Complexity Analysis of Massive MIMO Signal Detection Algorithms Based on Factor Graph

Zhichao Yao(✉), Chao Dong, Kai Niu, and Zhiqiang He

Key Lab of Universal Wireless Communications,
Beijing University of Posts and Telecommunications, Beijing, China
{mr_cuber,dongchao,niukai,hezq}@bupt.edu.cn

Abstract. Massive MIMO technology is one of the most promising concepts in 5G wireless system. In the uplink of a massive MIMO system, complexity and performance of signal detection are two key issues been concerned simultaneously. Many message passing algorithms based on factor graph have claimed to achieve nearly optimal performance at low complexity. A unified factor graph model is introduced to describe two typical message passing algorithm,approximate message passing (AMP) and message passing detection (MPD). By analyzing different message calculation methods in the two algorithms, their computational complexity and performance are given in detail. Simulation results have shown that MPD exceeds AMP in both complexity and performance.

Keywords: Massive MIMO · Signal detection · Factor graph
Message passing · Channel hardening · Computation complexity

1 Introduction

Massive MIMO with tens of hundreds of antennas at the BS can significantly improve the system capacity and spectrum efficiency, which is considered as a candidate technology for 5G standards [1]. However, as the number of antennas grows, the computational complexity of detectors has become the bottleneck of its hardware implementation. Some detectors that work well in traditional MIMO fail in massive MIMO. For example, the complexity of optimal maximum likelihood (ML) detector scales exponentially in the number of transmit antennas, and conventional linear detectors like zero-forcing (ZF) and minimum-mean-square-error (MMSE) involve complicated matrix inversion, are difficult to simultaneously satisfy high-performance and low-complexity requirement of massive MIMO. Thus, finding a low-complexity detection algorithm on the uplink in massive MIMO, while maintaining good performance, is necessary.

Researches have shown that iterative detection algorithms based on factor graph can achieve performance that close to ML, meanwhile the computation complexity scales linearly, rather than exponentially in the number of transmit antennas [2–4]. There are two main directions to solve this problem. One is message passing algorithm working in complex-value domain [2,3]. The other

K. Long et al. (Eds.): 5GWN 2017, LNICST 211, pp. 246–256, 2018.
https://doi.org/10.1007/978-3-319-72823-0_24

is message passing exploiting channel-hardening working in real-value domain [4]. These algorithms all claim that nearly ML performance has been achieved, but computation complexity comparison between these algorithms has not been done so far.

Therefore, in this paper, we select two typical algorithms, approximate message passing (AMP) and message passing detection (MPD) representing those two directions previously mentioned. First we described these two algorithms by using unified system model and message passing graphical model, then compared their computation complexity and performance in detail.

The rest of the paper is organized as follows. We first introduce the system model in Sect. 2. Two message passing detection algorithms based on factor graph are described in Sect. 3. Complexity analysis and simulation results are presented in Sects. 4 and 5, respectively. Section 6 concludes this paper.

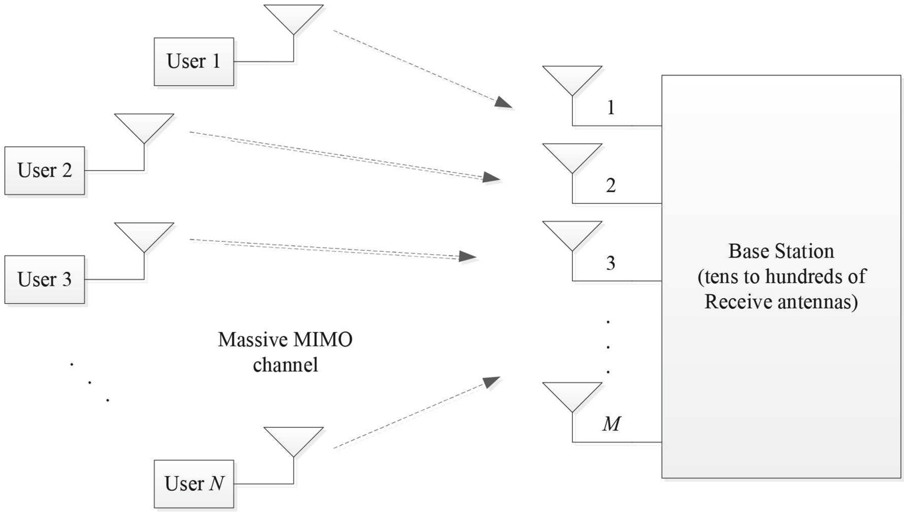

Fig. 1. Multi-user massive MIMO system on the uplink

2 System Model

Consider a multi-user massive MIMO system with N independent users, where each user is equipped with one transmit antenna, and the receiver is equipped with an array of M antennas, M is in the range of tens to hundreds [5]. System load factor φ is defined as N/M. The system model is illustrated in Fig. 1. For each user, every Q information bits are mapped to one modulation symbol. Let $\mathbf{x}_c = [x_1^c, x_2^c, ...x_N^c]^T$ be the transmitted vector from all the users, where $x_n^c \in \mathbb{B}$ is the symbol transmitted from the nth user and $\mathbb{B}$ is the modulation alphabet. Let $\mathbf{H}_c \in \mathbb{C}^{M\times N}$ denote the channel gain matrix and h_{ij}^c denote the complex channel gain from the jth user to the ith BS antenna. The elements of $\mathbf{H}_c$, which follow $\mathbb{C}\mathcal{N}(0,1)$, are assumed to be independent identically distributed (i.i.d.). It is assumed that the N transmitters and the receiver are perfect synchronized and all channel gains are known at the receiver. Then the received vector can be presented as

$$\mathbf{y}_c = \mathbf{H}_c\mathbf{x}_c + \mathbf{n}_c, \tag{1}$$

where $\mathbf{n}_c$ is the additive white Gaussian noise (AWGN) vector whose entries follow $\mathbb{C}\mathcal{N}(0,\sigma_n^2)$. The average received SNR per receive antenna is given by $\gamma = NE_s/\sigma_n^2$, where E_s is the average per transmitted symbol.

The MPD algorithm to be introduced works in real-value domain, so (1) can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where

$$\mathbf{H} \triangleq \begin{bmatrix} \Re(\mathbf{H}_c) & -\Im(\mathbf{H}_c) \\ \Im(\mathbf{H}_c) & \Re(\mathbf{H}_c) \end{bmatrix}, \mathbf{y} \triangleq \begin{bmatrix} \Re(\mathbf{y}_c) \\ \Im(\mathbf{y}_c) \end{bmatrix}, \mathbf{x} \triangleq \begin{bmatrix} \Re(\mathbf{x}_c) \\ \Im(\mathbf{x}_c) \end{bmatrix}, \mathbf{n} \triangleq \begin{bmatrix} \Re(\mathbf{n}_c) \\ \Im(\mathbf{n}_c) \end{bmatrix},$$

$\Re(\cdot)$,$\Im(\cdot)$ denotes the real and imaginary part, respectively.

For a QAM modulation alphabet $\mathbb{B}$, the elements of $\mathbf{x}$ will take value from the underlying pulse-amplitude modulation (PAM) alphabet $\mathbb{A}$.

3 Two Message Passing Algorithms

In this section, we introduce two message passing algorithms based on factor graphs dedicated to the detection of massive MIMO. 16-QAM gray-mapping modulation is considered.

3.1 Factor Graph Model of Message Passing Algorithm

Detection algorithms based on FG (Factor Graph) is briefly introduced in this section [2]. Consider the MIMO system model in (1) or (2), each entry of the received vector (or observation vector) is seen as a function node $f_j, j = 1, 2, n\_f$(number of function nodes) in a factor graph, and each transmitted symbol as a variable node $x_i, i = 1, 2, n\_x$(number of variable nodes). Figure 2 illustrates this graph model. The job of MIMO detection is using the knowledge of received vector and channel matrix to obtain an estimate of transmitted vector. Message passing algorithms are carried out on the factor graph by passing messages between the variable and function nodes.

3.2 MPD Algorithm

As proposed in [4], the MPD algorithm exploits channel hardening that occurs in massive MIMO channel.

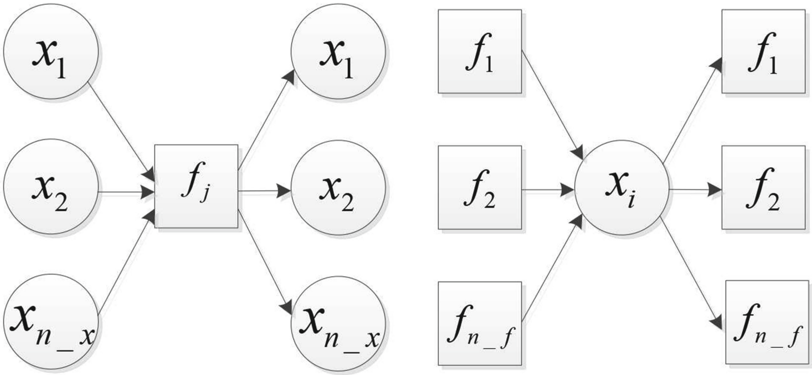

Fig. 2. Message passing between function and variable nodes on FG

Channel Hardening. As the dimension of the channel gain matrix $\mathbf{H}$ increases, the off-diagonal terms of the $\mathbf{H}^T\mathbf{H}$ matrix become increasingly weaker compared to the diagonal terms. This phenomenon is called channel hardening in [6].

In Sect. 3.2, we will work with approximations to the off-diagonal terms of the $\mathbf{H}^T\mathbf{H}$ matrix, which achieves very good performance in large dimensions at low-complexity.

MPD Algorithm. By performing matched filter operation on (2), we have

$$\mathbf{H}^T\mathbf{y} = \mathbf{H}^T\mathbf{H}\mathbf{x} + \mathbf{H}^T\mathbf{n}. \tag{3}$$

From (3), we write the following:

$$\mathbf{z} = \mathbf{G}\mathbf{x} + \mathbf{v}, \tag{4}$$

where

$$\mathbf{z} \triangleq \frac{\mathbf{H}^T\mathbf{y}}{M}, \mathbf{y} \triangleq \frac{\mathbf{H}^T\mathbf{H}}{M}, \mathbf{x} \triangleq \frac{\mathbf{H}^T\mathbf{n}}{M}.$$

The ith element of $\mathbf{z}$ can be written as

$$z_i = G_{ii}x_i + \underbrace{\sum_{j=1, j\neq i}^{2N} G_{ij}x_j + v_i}_{\triangleq k_i} \tag{5}$$

where G_{ij} is the (i,j)th of $\mathbf{G}$, x_i is the ith element of x, and

$$v_i = \sum_{j=1}^{2N} \frac{H_{ji}n_j}{M} \tag{6}$$

is the ith element of v, where H_{ji} is the (j,i)th element of H. The variable k_i defined in (5) denotes the interference-plus-noise term, which involves the off-diagonal elements of $\mathbf{G}$(i.e., H_{ji}, $i \neq j$). The distribution of k_i is approximated as $\mathbb{CN}(\mu_i, \sigma_i^2)$. By central limit theorem, this approximation is accurate for large M, N. Since the elements in $\mathbf{x}$, $\mathbf{v}$ are independent, the mean and variance in this approximation are given by

$$\mu_i = \mathbb{E}(k_i) = \sum_{j=1, j\neq i}^{2N} G_{ij}\mathbb{E}(x_j) \tag{7}$$

$$\sigma_i^2 = \mathrm{Var}(k_i) = \sum_{j=1, j\neq i}^{2N} G_{ij}^2 \mathrm{Var}(x_j) + \sigma_v^2. \tag{8}$$

Denoting the probability of the symbol x_j as $p_j(s)$,we have

$$\mathbb{E}(x_j) = \sum_{\forall s in \mathbb{A}} s p_j(s), \mathrm{Var}(x_j) = \sum_{\forall s in \mathbb{A}} s^2 p_j(s) - \mathbb{E}(x_j)^2$$

where $\sigma_v^2 = \sigma_n^2/2M$. Due to the above Gaussian approximation, the *a posteriori* probability (APP) of x_i being $s \in \mathbb{A}$ is computed as

$$p_i(s) \propto \exp\left(\frac{-1}{2\sigma_i^2}\left(z_i - G_{ii}s - \mu_i\right)^2\right). \tag{9}$$

Message Passing. First of all, because of the above matched filter operation, observation vector became a $2N \times 1$ vector, which means that the number of function and variable nodes is $2N$. As shown in Fig. 3, the messages passed from variable node x_j to any function node are $\mathbb{E}(x_j)$ and $\mathrm{Var}(x_j)$, the computation of which needs $p_i(s)$. Then, we use the knowledge of $\mathbf{G}$, i.e. channel matrix and the message passed to function node f_j to compute the mean μ_i and variance σ_i^2 of interference-plus-noise term k_j. Moreover, $p_i(s)$ is computed using μ_i, σ_i^2 and jth elements in observation vector. As we can see, the major difference in Fig. 3 comparing to Fig. 2 is that computation of messages coming from x_j only needs the messages from function node f_j, rather than all $2N$ function nodes. Therefore, computation complexity of MPD is significantly reduced.

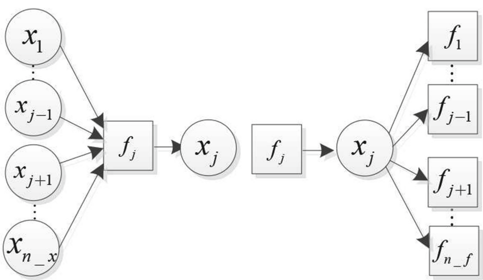

Fig. 3. Message passing between function and variable nodes for MPD

Damping Message. In the above graphical model, the message passing algorithm may fail to converge, and even if it does converge, the estimated probabilities may be far from exact. In [7], a damping method intended to improve the rate of convergence is proposed. The damped message to be passed in iteration t is computed as a weighted average of the message in iteration $t-1$ and the message computed at the tth iteration with a damping factor $\Delta \in [0, 1)$. In [3], it is shown that $\Delta = 0.33$ is optimal, Thus, let $\tilde{p}_i^t$ be the computed probability at the tth iteration, the message at the end of tth iteration is

$$p_i^t = (1 - \Delta)\,\tilde{p}_i^t + \Delta p_i^{t-1}. \tag{10}$$

The algorithm is initialized with $p_i(s) = 0.25, \forall s \in \mathbb{A}$ and terminates after a fixed number of iterations. Finally, the bit as

$$\Pr(b_i^p = 1) = \sum_{\forall s\in\mathbb{A}:\text{pth bit in s is 1}} p_i(s), \tag{11}$$

where b_i^p is the pth bit in the ith user's symbol. A hard estimate of bit b_i^p can be obtained as

$$b_i^p = \begin{cases} 1 \ if \ \Pr(b_i^p = 1) > 0.5 \\ 0 \ otherwise. \end{cases}$$

3.3 AMP Algorithm

Message passing algorithms referred as AMP (Approximate Message Passing) based on factor graph and its variants are proposed in [2]. In typical AMP algorithm, the message is modeled as Gaussian random variable. In addition, the messages of all the edges shown in Fig. 2 should be calculated. The focus of AMP is mainly on the mean and variance updating in message passing. In the following, $\mathcal{N}_{\mathbb{C}}(x; a, b) = (\pi b)^{-1}\exp(-|x-a|^2/b)$ denotes complex Gaussian function, where x, a, b denotes the random variable, the mean, the variance, respectively. The conditional probability $p(\mathbf{y}_c|\mathbf{x}_c)$ can be factorized into:

$$p(\mathbf{y}_c|\mathbf{x}_c) = \prod_j p_j(y_j^c|\mathbf{x}_c), \tag{12}$$

where

$$p_j(y_j^c|\mathbf{x}_c) = \frac{1}{\pi\sigma_n^2}\exp\left(-\frac{|y_j^c - \sum_i h_{ji}^c x_i^c|^2}{\sigma_n^2}\right). \tag{13}$$

This factorization is represented by the factor graph in Fig. 2. Let $\mu^t_{x_i\to f_j}(x_i^c)$ denotes the message sent from the variable node x_i to the function node f_j in the tth iteration, and let $\mu^t_{f_j\to x_i}(x_i^c)$ denotes the opposite message. The message-update rules are given by

$$\mu^t_{x_i\to f_j}(x_i^c) = \prod_{j'\neq j}\mu^{t-1}_{f_{j'}\to x_i}(x_i^c), \tag{14}$$

$$\mu^t_{f_j \to x_i}(x^c_i) = \sum_{\mathbf{x}\backslash x_i} p_j(y^c_j|\mathbf{x}_c) \prod_{i' \neq i} \mu^t_{x_{i'} \to f_j}(x^c_{i'}). \tag{15}$$

As the symbols take on values in the discrete set $\mathbb{B}$, the computation of $\mu^t_{f_j \to x_i}(x^c_i)$ in (15) requires exponential time to marginalize out the random vector $\mathbf{x}\backslash x_i$. To reduce complexity, x^c_i is considered as a continuous random variable and the message $\mu^t_{x_i \to f_j}(x^c_i)$ is approximated into a complex Gaussian function $\hat{\mu}^t_{x_i \to f_j}(x^c_i) = \mathcal{N}_{\mathbb{C}}(x^c_i; \bar{x}^t_{x_i \to f_j}, \bar{w}^t_{x_i \to f_j})$, where the parameters $\bar{x}^t_{x_i \to f_j}$ and $\bar{w}^t_{x_i \to f_j}$ are transmitted symbols' mean and variance, $\mu^t_{f_j \to x_i}(x^c_i)$ can be calculated by integration:

$$\begin{aligned} \mu^t_{f_j \to x_i}(x^c_i) &= \int_{\mathbf{x}\backslash x_i} p_j(y^c_j|\mathbf{x}_c) \prod_{i' \neq i} \mathcal{N}_{\mathbb{C}}(x^c_i; \bar{x}^{t-1}_{x_i \to f_j}, \bar{w}^{t-1}_{x_i \to f_j}) \\ &= \mathcal{N}_{\mathbb{C}}(h^c_{ji} x^c_i; \theta^t_{f_j \to x_i}, \gamma^t_{f_j \to x_i}) \end{aligned} \tag{16}$$

where the parameters $\theta^t_{f_j \to x_i}$ and $\gamma^t_{f_j \to x_i}$ are given by

$$\theta^t_{f_j \to x_i} = y_j - \sum_{i' \neq i} h_{ji} \bar{x}^t_{x_i \to f_j} = \underbrace{y_j - \sum_{i} h_{ji} \bar{x}^t_{x_i \to f_j}}_{\theta^t_{f_j}} + h_{ji} \bar{x}^t_{x_i \to f_j}, \tag{17}$$

and

$$\gamma^t_{f_j \to x_i} = \sigma^2_n + \sum_{i' \neq i} |h_{ji'}|^2 \bar{w}^t_{x_{i'} \to f_j} = \underbrace{\sigma^2_n + \sum_{i} |h_{ji}|^2 \bar{w}^t_{x_i \to f_j}}_{\gamma^t_{f_j}} - |h_{ji}|^2 \bar{w}^t_{x_i \to f_j}. \tag{18}$$

Then, by substituting $\mu^{t-1}_{f_j \to x_i}(x^c_i) = \mathcal{N}_{\mathbb{C}}(h^c_{ji} x^c_i; \theta^{t-1}_{f_j \to x_i}, \gamma^{t-1}_{f_j \to x_i})$ into (14), $\mu^t_{x_i \to f_j}(x^c_i)$ can be normalized as

$$\mu^t_{x_i \to f_j}(x^c_i) = \frac{\mathcal{N}_{\mathbb{C}}(x^c_i; \alpha^{t-1}_{x_i \to f_j}, \beta^{t-1}_{x_i \to f_j})}{\sum_{x^c_i \in \mathbb{B}} \mathcal{N}_{\mathbb{C}}(x^c_i; \alpha^{t-1}_{x_i \to f_j}, \beta^{t-1}_{x_i \to f_j})} \tag{19}$$

where $\alpha^{t-1}_{x_i \to f_j}$ and $\beta^{t-1}_{x_i \to f_j}$ are given by

$$\alpha^{t-1}_{x_i \to f_j} = \left(\sum_{j' \neq j} \frac{|h_{j'i}|^2}{\gamma^{t-1}_{f_{j'} \to x_i}} \right)^{-1} = \left(\underbrace{\sum_{j} \frac{|h_{ji}|^2}{\gamma^{t-1}_{f_j \to x_i}}}_{(\alpha^{t-1}_{x_i})^{-1}} - \frac{|h_{ji}|^2}{\gamma^{t-1}_{f_j \to x_i}} \right)^{-1} \tag{20}$$

$$\beta_{x_i \to f_j}^{t-1} = \alpha_{x_i \to f_j}^{t-1} \sum_{j' \neq j} \frac{h_{j'i}^* \theta_{f_{j'} \to x_i}^{t-1}}{\gamma_{f_{j'} \to x_i}^{t-1}} = \alpha_{x_i \to f_j}^{t-1} \left(\underbrace{\sum_{j} \frac{h_{ji}^* \theta_{f_j \to x_i}^{t-1}}{\gamma_{f_j \to x_i}^{t-1}}}_{\tau_{x_i}^{t-1}} - h_{ji}^* \theta_{f_j \to x_i}^{t-1} \right). \tag{21}$$

In (17–21), $\theta_{f_j}^t$, $\gamma_{f_j}^t$, $\alpha_{x_i}^t$, $\tau_{x_i}^t$ are used to reduce complexity. After Tth iterations, the probability of transmitted symbol from ith user been $x_i^c \in \mathbb{B}$ is

$$p(x_i^c) = \mathcal{N}_{\mathbb{C}}(x_i^c; \xi_{x_i}^T, \alpha_{x_i}^T), \tag{22}$$

where $\xi_{x_i}^T = \tau_{x_i}^T \alpha_{x_i}^T$. A hard estimate of symbol x_i^c can be obtained as

$$x_i^c = \arg\max p(x_i^c). \tag{23}$$

4 Complexity Analysis

The complexity is evaluated in terms of floating-point operations(FLOPs) as in [2]. A FLOP is assumed to be either a real multiplication or a real summation here. Transposition, Hermitian transposition, conjugate, and real/imaginary operator require no FLOP. It is also assumed that the operation of exponent can be implemented by a look-up table. Note that the multiplication of two complex numbers needs six FLOPs.

4.1 Complexity of MPD

In the preprocessing stage, the MPD algorithm requires $8MN$, $16MN^2$, $4N^2$ FLOPs for the calculation of $\mathbf{z}$, $\mathbf{G}$, $|\mathbf{G}|^2$ respectively. For computing the messages at nodes in each iteration, $\mathbb{E}(x_j)$ needs $14N$ FLOPs, $\mathrm{Var}(x_j)$ needs $18N$ FLOPs, $\{\mu_i, i \in \{1, 2, ...2N\}\}$ need $2N(4N-3)$ FLOPs, $\{\sigma_i^2,\ i \in \{1, 2, ...2N\}\}$ need $2N(4N-2)$ FLOPs, $\{\tilde{p}_i^t(s),\ i \in \{1, 2, ...2N\},\ s \in \mathbb{A}\}$ need $2N \times 4 \times 6$ FLOPs, the normalization of $\tilde{\mathbf{p}}$ needs $2N \times 7$ FLOPs. Finally, at the end of each iteration, damping of messages need $2N \times 4 \times 3$ FLOPs.

4.2 Complexity of AMP

In the preprocessing stage, the AMP algorithm requires $3MN$ FLOPs to compute $|\mathbf{H_c}|^2$. For computing the downward messages at the variable nodes, $\{\mu_{x_i \to f_j}^t(x_i^c), \forall i, \forall j\}$ and $\{\bar{x}_{x_i \to f_j}^t, \bar{w}_{x_i \to f_j}^t, \forall i, \forall j\}$ need $(11|\mathbb{B}| - 1)MN$ FLOPs and $(6|\mathbb{B}| + 1)MN$ FLOPs, respectively. For computing the upward messages at the function nodes, $\{\gamma_{f_j \to x_i}^t, \gamma_{f_j}^t, \theta_{f_j \to x_i}^t, \theta_{f_j}^t, \forall i, \forall j\}$ need $13MN$ FLOPs. For computing the messages at the variable nodes, $\{\alpha_{x_i \to f_j}^t, \alpha_{x_i}^t, \beta_{x_i \to f_j}^t, \tau_{x_i}^t, \forall i, \forall j\}$ need $16MN$-$3N$ FLOPs.

4.3 Complexity Comparison

Total complexity per iteration for MPD, AMP is listed in Table 1.

Table 1. Complexity Comparison of MPD and AMP

Algorithm	Preprocessing	Per iteration
MPD	$16MN^2$+$8MN$+$4N^2$	$16N^2$+$108N$
AMP	$3MN$	$(17\vert\mathbb{B}\vert + 29)MN$-$3N$

5 Simulation Results

Simulation results all base on the same assumption. System model (1) or (2) is used for simulation. For each simulated point, a minimum of 100 bit errors were counted.

First, a massive MIMO system with $N = 16$ users and $M = 128$ receiving antennas is considered. Figure 4 presents the BER performance of MPD and AMP with the number of iterations. It can be seen that 8 iterations are enough for both MPD and AMP to converge at SNR = 6 dB and SNR = 8 dB.

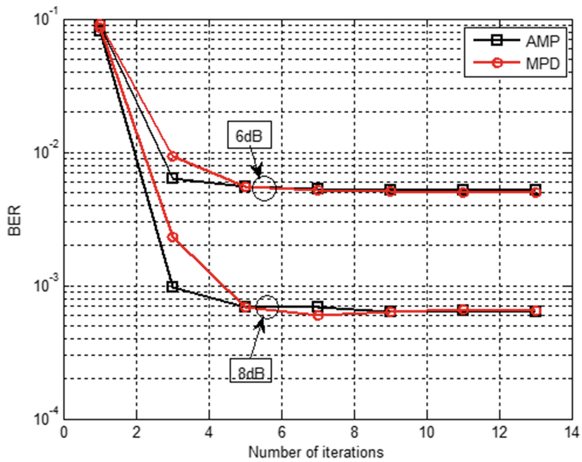

Fig. 4. BER performance versus number of iterations for MPD and AMP in a 16×128 system

Furthermore, Fig. 5 presented complexity versus number of users with fixed receiving antenna number, M = 128. The number of iterations is set to be 8. It is shown in Fig. 5 that AMP needs more than ten times FLOPs than MPD at lightly loaded system (let's say, $\varphi < 0.1$). However, as the load factor grows, the complexity of MPD increase faster than AMP. In [3], several variants of AMP that can significantly reduce complexity are proposed.

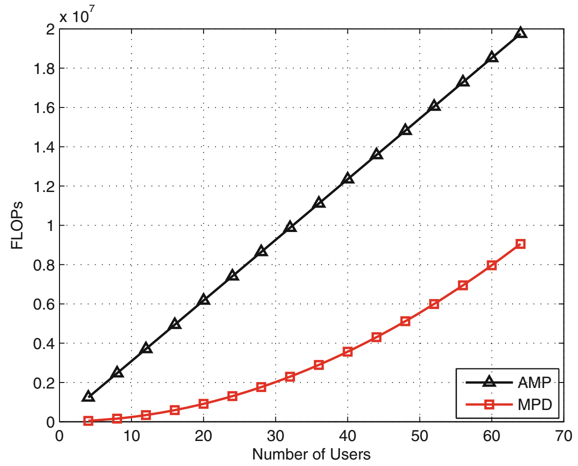

Fig. 5. Complexity versus number of users with $M = 128$

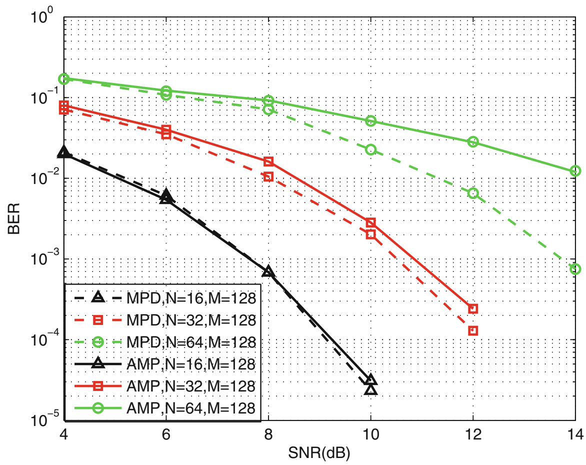

Fig. 6. BER performance of MPD and AMP for different values of N(=16, 32, 64) and fixed $M = 128$

Figure 6 presents the BER performance of MPD and AMP for a fixed number of receiver antennas at the BS ($M = 128$) and varying number of users ($N = 16$, 32, 64). The number of iterations is big enough to converge. It can be observed that when $N = 16$, MPD performs as well as AMP, when $N = 32$, MPD outperforms AMP by about 0.2 dB to achieve BER of 10^{-3}, when $N = 64$, MPD performs much better than AMP.

6 Conclusions

In this paper, we introduced a unified message passing graphical model based on factor graph to describe two detection algorithms dedicated to massive MIMO. Then detailed computation complexity is analyzed in detail in the terms of FLOP. Simulation results had shown that thanks to channel hardening phenomenon, the computation complexity of MPD is far less than AMP. In addition, as the load factor grows, MPD outperformed AMP. Therefore, MPD is a promising signal detection algorithm for massive MIMO.

Acknowledgement. This paper is supported by the National Natural Science Foundation of China (No. 61171099 & No. 61671080).

References

1. Larsson, E., Edfors, O., Tufvesson, F.: Massive MIMO for next generation wireless systems. IEEE Commun. Mag. **52**(2), 186–195 (2013)
2. Som, P., Datta, T., Srinidhi, N., Chockalingam, A.: Low-complexity detection in large-dimension MIMO-ISI channels using graphical models. IEEE J. Sel. Top. Sig. Process. **5**(8), 1497–1511 (2011)
3. Wu, S., Kuang, L., Ni, Z., Lu, J., Huang, D., Guo, Q.: Low-complexity iterative detection for large-scale multiuser MIMO-OFDM systems using approximate message passing. IEEE J. Sel. Top. Sig. Process. **8**(5), 902–915 (2014)
4. Narasimhan, T.L., Chockalingam, A.: Channel hardening-exploiting message passing (CHEMP) receiver in large-scale MIMO systems. IEEE J. Sel. Top. Sig. Process **8**(5), 847–860 (2013)
5. Guo, R., Li, X., Fu, W., Hei, Y.: Low-complexity signal detection based on relaxation iteration method in massive MIMO systems. Commun. China **12**(Supplement), 1–8 (2015)
6. Hochwald, B.M., Marzetta, T.L., Tarokh, V.: Multiple-antenna channel hardening and its implications for rate feedback and scheduling. IEEE Trans. Inf. Theory **50**(9), 1893–1909 (1909)
7. Pretti, M.: A message-passing algorithm with damping. J. Stat. Mech. Theory Exp. **2005**(11), 283–292 (2005)