LR-RZF Pre-coding for Massive MIMO Systems Based on Truncated Polynomial Expansion

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Abstract. In order to effectively eliminate multi-user interference at the transmitter, the transmit signals are needed to be pre-processing, which is called pre-coding. Traditional linear pre-coding algorithms, especially regularized zero forcing (RZF) pre-coding, are famous for good performance and low computation complexity. However, they cause noise amplification which requires high transmit power to prevent. Lattice-reduction aided (LR-aided) technique is used to deal with the row/column of matrix, which can make the matrix orthogonality better. Therefore, to avoid noise amplification as well as effectively eliminate the multi-user interference, we propose a LR-RZF pre-coding algorithm which based on matrix truncated polynomial expansion (TPE) with J terms. TPE method can decrease the complexity of matrix inversion in RZF. Compared with RZF pre-coding, LR-RZF pre-coding has lower bit error rate.

Keywords: LR · RZF · TPE · Massive MIMO

1 Introduction

Massive MIMO systems is able to cope with the exponential growth in number of user terminals (UTs) and data traffic, so recently the deployment of massive MIMO has received a lot of attention. In this system, in order to effectively eliminate multi-user interference at the transmitter, the transmit signals are needed to be pre-processing, which is pre-coding. Pre-coding can be divided into linear and nonlinear pre-coding, where the nonlinear pre-coding mainly has the dirty paper pre-coding [1] and the constant envelope pre-coding [2, 3]. The basic idea of the dirty paper pre-coding is to process the signals at the transmitter, so that the receiver cannot think of the interference exiting between users in the receive signals, which can increase the total system capacity. While the basic idea of the constant envelope pre-coding is to pre-process the

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information symbols, so that the signal amplitudes are the same, and the receiver can recover the signals according to the signal phases. Unfortunately, the complexity of two nonlinearly pre-coding algorithms are very high. Therefore, some scholars proposed some low complexity pre-coding algorithms, in which the most famous is the regularized zero forcing (RZF) pre-coding algorithm. However, its complexity increases sharply when the number of antennas is large as it needs to compute matrix inversion. Therefore, scholars proposed to replace the matrix inversion with approximate algorithm. In [4, 5], authors used the matrix truncated polynomial expansion (TPE) to replace the matrix inverse operation in RZF pre-coding algorithm to reduce the computation complexity. But in this two references, only the sum of the mean square error is minimized and the optimal polynomial coefficients through optimization of power allocation is obtained, respectively. The coherence of the base station transmit antennas is considered [6, 7], in which the matrix TPE is used to carry out the linear pre-coding, so as to avoid the computation of matrix inverse in the RZF pre-coding algorithm.

Lattice-Reduction (LR) technology is a kind of mathematical processing method [8, 9]. The basic idea is that, the columns of a matrix A can be interpreted as the basis of a lattice. The aim of the lattice reduction is to transform a given basis A into a new basis consisting of roughly orthogonal basis vectors. For the channel matrix, because each row/column vector is not orthogonal, and after the lattice reduction technique processing, it can makes the row/column vector orthogonality better. In this paper, we propose a LR-RZF pre-coding algorithm based on matrix TPE. The channel matrix is handled by LR first and gets good orthogonality. Then it is applied in RZF and RZF based on the TPE pre-coding. Furthermore, we simulate the bit error rate of those pre-coding algorithms.

The paper is organized as follows. In Sect. 2, we describe the system model. The LR-RZF pre-coding algorithm based on matrix TPE is presented in Sect. 3. The simulation results are in Sect. 4. Some conclusions are made in Sect. 5.

2 System Model

We consider a downlink massive MIMO system in which a base station (BS), equipped with N transmit antennas, serves K single-antenna UTs, where $N \gg K$. Suppose $\mathbf{h}_k \sim \mathrm{CN}(\mathbf{0}_{N\times 1}, \mathbf{\Phi}/\mathrm{K})$ represents the random channel vector between the transmit antennas at BS and the kth UT ($1 \leq k \leq K$), where Φ is the channel covariance matrix with $N \times N$, and $\mathbf{0}_{N\times 1}$ is a zero vector. The signals needed by the K single-antenna UTs can be represented by $\mathbf{s} = [s_1,\ldots,s_k,\ldots,s_K]^{\mathrm{T}}$, where s_k is the signal of the kth UT which satisfies $s_k \sim CN(0,1)$, and $(\cdot)^{\mathrm{T}}$ denotes the transpose of a matrix. Suppose $\mathbf{G} = [\mathbf{g}_1,\ldots,\mathbf{g}_k]$ is the pre-coding matrix with $N \times K$. The signal $\mathbf{s}$ is transmitted to the mobile UTs via the base station antennas after the pre-coding. So the received signal of the kth UT is

$$y_k = \mathbf{h}_k^{\mathrm{H}}\mathbf{G}\mathbf{s} + n_k = \mathbf{h}_k^{\mathrm{H}}\mathbf{g}_k s_k + \sum_{n=1,n\neq k}^{K} \mathbf{h}_k^{\mathrm{H}}\mathbf{g}_n s_n + n_k \tag{1}$$

where n_k is the additive circularly symmetric complex Gaussian noise at the kth UT, which satisfies $n_k \sim CN(0,\sigma^2)$, where σ^2 is the receiver noise variance. And $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose of a matrix or a vector. Suppose that the total transmit power satisfies as follow

$$\frac{1}{K}\mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}}) = P \tag{2}$$

where $\mathrm{tr}(\cdot)$ is the trace of a matrix.

Then receive signal is detected as

$$\hat{\mathbf{s}} = G^{-1}\Omega(y_k) \tag{3}$$

where $\Omega(\bullet)$ is detection rule.

We suppose that the transmitter does not know the perfect instantaneous channel state information (CSI) $\hat{\mathbf{h}}_k$ of each mobile UT. Based on the model in [10, 11], we suppose that the instantaneous channel of the kth UT obey Gauss-Markov distribution. In other words, $\hat{\mathbf{h}}_k$ can be expressed as follow

$$\hat{\mathbf{h}}_k = \sqrt{1-\tau^2}\mathbf{h}_k + \tau\mathbf{n}_k \tag{4}$$

where τ is a scalar parameter which indicates the quality of the instantaneous CSI, which satisfies $\tau \in [0, 1]$. We can get perfect instantaneous CSI and only statistical knowledge of channel for $\tau = 0$ and $\tau = 1$, respectively. Assuming that the joint imperfect channel matrix of all user channels can be denoted by $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \cdots, \hat{\mathbf{h}}_K]$, with $N \times K$.

3 LR-RZF Pre-coding Algorithm Based on the TPE

We conducted LR decomposition of the channel matrix $\hat{\mathbf{H}}$, assuming that the decomposition of the matrix $\hat{\mathbf{H}}$ into matrix $\tilde{\mathbf{H}}$, the two matrices are satisfied [8, 9]

$$\tilde{\mathbf{H}} = \hat{\mathbf{H}}\mathbf{T} \tag{5}$$

where $\mathbf{T}$ is a unimodular matrix with $K \times K$, it contains only Gaussian integers and det $(\mathbf{T}) = \pm 1$ or $\pm j$. The LR decomposition algorithm is listed in Algorithm 1.

Algorithm 1. LR algorithm

1. Initialize: $\mathbf{T}=\mathbf{I}$ and $\tilde{\mathbf{H}}=\mathbf{H}$
2. Set $[\mathbf{S}]_{ij}=[\mathbf{H}^H\mathbf{H}]_{ij}=s_{ij},[\mathbf{S}']_{ij}=[(\mathbf{H}^H\mathbf{H})^{-1}]_{ij}=s'_{ij}\ \forall i,j$
3. for i=1 to K do
4. for j=1 to K do
5. $v_{ij}\leftarrow\frac{s'_{ji}}{2s'_{ii}}-\frac{s_{ji}}{2s_{jj}}$
6. $\lambda_{ij}\leftarrow\lfloor v_{ij}\rceil$
7. $\Delta_{ij}\leftarrow 2s_{jj}s'_{ii}\{\mathrm{real}(2\lambda^*_{ij}v_{ij}\text{-}|\lambda_{ij}|^2)\}$
8. end for
9. end for
10. $(n,l)\leftarrow\arg\max_{(i,j)}\Delta_{ij}$
11. $\mathbf{t}^u_n\leftarrow\mathbf{t}_n+\lambda_{n,l}\mathbf{t}_l$
12. $\mathbf{T}\leftarrow[\mathbf{t}_1,\cdots,\mathbf{t}_{n-1},\mathbf{t}^u_n,\mathbf{t}_{n+1},\cdots,\mathbf{t}_N]$
13. $\tilde{\mathbf{h}}^u_n\leftarrow\mathbf{h}_n+\lambda_{n,l}\tilde{\mathbf{h}}_l$
14. $\tilde{\mathbf{H}}\leftarrow[\tilde{\mathbf{h}}_1,\cdots,\tilde{\mathbf{h}}_{n-1},\tilde{\mathbf{h}}^u_n,\tilde{\mathbf{h}}_{n+1},\cdots,\tilde{\mathbf{h}}_N]$
15. Repeat 1 to 14, until all $\Delta_{ij}=0$, we can get $\tilde{\mathbf{H}}$ and $\mathbf{T}$, which are the last $\tilde{\mathbf{H}}$ and the product of all $\mathbf{T}$, respectively.

Similar to [6, 11], we define LR-RZF pre-coding algorithm as

$$\mathbf{G}_{\mathrm{LR-RZF}}=\beta\tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}^{\mathrm{H}}\tilde{\mathbf{H}}+\xi\mathbf{I}_K\right)^{-1}=\beta\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathrm{H}}+\xi\mathbf{I}_N\right)^{-1}\tilde{\mathbf{H}} \tag{6}$$

where β is the power normalization parameter, which is set such that the pre-coding matrix $\mathbf{G}_{\mathrm{RZF}}$ satisfies the power constraint in (2). The parameter ξ depends on the total transmit power P, the noise variance σ^2, the scalar parameter τ affecting the instantaneous CSI, and system dimensions [11, 12].

Inspecting (6) shows that, when the base station antenna number N is very large, the complexity of the $N\times N$ matrix $\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathrm{H}}+\xi\mathbf{I}_N$ inversion is very high. In order to reduce the computational complexity, the matrix inversion can be replaced by a matrix TPE. From [11] we know that for any positive definite Hermitian matrix $\mathbf{X}$, when the parameter α is selected such that $0<\alpha<\frac{2}{\max_n\lambda_n(\mathbf{X})}$, the inverse of the matrix $\mathbf{X}$ can be expressed as

$$\mathbf{X}^{-1} = \alpha(\mathbf{I} - (\mathbf{I} - \alpha\mathbf{X}))^{-1} = \alpha\sum_{l=0}^{\infty}(\mathbf{I} - \alpha\mathbf{X})^{l} \tag{7}$$

By substituting (7) and the expression $(a+b)^l = \binom{l}{n} a^{l-n} b^n$ into (6), we can get the polynomial expansion of the pre-coding matrix $\mathbf{G}_{\text{LR-RZF}}$, and we just only consider the first J terms because of the low-order terms are the most influential ones. So we have

$$\mathbf{G}_{\text{LR}-\text{RZF}} = \beta\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\text{H}} + \xi\mathbf{I}_N\right)^{-1}\tilde{\mathbf{H}} \approx \sum_{l=0}^{J-1}\omega_l\left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\text{H}}\right)^{l}\tilde{\mathbf{H}} \tag{8}$$

where ω_l is a scalar parameter, and its value is seen in [11].

4 Simulation results

In this section, we calculate the bit error rate performance of pre-coding algorithms, including RZF, LR-RZF, RZF based on the TPE and LR-RZF based on the TPE. The MIMO system has 8 transmit antennas in the BS and 8 antennas at receiver. The transmit symbols are modulated by BPSK and QPSK respectively. The detection method is MMSE.

According to Fig. 1, it can be seen that the BER performance of LR-RZF is better than RZF in the same condition using BPSK modulation. The channel matrix **H** is generated randomly which is independent and identically complex Gaussian random variables. When **H** is handled by LR algorithm, it is more orthogonal and the channel condition is improved. Therefore, at the target BER of 10^{-4}, LR-RZF gains about 4 dB transmit power compared with RZF which needs 25 dB SNR. What is more, LR-RZF can get 10^{-6} BER performance at 45 dB. However, the BER performance of RZF

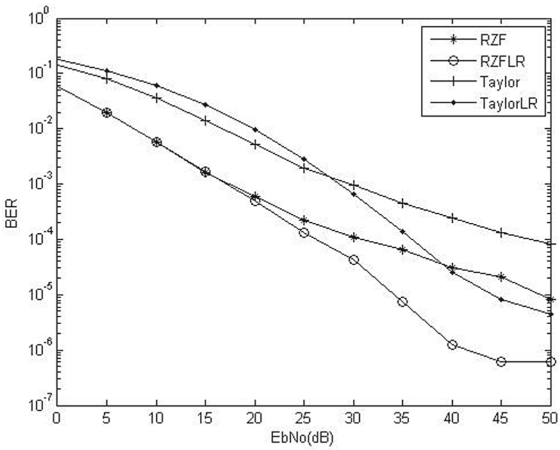

Fig. 1. BER performances of RZF pre-coding schemes with BPSK modulation

based on the TPE is worse than RZF because it estimates the matrix inversion with 3-order TPE. At low SNR, the performance of LR-RZF based on the 3-order TPE is worse than RZF based on the 3-order TPE. When SNR is 27 dB, two pre-coding has similar BER performance. With the increase of SNR, LR-RZF based on the 3-order TPE gains better performance. Besides, LR-RZF based on the 3-order TPE is able to get 10^{-5} BER performance at 50 dB.

According to Fig. 2, the transmit signal is modulated by QPSK. At the target BER of 10^{-3}, RZF needs about 28 dB SNR. LR-RZF gains about 3 dB transmit power compared with RZF. LR-RZF can get nearly 10^{-5} BER performance at 45 dB. At low SNR, LR-RZF based on 3-order TPE is not good. When SNR is 26 dB, LR-RZF based on 3-order TPE has similar performance compared with RZF based on the 3-order TPE. With the increase of SNR, LR-RZF based on 3-order TPE has better BER performance. At about 50 dB, it can get 10^{-5} BER performance.

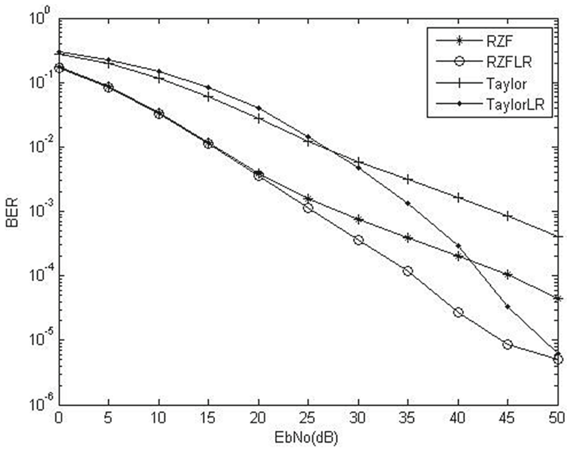

Fig. 2. BER performances of RZF pre-coding schemes with QPSK modulation

5 Conclusion

In this paper, we proposed a LR-RZF pre-coding algorithm based on the TPE. We apply LR algorithm to deal with channel matrix and make it more orthogonal. Then the improved channel matrix is used in RZF and RZF based on the TPE pre-coding. Therefore, improved pre-coding algorithm can decrease the noise amplification and multi-user interference. It can be seen that LR-RZF gets good BER performance than RZF. LR-RZF based on the TPE is not good when SNR is low, but gets better BER performance with the increase of SNR.

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