^2}\right) \tag{3}$$

where $SINR_i = \frac{\alpha_i P|h_i|^2}{P|h_i|^2 \sum_{k=i+1}^{N} \alpha_k + \sigma_n^2}, i = 1, 2, \cdots, N-1.$

3 NOMA Power Allocation Problem Formulation

The different optimization power allocation coefficients can be obtained with different optimization criteria, and the following describes two different optimization criteria. One is to maximize the achievable sum rate to get the optimal power allocation; the other is to maximize the weighted sum rate for obtaining the power allocation coefficients to calculate the capacity.

3.1 Maximize the Achievable Sum Rate

The optimal capacity is obtained by maximizing the achievable sum rate when each user meets its quality of service (QoS) that refers to the minimum rate requirement. For instance, the *i-th* user has to satisfy the inequality $\mathrm{SINR}_i \geq \gamma_i$, and the optimization problem can be formulated as follows:

$$\begin{aligned} &\max_{\alpha_i} R_{sum} \\ &\text{s.t.}\ (i)\ \sum_{i=1}^{N} \alpha_i = 1 \\ &\quad (ii)\ 0 \leq \alpha_i \leq 1 \\ &\quad (iii)\ \alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_N \\ &\quad (iv)\ SINR_i \geq \gamma_i, i = 1, 2, \cdots N \end{aligned} \tag{4}$$

where $(4, i)$ represents the sum of all the users' power is P; $(4, ii)$ represents that the lower bound and the upper bound of all the users' power allocation coefficients; $(4, iii)$ represents the NOMA principle that power allocated to the weaker user must be more than that of the stronger user; and $(4, iv)$ expresses the constraints that the SINR of each user must meet the targeted SINR γ_i to guarantee the QoS.

Subsequently, the constraint $(4, iv)$ is analyzed and can be described in detail as (5).

$$\begin{cases} \frac{\alpha_i P|h_i|^2}{P|h_i|^2 \sum_{k=i+1}^{N} \alpha_k + \sigma_n^2} \geq \gamma_i, (i = 2, 3, \ldots, N-1) \\ \frac{\alpha_N P|h_N|^2}{\sigma_n^2} \geq \gamma_N \end{cases} \tag{5}$$

The bound of the power allocation coefficients α can be obtained and formulated as follows:

$$\begin{cases} 1 \geq \alpha_i \geq \frac{\gamma_i\left(|h_i|^2\theta + \frac{1}{\rho}\right)}{(1+\gamma_i)|h_i|^2}, (i = 1, 2, \ldots, N-1) \\ 1 \geq \alpha_N \geq \frac{\gamma_N}{\rho|h_N|^2} \end{cases} \tag{6}$$

where ρ is the transmission SNR, $\rho = \frac{P}{\sigma_n^2}$, set $\alpha_0 = 0$, and get $\theta = 1 - \sum_{k=0}^{i-1} \alpha k$, $(0 \leq \theta \leq 1)$. The inequalities in (6) show the constraints between the users' power allocation coefficients induced by the users' QoS constraints. The lower bound of $\alpha_i (i = 1, 2, \ldots, N)$ are denoted as $\beta_i (i = 1, 2, \ldots, N)$. If $\beta_i \geq 1, (i = 1, 2, \ldots, N)$, it means that the *i-th* user can't be supported to meet the QoS, even if the BS allocates the total power to the user.

We utilize 4 users to analyze the problem in detail as follows:

$$\begin{aligned} &\max \sum_{i=1}^{3} log_2(1 + SINR_i) + log_2\left(1 + \frac{\alpha_4 P|h_4|^2}{\sigma_n^2}\right) \\ &s.t.\ (i)\ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1 \\ &\quad (ii)\ 0 \leq \alpha_1 \leq 1, 0 \leq \alpha_2 \leq 1, 0 \leq \alpha_3 \leq 1, 0 \leq \alpha_4 \leq 1 \\ &\quad (iii) - \alpha_1 + \alpha_2 \leq 0, -\alpha_2 + \alpha_3 \leq 0, -\alpha_3 + \alpha_4 \leq 0 \\ &\quad (iv) - \alpha_1 \leq -\eta_1, \\ &\qquad -\lambda_2\alpha_1 - \alpha_2 \leq -\eta_2, \\ &\qquad -\lambda_3\alpha_1 - \lambda_3\alpha_2 - \alpha_3 \leq -\eta_3, \\ &\qquad -\alpha_4 \leq -\frac{\gamma_4}{\rho|h_4|^2} \end{aligned} \tag{7}$$

where $\lambda_i = \frac{\gamma_i}{1+\gamma_i}$, $\eta_i = \frac{\gamma_i\left(|h_i|^2 + \frac{1}{\rho}\right)}{(1+\gamma_i)|h_i|^2}$, and the constraints in (7) correspond to that in (4) respectively.

3.2 Maximize the Weighted Sum Rate

We consider the weighted sum rate as the optimization objective function to allocate power for multi-users and the objective function is shown as (8):

$$\max_{\alpha_i} R_{weighted\_sum} = \sum_{i=1}^{N} \frac{R_{i-NOMA}}{R_{i-OMA}} \tag{8}$$

where R_{i-NOMA} equals to R_i shown as Eqs. (1) and (2), $R_{i-OMA} = log_2\left(1 + P|h_i|^2/\sigma_n^2\right)$ represents the users' OMA capacity. The constraints of the problem (8) are the same as (4). The optimal power allocation coefficients are obtained by optimizing the problem (8) and substituted into (3) to obtain the optimal achievable sum rate.

In general, there is no analytical solutions for the multivariable optimization problem, and the GA function in MTLAB can be used to obtain optimal power allocation coefficients, but the computational process of the genetic algorithm is complex and time-consuming.

4 Genetic Algorithm

Genetic algorithm is an optimization method inspired by the process of natural selection that belongs to the evolutionary algorithms [15]. Traditionally, a population is represented in binary as strings of 0s and 1s. In the genetic algorithm, a population of candidate solutions to an optimization problem evolves towards to better solutions. The solutions selected based on their fitness will be mutated and altered, and offspring will be used to form a new population. The new population will be better than the old one. The process will be repeated until there's a solution satisfied.

The genetic algorithm process is as follows [16] and the flowchart of the algorithm is shown as Fig. 2.

Step 1: Represent the problem domain as a chromosome of fixed length and determine the number of chromosomes, generations, and mutation rate and crossover rate value;
Step 2: Choose the initial population;
Step 3: Evaluate the fitness of each individual chromosome by calculating the objective function;
Step 4: Select a pair of chromosomes from the current population for mating, based on their fitness scores (the better fitness, the bigger chance to be selected);
Step 5: Crossover from those parents to create a pair of offspring chromosomes;
Step 6: Mutation (maintain genetic diversity from one generation of a population to the next);
Step 7: Return to Step3 and repeat the process until the termination (or optimization) criterion is met;
Step 8: Get the solution.

The general iterative algorithm can easily fall into the local minimum, But GA is a good way to overcome the drawback due to its good global search capabilities that can find the best possible solution with a high probability. Compared with the traditional optimization methods (enumeration, heuristic, etc.), GA has a good convergence and high explorative ability. In addition, GA is widely used to solve function optimization problems, combinatorial optimization problems, production scheduling problems, adaptive control, robotics, image processing, genetic programming, data mining, robotic learning, and artificial life. Although the genetic algorithm is applied in various fields, it has its own shortcomings, for example, the local search ability and

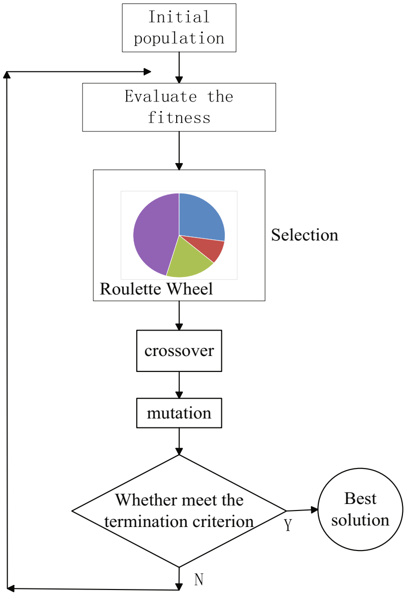

Fig. 2. The flowchart of genetic algorithm.

convergence is poor, and it takes a long time to find the optimal solution. The primary problem is to improve the search ability and the convergence speed of the algorithm.

5 Numerical Results

In this section, the performance of the downlink NOMA systems with power allocation using genetic algorithm is compared to that achieved by the OMA systems. For a given downlink NOMA scheme with a base station and N users, the channel gains are generated as $h_i = \sqrt{d_i^{-\mu}} g_i$, where $g_i \sim CN(0,1)$ (i.e. $\sigma_{h_n}^2 = d^{-\mu}$), μ is the pass-loss exponent $\mu = 2$, and the distances between the base station and the users are fixed and uniformly distributed between 1 and D. The noise power for each user is normalized to $\sigma_n^2 = 1$. The *i-th* user's OMA capacity is given as: $Coma_i = (1/N)log_2(1 + \left(P|h_i|^2\right)/\sigma_n^2), (i = 1, 2, \ldots, N)$.

Figures 3 and 4 show the capacity of a user and the sum capacity versus P (dB) for the NOMA schemes with power allocation using genetic algorithm and for the OMA schemes.

Figure 3 compares three users' achievable rate and the maximum sum rate of the NOMA scheme acquired by maximizing the sum rate to that of the OMA scheme. Simulation parameters for performance evaluation are given as follows. We will take D = 11 and the distance vector between the three users and the base station is $d' = [11, 6, 1]$ meters. The vector of the users' QoS is $\gamma' = [-20, -15, -5]$ dB. The channels

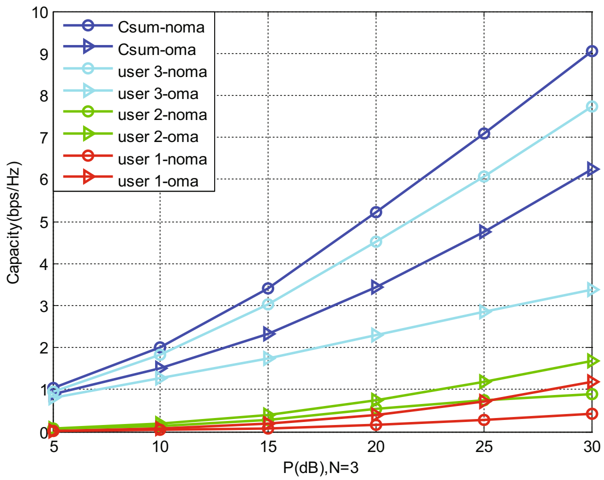

Fig. 3. Three users' capacity analysis of NOMA and OMA by maximizing the sum rate.

need to satisfy the order of $|h_1|^2 \leq |h_2|^2 \leq |h_3|^2$ to ensure that the users' signal information can be decoded. The sum capacity and the third user's capacity of the NOMA system is higher than the capacity of the OMA system. The NOMA capacity of the first user and the second user is lower than the OMA capacity.

Figure 4 depicts the four users' capacity and the sum capacity for the NOMA and OMA systems, and the NOMA power allocation is optimized by maximizing the sum rate. Simulation parameters are set as follows. The distance vector is $d'' = [11, 7.67, 4.33, 1]$ meters and the users' QoS vector is $\gamma'' = [-30, -25, -20, -10]\,\text{dB}$. The channels need to satisfy the order of $|h_1|^2 \leq |h_2|^2 \leq |h_3|^2 \leq |h_4|^2$. The sum capacity and the fourth user's capacity of the NOMA system are higher than the capacity of the OMA system while the others are lower than the capacity of the OMA system respectively.

Figures 5 and 6 depict the achievable rate of 3 users and 4 users with power allocation obtained by maximizing the weighted sum rate under the same simulation parameters as Figs. 3 and 4, respectively. In both Figs. 5 and 6, the sum capacity of the NOMA system is higher than the capacity of the OMA system. In Fig. 5, the NOMA capacity of the first user and the third user is higher than the OMA capacity while the second user's capacity of the NOMA system is lower than the capacity of the OMA system. In the Fig. 6, the NOMA capacity of the first user and the fourth user is higher than the OMA capacity while the NOMA capacity of the second user and the third user is lower than the OMA capacity.

Figure 7 compares the NOMA sum capacity optimized by maximizing sum rate and weighted sum rate for three users and four users. The sum capacity obtained by maximizing the weighted sum rate is lower than that obtained by maximizing the sum rate for both three users and four users.

For the maximizing sum rate scenario, only the strongest user's capacity and the sum capacity of the NOMA system are higher than the capacity of the OMA system,

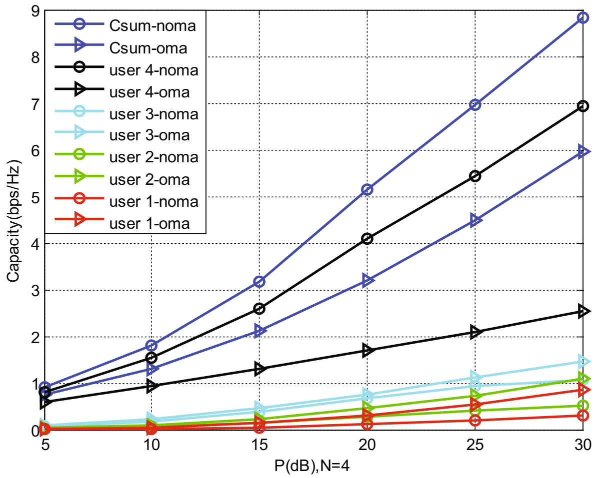

Fig. 4. Four users' capacity analysis of NOMA and OMA by maximizing the sum rate.

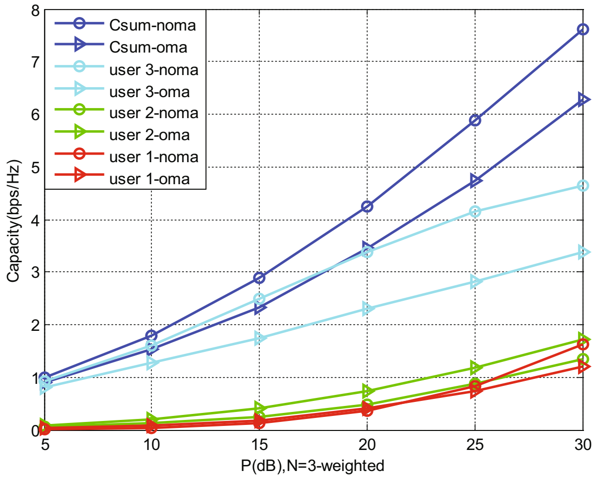

Fig. 5. Three users' capacity analysis of NOMA and OMA by maximizing the weighted sum rate.

respectively, while the other users' capacity of the NOMA system is lower than that of the OMA system, grows slowly and only satisfy the required SNR. For the maximizing weighted sum rate scenario, except for the strongest user, the weakest user's NOMA capacity is also higher than the OMA capacity at the cost of the reduction of the sum capacity shown as Fig. 7. Different optimal criteria will lead to different results, but the NOMA systems is better than the OMA systems in terms of the sum capacity whatever criteria is chosen.

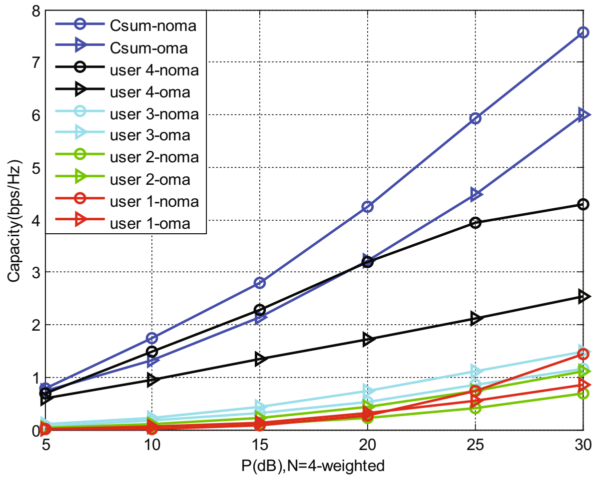

Fig. 6. Four users' capacity analysis of NOMA and OMA by maximizing the weighted sum rate.

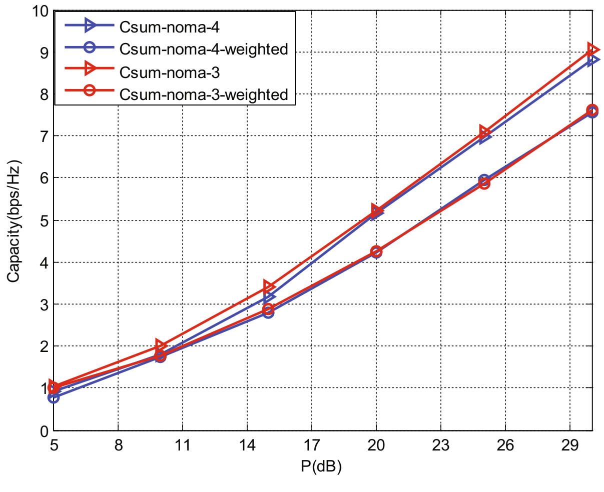

Fig. 7. The comparison of NOMA capacity optimized by maximizing the sum rate and weighted sum rate.

6 Conclusion

In this paper, we studied the capacity maximization problem under a total power constraint and users' QoS constraints for power allocation by utilizing GA in NOMA downlink systems. We derived the optimal power allocation and obtained the optimal capacity by maximizing the sum rate and the weighted sum rate. The simulation results show that the performance of the NOMA system based on GA can achieve higher gain than the traditional OMA schemes when the channel state information is available to the transmitters and different optimization criteria will induce different results. The values of the users' QoS is fixed in this paper and the dynamic QoS will be considered in the future work. On the other hand, the solution obtained by a simple genetic algorithm is time-consuming and the genetic algorithm is prone to premature convergence in practical application. Therefore, the combination of genetic algorithm and other algorithms will be studied in the future work.

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