Probabilistic Caching in Wireless Device to Device Networks with Contention Based Multimedia Delivery

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Abstract. This paper studies the optimal probabilistic caching placement in large-scale cache-enabled D2D networks to maximize the cache hit performance, which is defined as the probability that a random user request can be served by mobile helpers (MHs) in the vicinity. To avoid collisions of the concurrent transmissions, a contention based multimedia delivery protocol is proposed, under which a MH is allowed to transmit only if its back-off timer is the smallest among its associated contenders. By applying tools from stochastic geometry, the optimal caching probability is derived and analyzed. It is shown that the optimal solution of the probabilistic caching placement depends on the density of MHs, the D2D communication range, and the user request distribution. With the derived optimal caching probabilities, we further characterize the transmission probability of MHs and thereby the successful content delivery probability of the cache-enabled D2D network. Simulations are provided to validate our analysis.

Keywords: Cache-enabled D2D networks Contention based multimedia delivery protocol Optimal probabilistic caching strategy · Stochastic geometry
Transmission probability · Cache hit probability Transmission probability • Cache hit probability
Successful content delivery probability Successful content delivery probability

1 Introduction

The proliferation of smart mobile devices has triggered an explosive increase of data traffic over recent years, mainly driven by the ever-growing demand of bandwidthintensive multimedia services. Predicted by Cisco [\[1\]](#page-14-0), the mobile data traffic is expected to reach 30.6 exabytes per month by 2020, an eightfold increase over 2015, in which more than 80% would be contributed by multimedia streaming. Facing such an unprecedented growth of multimedia data traffic over the air, it is crucial for mobile operators to seek and leverage more advanced techniques to facilitate the contentcentric design of next generation wireless networks [\[2](#page-14-0)–[4](#page-14-0)].

The emerging cache-enabled device-to-device (D2D) communication paradigm is considered to be an effective approach to tackle the mobile data tsunami induced by massive demands on multimedia content delivery [\[5](#page-14-0)–[9](#page-14-0)]. Particulary, with caching abilities enabled at the proximate mobile devices (helpers), the D2D communication

can cost-effectively bring the multimedia contents closer to users, and simultaneously exploit the spatial recuse and (coded or uncoded) multicasting opportunities in data dissemination to relieve the heavy burden of the fast growing traffic.

Existing works on the modeling and analysis of cache-enabled D2D communication has taken two main directions. The first line of work $[10, 11]$ $[10, 11]$ $[10, 11]$ $[10, 11]$ focuses on the theoretical analysis of asymptotic scaling laws for cache-enabled D2D networks under the classic protocol model, where the reception of a data packet from source u to destination v is assumed to be successful only if their transmission distance $d(u, v)$ is less than or equal to a predefined collaboration range r , and no other concurrent transmissions is within distance $(1 + \Delta)r$ from destination v for $\Delta > 0$. Particularly, in [[10\]](#page-14-0), Ji et al. characterized the optimal throughput-outage tradeoff for cache-enabled D2D networks in terms of scaling laws, under the assumption that both the number of users n and the library size m grow to infinity. With decentralized random caching and unicast delivery, it was shown in [[10\]](#page-14-0) that, for arbitrary outage probability $\rho \in (0, 1)$, the per-user throughput of the cache-enabled D2D networks can achieve a bit rate of order $\frac{M}{m}$, where M denotes the caching capacity of each device and $M \gg \frac{M}{m}$. Based on $[10]$ in [11] li et al. further characterized the scaling laws of the same cache enabled [[10\]](#page-14-0), in [[11\]](#page-14-0), Ji et al. further characterized the scaling laws of the same cache-enabled D₂D network by applying both spatial reuse and coded multicasting. Interestingly and somehow counterintuitively, it was shown in [[11\]](#page-14-0) that the gain of spatial reuse and the gain of the coded multicasting do not accumulate in the order sense.

The second line of work $[12–17]$ $[12–17]$ $[12–17]$ $[12–17]$ considers the physical model, where the successful communication between two nodes is based on the received signal-tointerference ratio (SIR) or signal-to-interference-plus-noise ratio (SINR). Tools from stochastic geometry have been widely applied in this category for tractable characterization of key network performance metrics, such as coverage, rate, and spatial throughput. Particularly, in [\[12](#page-14-0)], Jarray et al. studied the hit performance of caching in D2D networks for different degrees of node mobility. In [\[13](#page-14-0)], Malak et al. developed the optimal spatially-independent content caching strategies that aim to maximize the average density of successful receptions under different fading distributions. In [[14\]](#page-15-0), Malak et al. further investigated the optimal geographic content placement problem for D2D networks, and proposed spatially correlated caching strategies to maximize the D2D cache hit probability. In [\[15](#page-15-0)], Afshang et al. characterized the performance of cluster-centric content placement in a cache-enabled D2D network by developing a comprehensive analytical framework with foundations in stochastic geometry. In [[16\]](#page-15-0), Chen et al. provided analytical and numerical results to compare the performance of caching at mobile devices and caching at small cells, in terms of the cache hit probability, the density of cache-served requests and average power consumption. In [[17\]](#page-15-0), Chen et al. studied the optimal caching probabilities with numerical optimization by analyzing a closed-form approximation of cache-aided throughput, which measures the density of successfully served requests by local device caches.

Interference management is of critical importance in the design of cache-enabled D2D networks. With elaborate interference management schemes, the effective spatial reuse and thereby the overall performance of cache-enabled D2D networks can be significantly improved. It is worth noting that in the above prior works $[10-17]$ $[10-17]$ $[10-17]$ $[10-17]$, the interference management issue of cache-enabled D2D networks has not been well addressed.

Particularly, in [\[10](#page-14-0), [11](#page-14-0)], the studied protocol model and the corresponding guard-zone based interference avoidance strategy failed to capture the effect of small-scale fading and aggregate interference on the performance of successful reception. While in $[12-17]$ $[12-17]$ $[12-17]$ $[12-17]$, though tractable analytical results on network performance were characterized under the physical model, the interference management problem was not discussed. As such, the benefits of advanced interference management mechanisms in enhancing the performance of cache-enabled D2D networks remain to be explored.

In this paper, we investigate the optimal probabilistic caching strategy in large-scale cache-enabled D2D networks under the physical model to maximize the cache hit performance, which is defined as the probability that a random user request can be served by mobile helpers (MHs) in the vicinity. Different from that in $[12-17]$ $[12-17]$ $[12-17]$ $[12-17]$, a contention based multimedia delivery protocol is proposed to tackle the interference management issue, which is described as follows.

Contention based Multimedia Delivery Protocol: In the content delivery phase, to avoid collisions among the concurrent transmissions, the MHs under requests are assumed to each start with a random back-off timer, which is uniformly distributed on [0, 1]. By monitoring the medium, a MH then makes decision to initiate its transmission if no contender with contention threshold N_d is detected prior to the expiration of its back-offer timer, while otherwise it defers. In other words, in the content delivery phase, under the proposed contention based multimedia delivery protocol, a MH under request is allowed to transmit only if it has the minimal back-off timer among its contenders with contention threshold N_d .

It is worth noting that under the proposed contention based multimedia delivery protocol, due to the interactions among the concurrent transmissions, the positions of the active MHs are in general dependent. As a result, how to effectively characterize the dependencies among the active MHs is the major challenge to be tackled in this paper.

The remainder of this paper is organized as follows. The system model and performance metrics are described in Sect. 2. The optimal probabilistic caching placement is characterized in Sect. [3.](#page-5-0) The transmission probability of MHs is derived in Sect. [4](#page-7-0). The successful content delivery probability of the cache-enabled D2D networks is analyzed in Sects. [5.](#page-9-0) Simulation results are presented in Sect. [6](#page-11-0). Finally, we conclude the paper in Sect. [7](#page-13-0).

2 Model and Metrics

2.1 System Model

We consider a large-scale cache-enabled D2D network formed by dedicated MHs, intended UEs, and a library of multimedia files $\mathcal{F} := \{1, 2, \dots, F\}$ on \mathbb{R}^2 as illustrated in Fig. [1](#page-3-0). The locations of MHs and UEs are modeled as two independent HPPPs with density λ_m and λ_u , respectively. To simplify the analysis, it is assumed that the files in library $\mathcal F$ are of the same size, and each MH has a cache memory of $M = 1$ file. A decentralized probabilistic caching strategy is investigated in this paper, under which the MH randomly caches the f-th file in $\mathcal F$ with probability c_f . We further assume that the popularity of the *f*-th file in $\mathcal F$ follows the Zipf distribution as

Fig. 1. Cache-enabled D2D network formed by dedicated MHs, intended UEs, and a library of multimedia files F.

$$
pf = \frac{1/f^{\gamma}}{\sum_{j=1}^{F} 1/j^{\gamma}},
$$
\n(1)

where $0 < \gamma < 1$ denotes the Zipf parameter.

The propagation channel is modeled as the combination of the small-scale Rayleigh fading and the large-scale path-loss given by

$$
g(d) = hd^{-\alpha},\tag{2}
$$

where h denotes the exponentially distributed power coefficient with unit mean, d denotes the propagation distance, and α denotes the path-loss exponent. The transmit power of MHs is denoted by P_d . For the sake of simplicity, we ignore the thermal noise in the regime of interest and simply focus on the received signal-to-interference ratio (SIR). Let R_d denote the collaboration distance of D2D transmission between MH and UE. We further denote θ_d as the SIR target for successful data receptions under D2D communication.

To avoid collisions among the concurrent transmissions, a contention based multimedia delivery protocol with contention threshold N_d is proposed, under which a content request from the UE can be served iff:

- 1. The requested file is available at the MHs (denoted by eligible MHs) within a distance of R_d .
- 2. At least one of the eligible MHs has the minimal back-off timer among its contenders¹ with contention threshold N_d .

Upon finding the eligible MHs, the UEs then associate with the nearest ones for data transmission.

2.2 Performance Metric

The performance metrics studied in this paper are specified as follows.

Transmission Probability: The transmission probability of MH, denoted by q_d , is defined as the probability that a MH is eligible to launch the transmission.

Cache Hit Probability: The cache hit probability, denoted by ξ_d , is defined as the probability that a randomly requested file can be found at the caches of eligible MHs which are able to launch the transmissions within a distance of R_d Let ζ_f denote the conditional cache hit probability for the f-th file in $\mathcal F$. Then, the cache hit probability ζ_d of the cache-enabled D2D network is given by

$$
\xi_d = \sum_{f=1}^F p_f \cdot \xi_f. \tag{3}
$$

Coverage Probability: The coverage probability of a randomly requested file is defined as the probability that a UE succeeds in decoding the received data packets of the file from its associated MH. In particular, for the f-th file in \mathcal{F} , given the received SIR SIR_f, and the SIR target θ_d , the coverage probability C_f is defined as

$$
c_f = \Pr(SIR_f \ge \theta_d). \tag{4}
$$

Successful Content Delivery Probability: The successful content delivery probability in cache-enabled D2D networks, denoted by τ_d , is defined as the probability that a random UE request is successfully served by MHs within a distance of R_d . In particular, for the f-th file in F, given the conditional cache hit probability ξ_f , and the coverage probability C_f , the successful content delivery probability τ_d of the cache-enabled D2D network is defined as

$$
\tau_d = \sum_{f=1}^F p_f \cdot \zeta_f \cdot C_f \tag{5}
$$

¹ For two eligible MHs located at **x** and **y**, we say **y** is a contender of **x** with contention threshold N_d if $P_d h |\mathbf{y} - \mathbf{x}|^{-\alpha} \ge N_d$.

3 Optimal Probabilistic Caching Placement

In this section, we derive the optimal probabilistic caching placement to maximize the cache hit performance. We first obtain the following lemma.

Lemma 1. The cache hit probability of the studied large-scale cache-enabled D2D network is given by

$$
\rho_d = 1 - \sum_{f=1}^{F} p_f e^{-\lambda_m c_f \pi R_d^2}, \tag{6}
$$

where F denotes the size of library F

Proof. Given p_f , c_f , and R_d , (6) can be immediately obtained by characterizing the void probability that none of the MHs within a distance of R_d caches the requested file. This thus completes the proof of Lemma [1](#page-7-0).

Based on (6), the optimization problem for maximizing the cache hit probability is defined as

$$
\max_{cf} \rho d \tag{7}
$$

$$
s.t. \ c_f \ge 0,\tag{8}
$$

$$
\sum_{f=1}^{F} c_f = 1.
$$
\n(9)

It is worth noting that the second order derivative of the objective function (7) is strictly negative over the feasible set of c_f . As such, ρ_d is a concave function with respect to c_f . By introducing the Lagrangian multipliers γ_f and v on (8) and (9), respectively, we obtain the Lagrangian function of (7) as

$$
L = -\rho_d - \sum_{f=1}^{F} \gamma_f c_f + v(\sum_{f=1}^{F} c_f - 1),
$$
\n(10)

which leads to the following Karush-Kuhn-Tucker (KKT) optimality conditions [\[18](#page-15-0)]

$$
-\lambda_m p_f \pi R_d^2 \cdot e^{-\lambda_m c_f \pi R_d^2} - \gamma_f + \nu = 0, \forall f,
$$
\n(11)

$$
\sum_{f=1}^{F} c_f - 1 = 0,\t\t(12)
$$

$$
\gamma_f.c_f = 0, \forall f,\tag{13}
$$

$$
c_f \ge 0, \forall f,\tag{14}
$$

$$
\gamma_f \ge 0, \forall f. \tag{15}
$$

Further, with (11) (11) , (13) (13) and (15) , it can be easily verified that

$$
\nu - \lambda_m p_f \pi R_d^2 \cdot e^{-\lambda_m c_f \pi R_d^2} \ge 0, \forall f,
$$
\n(16)

and

$$
(\nu - \lambda_m p_f \pi R_d^2 \cdot e^{-\lambda_m c_f \pi R_d^2}). c_f = 0, \forall f,
$$
 (17)

respectively.

It is worth noting that for $v < \lambda_m p_f \pi R_d^2$, (16) holds only if $c_f > 0$. As such, based on (17) , we have

$$
c_f = -\frac{1}{\lambda_m \pi R_d^2} \ln \left(\frac{v}{\lambda_m p_f \pi R_d^2} \right),\tag{18}
$$

for $v < \lambda_m p_f \pi R_d^2$. On the other hand, for $v \ge \lambda_m p_f \pi R_d^2$, it can be easily verified from (17) that

$$
c_f = 0.\t\t(19)
$$

By combining the results derived in (18) and (19), we then obtain that

$$
c_f = \left(-\frac{1}{\lambda_m \pi R_d^2} \ln \left(\frac{v}{\lambda_m p_f \pi R_d^2}\right)\right)^+, \tag{20}
$$

where $(z)^{+}$ = max {0, z}. To characterize the optimal solutions of c_f , we substitute (20) for c_f into [\(12](#page-5-0)) and obtain that

$$
\sum_{f=1}^{F} \left(-\frac{1}{\lambda_m \pi R_d^2} \ln \left(\frac{v}{\lambda_m p_f \pi R_d^2} \right) \right)^+ = 1. \tag{21}
$$

Then, based on (21), by applying a computational procedure, the optimal value of c_f , which is denoted by c_f^* , is readily obtained.

Remark 3.1. It can be observed from the analysis that the optimal solution c_f^* , depends on the density of MHs λ_m , the D2D communication range R_d , and the user request distribution p_f .

4 Transmission Probability

In this section, we characterize the transmission probability of MHs in large-scale cache-enabled D2D networks under the proposed contention based multimedia delivery protocol. Particularly, let ζ_d denote the probability that a MH is under request. We first derive ζ_d in the following lemma.

Lemma 2. For large-scale cache-enabled D2D networks, the probability that a MH is under request is given by

$$
\varsigma_d \sum_{f=1}^F c_f \cdot \left(1 - e^{-\lambda_u p_f \pi R_d^2}\right). \tag{22}
$$

Proof. Given p_f , c_f , and R_d , (22) can be immediately obtained by considering the void probability that there is no UE request of the file cached at the tagged MH within a distance of R_d . This thus completes the proof of Lemma 2.

Let Ψ_m^r be the point process formed by the MHs under request and let λ_m^r be the responding density. Then based on Lemma 2, we obtain the following corollary corresponding density. Then, based on Lemma 2, we obtain the following corollary.

Corollary 1. For large-scale cache-enabled D2D networks, the density of Ψ_m^r is given by by

$$
\lambda_m^r = \lambda_m . \varsigma_d. \tag{23}
$$

It is worth noting that for MHs within a distance of R_d , the requests of UEs are spatially correlated. As such, Ψ_m^r does not follow a HPPP. Furthermore, since the higher order statistics of Ψ^r are intractable, the transmission probability of MHs, which higher order statistics of Ψ_m^r are intractable, the transmission probability of MHs, which
depends on the probability generating functional (PGFL) [19] of Ψ^r is difficult to be depends on the probability generating functional (PGFL) [\[19](#page-15-0)] of Ψ_m^r is difficult to be
characterized exactly. To tackle this difficulty, similar to [20, 22], we make the folcharacterized exactly. To tackle this difficulty, similar to $[20-22]$ $[20-22]$ $[20-22]$ $[20-22]$, we make the following approximation on Ψ_m^r , which will be verified later by simulations in Sect. [6](#page-11-0).

Assumption 1. For large-scale cache-enabled D2D networks, Ψ_m^r follows a HPPP with density \mathcal{V}^r with density λ_m^r
With (22) s

With (22) and (23), based on Assumption 1, we are ready to evaluate the transmission probability of MHs under the proposed contention based multimedia delivery protocol, as given by following theorem.

Fig. 2. Conditional distribution of active MH under the proposed contention based multimedia delivery protocol.

Theorem 1. For large-scale cache-enabled D2D networks, under the proposed contention based multimedia delivery protocol, based on Assumption [1](#page-7-0), the transmission probability of MHs is given by

$$
qd = \zeta d \cdot \vartheta d,\tag{24}
$$

where ϑ_d denotes the probability that a MH in Ψ_m^r is eventually allowed to launch the transmission as transmission as

$$
\vartheta_d = \frac{1 - e^{-\pi \lambda_m^r \Gamma \left(1 + \frac{2}{\alpha}\right) \left(\frac{N_d}{p_d}\right)^{-\frac{2}{\alpha}}}}{\pi \lambda_m^r \Gamma \left(1 + \frac{2}{\alpha}\right) \left(\frac{N_d}{p_d}\right)^{-\frac{2}{\alpha}}}.
$$
\n(25)

Proof. The proof of Theorem 1 is omitted due to space limitation.

Remark 4.1. It can be easily verified from Theorem 1 that $\xi_d < 1$ and thereby $q_d < \zeta_d$.

Let Ψ_m^a be the point process formed by active MHs under the proposed contention
ed multimedia delivery protocol and let λ^a be the corresponding density. Based on based multimedia delivery protocol and let λ_m^a be the corresponding density. Based on Theorem 1, we obtain the following corollary Theorem 1, we obtain the following corollary

Corollary 2. For large-scale cache-enabled D2D networks, under the proposed contention based multimedia delivery protocol, based on Assumption [1](#page-7-0), the density of Ψ_m^a is given by

$$
\lambda_m^a = \lambda_m.q_d. \tag{26}
$$

In the following section, with Theorem [1](#page-8-0), we characterize the successful content delivery probability of large-scale cache-enabled D2D networks under the proposed contention based multimedia delivery protocol.

5 Successful Content Delivery Probability

To analyze the successful content delivery probability of the studied large-scale cache-enabled D2D networks, thanks to the stationarity of the point processes formed by the MHs and UEs, we focus on a typical UE at the origin denoted by U with its associated MH at a random distance of d_f away denoted by M. Let $\Psi_m^{\text{U}}(u)$ and $\Psi_m^{\text{M}}(u)$
be the point processes formed by the active MHs on a circle of radius u centered at be the point processes formed by the active MHs on a circle of radius u centered at **U** and **M**, respectively, as illustrated in Fig. [2](#page-8-0). Further, let $\lambda_m^{\text{U}}(u)$ and $\lambda_m^{\text{M}}(u)$ be the overage density of $W^{\text{U}}(u)$ and $W^{\text{M}}(u)$ respectively. average density of $\Psi_m^{\text{U}}(u)$ and $\Psi_m^{\text{M}}(u)$, respectively.

In the following, we first characterize the distributions of $\Psi_m^U(u)$ and $\Psi_m^M(u)$,
pectively and then derive the successful content delivery probability of the respectively, and then derive the successful content delivery probability of the large-scale cache-enabled D2D network.

Lemma 3. Under the proposed contention based multimedia delivery protocol, based on Assumption [1](#page-7-0), conditioned on a typical UE at the origin, $\Psi_m^{\text{M}}(u)$ is isotropic² with
respect to M with density $i^{\text{M}}(u)$ given by respect to **M** with density $\lambda_m^{\mathbf{M}}(u)$ given by

$$
\lambda_m^{\mathbf{M}}(u) = \lambda_m^a \left(1 - e^{-\frac{N_d u^2}{P_d}} \right). \tag{27}
$$

Proof. Conditioned on a typical UE at the origin, under Assumption [1](#page-7-0), the probability that a MH on a circle of radius u centered at **M** is active is given by q_d . Pr $\left(h \leq \frac{N_d u^2}{P_d}\right)$, where h denotes an exponentially distributed random variable with unit mean. Based on this result, it can be easily verified that $\Psi_m^M(u)$ is isotropic around **M** with density $\lambda_m^{\mathcal{M}}(u)$ given by (27). This thus completes the proof of Lemma 3.
It is worth noting that due to the contentions among the

It is worth noting that due to the contentions among the MHs, even under Assumption [1,](#page-7-0) $\Psi_m^M(u)$ does not follow a HPPP. As a result, with only the first-order
moment measures (overage densities) of $\Psi_m^M(u)$ being obtained, the successful content moment measures (average densities) of $\Psi_m^M(u)$ being obtained, the successful content
delivery probability of the studied large-scale cache-enabled D2D network is difficult to delivery probability of the studied large-scale cache-enabled D2D network is difficult to be characterized exactly. To tackle this difficulty, we make the following approximation on the conditional distribution of the active MHs, which will be verified later by simulations in Sect. [6](#page-11-0).

Assumption 2. Under the proposed contention based multimedia delivery protocol, based on Assumption 2, conditioned on a typical UE at the origin, $\Psi_m^{\text{M}}(u)$ follows a
HPPP with density $\mathcal{M}(\mu)$ HPPP with density $\lambda_m^{\mathbf{M}}(u)$.

² A point process $\mathcal N$ is isotropic if its characteristics are invariant under rotation [[24\]](#page-15-0).

Based on Lemma [3](#page-9-0) and Assumption [2](#page-9-0), we characterize the distribution of $\Psi_m^{\text{U}}(u)$ in following two lemmas the following two lemmas.

Lemma 4. Under the proposed contention based multimedia delivery protocol, conditioned on a typical UE at the origin, an upper bound on $\lambda_m^{\text{U}}(u)$ is given by

$$
\lambda_m^{\text{U}}(u) \le \lambda_m^a \cdot \left(1 - e^{-\frac{N_d(u + d_f)^2}{p_d}}\right). \tag{28}
$$

Proof. The proof immediately follows from Fig. [2](#page-8-0) by observing that the highest density of $\Psi_m^{\text{U}}(u)$ is $\lambda_m^{\text{M}}(u+d_f)$.

Lemma 5. Under the proposed contention based multimedia delivery protocol, based on Assumption [2](#page-9-0), conditioned on a typical UE at the origin, the following inequality on $\lambda_m^{\rm U}(u)$ holds:

$$
\int_0^\infty \frac{\lambda_m^{\mathrm{U}}(u)}{1 + \frac{u^2}{\theta_d d_f^2}} u du \ge \int_0^\infty \frac{\lambda_m^{\mathrm{M}}(u)}{1 + \frac{u^2}{\theta_d d_f^2}} u du. \tag{29}
$$

Proof. The proof of Lemma 5 is omitted due to space limitation.

With Lemmas 4 and 5, we are ready to evaluate the successful content delivery probability of this D2D network, as given by the following theorem.

Theorem 2. For large-scale cache-enabled D2D networks, under the proposed contention based multimedia delivery protocol, based on Assumption [2](#page-9-0), the successful content delivery probability is lower-bounded and upper-bounded, respectively, by

$$
\tau_d \ge \sum_{f=1}^F p_f \int_0^{R_d} \exp\left\{ -(1 - c_f) \int_0^{\infty} \eta(u + d_f)u \, du \right\}
$$
\n
$$
\times \exp\left\{ -c_f \int_{d_f}^{\infty} \eta(u + d_f)u \, du \right\} \cdot \overline{\omega}_f(d_f) \, d_f,
$$
\n
$$
\tau_d \le \sum_{f=1}^F p_f \int_0^{R_d} \exp\left\{ -(1 - c_f) \int_0^{\infty} \eta(u)u \, du \right\}
$$
\n
$$
\times \exp\left\{ -c_f \int_{d_f}^{\infty} \eta(u - d_f)u \, du \right\} \cdot \overline{\omega}_f(d_f) \, d_f,
$$
\n(31)

where $\eta(x)$ is given by

$$
\eta(x) = 2\pi\lambda_m^a \cdot \frac{\left(1 - e^{-\frac{N_d x^2}{P_d}}\right)}{1 + \frac{u^2}{\theta_d d_f^2}},\tag{32}
$$

and $\varpi_f(d_f)$ is given by

$$
\varpi_f(d_f) = 2\lambda_m^a c_f \pi d_f \cdot e^{-\lambda_m^a c_f \pi d_f^2}.
$$
\n(33)

Proof. The proof of Thereom [2](#page-10-0) is omitted due to space limitation.

6 Numerical Results

In this section, we present simulation results on the performance of the cache-enabled D2D network to validate our analytical results. Throughout this section, unless specified otherwise, we set up $P_d = 10, P_d/N_d = 5, \theta_d = 2, R_d = 25, \gamma = 1$, and $\alpha = 4$.

6.1 Cache Hit Probability

Figure 3 shows the cache hit probability ρ_d versus the number of files in library F when λ_m = 0.05 and 0.01, respectively. It is observed in Fig. 3 that cache hit probability is a decreasing function of F and λ_m . We also compare the proposed optimal probabilistic caching strategy with the random caching strategy in terms of the cache hit performance. It is also observed that the proposed optimal probabilistic caching strategy outperforms the Zipf-like random caching strategy.

Fig. 3. Cache hit probability ρ_d versus the number of files in library F.

Fig. 4. Transmission probability q_d versus the density of UEs λ_u .

6.2 Transmission Probability

Figure 4 shows the analytical and simulated results on the transmission probability q_d versus the density of UEs λ_u when $\lambda_m = 0.005, 0.01$, and 0.05, respectively, with $F = 20$. It is observed that the transmission probability in the cache-enabled D2D network is an increasing function of λ_{u} , while a decreasing function of λ_{m} , which are intuitively expected according to Theorem 4.1. It is also observed that the simulation results fit closely to our analytical results.

6.3 Successful Content Delivery Probability

Figure [5](#page-13-0) compares the analytical and simulated results on the successful content delivery probability τ_d versus the transmission probability λ_u , when $\lambda_m = 0.005$, and 0.01, respectively. It is observed that the simulated successful content delivery probability of the cache-enabled D2D network falls between the upper and lower bounds derived in Theorem 5.1 as expected. It is also observed that the successful content delivery probability τ_d is an increasing function of the density of UEs λ_u .

Fig. 5. Successful content delivery probability τ_d versus the density of UEs λ_u for $F = 5$, where CBMD stands for the contention based multimedia delivery protocol.

7 Conclusion

This paper has studied the performance of large-scale cache-enabled D2D networks with contention based multimedia delivery protocol. By applying tools from stochastic geometry, the optimal caching probability is derived and analyzed. It is shown in the

analytical results that the optimal solution of the probabilistic caching placement depends on the density of MHs, the D2D communication range, and the user request distribution. With the derived optimal caching probabilities, we further characterize the transmission probability of MHs and thereby the successful content delivery probability of the cache-enabled D2D network. Simulations are provided to validate our analysis. It is hoped that the results in this paper will provide new insights to the optimal design of large-scale cache-enabled D2D networks.

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