

Throughput Maximization for Two-Hop Decode-and-Forward Relay Channels with Non-ideal Circuit Power

Hengjing Liang^{1,2}(✉), Xiaojie Wen², Chuan Huang^{1,2}, Zhi Chen^{1,2},
and Shaoqian Li^{1,2}

¹ National Key Laboratory of Science and Technology on Communications,
University of Electronic Science and Technology of China,
Chengdu, People's Republic of China

lianghj@hotmail.com, {huangch,chenzhi,lsq}@uestc.edu.cn

² Beijing Institute of Satellite Information Engineering,
Beijing, People's Republic of China
ziwen7189@aliyun.com

Abstract. This paper studies the throughput maximization problem for a two-hop relay channel considering non-ideal circuit power. In particular, the relay operates in a half-duplex manner, and the decode-and-forward (DF) relaying scheme is adopted. Considering the extra power consumption by the circuits, the optimal power allocation to maximize the throughput of the considered system over the infinite time horizon is investigated. By transforming the non-convex problem into the quasiconcave one, the closed-form solution shows that the source and the relay transmit with certain probability, which is determined by the average power budget, circuit power consumption, and channel gains. Numerical results show that the optimal power allocation scheme outperforms other conventional schemes.

Keywords: Green communication · Relay channel
Throughput maximization · Optimal power allocation
Decode-and-forward (Df)

1 Introduction

Green communication has drawn great attention during the past years. The energy consumed by communication networks constitute a large portion of total energy consumption, and will still go up in the future [1]. The growing cost of fossil fuel energy calls for both environmental and economical demands and motivations for the design of green communications.

Circuit energy consumption amounts for a significant part of the total energy consumption [2]. In order to save energy, green communication associated with

This work was supported by the National Natural Science Foundation of China under Grand No. 61401030.

non-ideal circuit power needs to be designed both energy and spectrum efficiently. In [3], a link adaptation scheme that balances circuit power consumption and transmission power was proposed in frequency-selective channels. Energy efficiency (EE) maximization problems with circuit energy consumption was also considered in orthogonal frequency division multiple access in [4]. A throughput optimal policy considering circuit power was proposed for point-to-point channels with energy harvesting transmitter [5].

Relaying has been considered as a promising technique to mitigate fading and extend coverage in wireless networks [6]. The capacity of a classical three-node relay channel, consisting of a source, a destination, and a single half-duplex DF relay, was investigated in [7]. Green communication problems in relay networks were discussed in [8–10]. In [8,9], energy minimization problems considering channel state information acquiring energy and signaling overhead were investigated in single relay selection schemes, respectively. In [10], non-zero circuit power consumption was considered for a total energy minimization problem in multihop relay channels. Most of the existing problems associated with the non-ideal circuit power consumptions focused on the total energy consumption minimization, whereas the throughput maximization problems were rarely investigated.

In this paper, throughput maximization for a two-hop half-duplex Gaussian relay channels considering non-ideal circuit power is studied over the infinite time horizon. The transceiver circuitry consumes a constant amount of power in the active mode and negligible power in the sleep mode. Under this setup, the optimal power allocations for the throughput maximization of the two-hop relay channel is investigated. By solving a max-min problem, the optimal power allocation shows that the source and the relay transmit either at a certain portion of time slots or constantly according to different average power budgets, circuit power consumptions, and channel power gains. Then, the average throughput and asymptotic analysis of the relay channel is studied. Finally, simulation results show that the optimal power allocation scheme outperforms other conventional schemes.

The rest of this paper is organized as follows. Section 2 introduces the signal model of this paper. Section 3 introduces the power consumption model. Section 4 studies the optimal power allocation and the throughput performances in low SNR and high SNR regimes. Section 5 evaluates the throughput performances by simulations and finally Sect. 6 concludes the paper.

2 Signal Model

This paper considers a two-hop relay channel as shown in Fig. 1, which consists of a source, a destination, and a half-duplex relay. The source sends information to the destination via the relay, and the direct link is unavailable. Slotted transmission scheme is adopted, and each time slot is with duration T .

Then, channel input and output relationship of the considered relay channel is introduced. Denote the channel coefficients of the source-relay and the relay-destination links as g_{SR} and g_{RD} , respectively, and then the channel power gains of the two links are given by

$$h_{\text{SR}} = |g_{\text{SR}}|^2, \quad h_{\text{RD}} = |g_{\text{RD}}|^2, \quad (1)$$

which are all constants across the time slots.

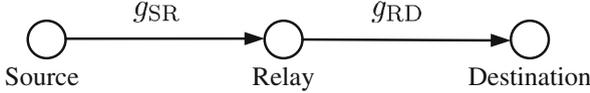


Fig. 1. A two-hop relay channel.

When the DF relaying scheme is adopted, it operates in a half-duplex manner (one time slot is then divided into two phases), and the information encoding and decoding processes are described as follows:

1. In the first phase of time slot i , the source transmits $x(i)$ to the relay with power $P_{\text{S}}(i)$;
2. Then, the received signal at the relay during the first phase of time slot i is given as

$$y_{\text{R}}(i) = g_{\text{SR}}x(i) + n_{\text{R}}(i), \quad (2)$$

where $n_{\text{R}}(i)$ is the independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) noise with zero mean and unit variance. Next, the relay decodes the source message, re-encodes it into a new signal $\tilde{x}(i)$, and forwards $\tilde{x}(i)$ to the destination with power $P_{\text{R}}(i)$.

3. Finally, the destination receives the signals over the second phase of time slot i , and the received signal $y_{\text{D}}(i)$ at the destination is given as

$$y_{\text{D}}(i) = g_{\text{RD}}\tilde{x}(i) + n_{\text{D}}(i), \quad (3)$$

where $n_{\text{D}}(i)$ is the i.i.d. CSCG noise with zero mean and unit variance.

For the purpose of exposition, consider the case that the two phases in one time slot are with equal length. Thus, the transmission rate for the DF relaying scheme at time slot i is given as [7]

$$R(i) = \frac{1}{2} \min \{ \mathcal{C}(P_{\text{S}}(i)h_{\text{SR}}), \mathcal{C}(P_{\text{R}}(i)h_{\text{RD}}) \}, \quad (4)$$

where $\mathcal{C}(x) = \log_2(1+x)$ denotes the capacity of the additive white Gaussian noise channel, where x is the signal-to-noise ratio (SNR) of the channel.

3 Power Consumption Model

In this subsection, power consumption model considering the non-ideal circuit power is discussed. The transceiver circuitry works in two modes: when a signal is transmitting, all circuits work in the *active mode*; and when there is no signal to transmit, they work in the *sleep mode*.

Active mode: The consumed power is mainly comprised of the transmission power and the circuit power. The transmission power is determined by the power allocation $P_S(i)$ and $P_R(i)$. The circuit power consists of the following two parts: the transmitting circuit power P_{ct} comes from the power consumed by the mixer, frequency synthesizer, active filter, and digital-to-analog converter [2]; and the receiving circuit power P_{cr} is composed of the power consumption of the mixer, frequency synthesizer, low noise amplifier, intermediate frequency amplifier, active filter, and analog-to-digital converter [2]. Constant circuit power model is considered in this paper, i.e., P_{ct} and P_{cr} are constants [2]. In the sequel, superscripts “S”, “R”, and “D” are added to P_{ct} and P_{cr} to distinguish the power consumed at the source, relay, and destination, respectively.

Sleep mode: It has been shown that the power consumption P_{sp} in the sleep mode is dominated by the leaking current of the switching transistors and is usually much smaller than that in the active mode [2]. Therefore, the power consumption in the sleep mode is set as $P_{sp} = 0$. It is worth pointing out that the results of this paper can be readily extended to the case of $P_{sp} \neq 0$ by deducting P_{sp} from the average power budget and the power consumption in the active mode.

In general, the circuit power consumed in the active mode is larger than that in the sleep mode, i.e.,

$$P_{cr} > P_{ct} > P_{sp}. \quad (5)$$

Thus, smartly operating between the two modes can potentially save a significant amount of energy.

Based on the power model discussed above, the power consumptions for the considered DF relaying is computed as: Denote α as the total circuit power consumption in the active mode, and it is the sum of the transmitting circuit power at the source and the relay, the receiving circuit power at the relay and the destination, i.e.,

$$\alpha = \frac{1}{2} (P_{ct}^S + P_{cr}^R) + \frac{1}{2} (P_{ct}^R + P_{cr}^D), \quad (6)$$

where the $\frac{1}{2}$ penalty is due to the half-duplex constraint for the considered relaying scheme. With the defined α and $P_{sp} = 0$, the total power consumption at time slot i is given as

$$P_{\text{total}}(i) = \begin{cases} 0 & P_S(i) = 0, P_R(i) = 0 \\ \frac{1}{2} (P_S(i) + P_R(i)) + \alpha & P_S(i) > 0, P_R(i) > 0, \end{cases} \quad (7)$$

where the $\frac{1}{2}$ penalty is also due to the half-duplex constraint for the considered relaying scheme. Then, the average power constraint is defined over N time slots, as N goes to infinity, i.e.,

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N P_{\text{total}}(i) \leq P_0, \quad (8)$$

where $P_0 \geq 0$ is the power budget.

4 Optimal Power Allocation

In this section, the optimal power allocation and throughput performance of the considered relaying scheme is studied.

4.1 Problem Formulation

The goal is to determine $\{P_S(i)\}$ and $\{P_R(i)\}$ such that the long term average throughput subject to the average power constraint defined in (8) is maximized over N time slots as $N \rightarrow \infty$, i.e., solve the following optimization problem

$$\mathcal{C}_{\text{DF}}(P_0) = \max_{\{P_S(i)\}, \{P_R(i)\}} \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N R(i) \quad (9)$$

$$\text{s.t.} \quad (8), P_S(i) \geq 0, P_R(i) \geq 0, \quad (10)$$

where $R(i)$ is given in (4).

It can be checked that the objective function (9) is nonnegative and concave [11]. Thus, it is easy to check [5] that the optimal power allocation of problem (9)–(10) is given as: Transmit with power $P_S(i) = P_S > 0$ and $P_R(i) = P_R > 0$ over p portion of time slots and keep silent for the rest of the slots, where P_S and P_R are constants. As a result, problem (9)–(10) can be reformulated as

$$\mathcal{C}_{\text{DF}}(P_0) = \max_{\{P_S, P_R, p\}} \frac{p}{2} \min \{ \mathcal{C}(P_S h_{\text{SR}}), \mathcal{C}(P_R h_{\text{RD}}) \} \quad (11)$$

$$\text{s.t.} \quad \left(\frac{1}{2} P_S + \frac{1}{2} P_R + \alpha \right) \cdot p \leq P_0, \quad (12)$$

$$0 \leq p \leq 1, P_S \geq 0, P_R \geq 0, \quad (13)$$

where (12) is obtained from (8).

It is easy to check that to achieve the optimal value of problem (11)–(13), constraint (12) must be satisfied with equality; otherwise, increasing the transmission power or transmission probability could still boost the average throughput. Thus it follows that the optimal transmission probability $p^* = \frac{2P_0}{P_S + P_R + 2\alpha}$. Hence, problem (11)–(13) can be further simplified as

$$\mathcal{C}_{\text{DF}}(P_0) = \max_{\{P_S, P_R\}} \frac{\min \{ \mathcal{C}(P_S h_{\text{SR}}), \mathcal{C}(P_R h_{\text{RD}}) \}}{P_S + P_R + 2\alpha} \cdot P_0 \quad (14)$$

$$\text{s.t.} \quad P_S + P_R \geq 2P_0 - 2\alpha, \quad (15)$$

$$P_S \geq 0, P_R \geq 0, \quad (16)$$

where (15) is obtained by substituting p^* into the constraint $0 \leq p \leq 1$.

4.2 Optimal Power Allocation

In this subsection, the optimal power allocation for problem (14)–(16) is investigated.

For objective function (14), the maximum value is achieved when $\mathcal{C}(P_S h_{SR}) = \mathcal{C}(P_R h_{RD})$; otherwise, decrease the source or relay power could still achieve the same average throughput. Thus, it can be obtained from $\mathcal{C}(P_S h_{SR}) = \mathcal{C}(P_R h_{RD})$ that $P_R = \frac{P_S h_{SR}}{h_{RD}}$. Substituting $P_R = \frac{P_S h_{SR}}{h_{RD}}$ into (14) and (15), problem (14)–(16) can be rewritten as

$$\mathcal{C}_{DF}(P_0) = \max_{P_S \geq 0} \frac{h_{RD} \mathcal{C}(P_S h_{SR})}{(h_{SR} + h_{RD}) P_S + 2h_{RD} \alpha} \cdot P_0 \quad (17)$$

$$\text{s.t. } (h_{SR} + h_{RD}) P_S \geq 2h_{RD} (P_0 - \alpha). \quad (18)$$

Since objective function (17) is a concave function divided by a linear function, it is quasiconcave over $P_S > 0$ [11]. Define

$$P_{ee} \triangleq \arg \max_{P_S \geq 0} \frac{P_C h_{RD} \mathcal{C}(P_S h_{SR})}{(h_{SR} + h_{RD}) P_S + 2h_{RD} \alpha}, \quad (19)$$

which achieves the maximum value of (17) without considering constraint (18). Then, the optimal power allocation of problem (9)–(10) and the average throughput for the considered relaying scheme are given in the following proposition.

Proposition 1. *The optimal power allocation for problem (9)–(10) is given as: Transmit with power value (P_S^*, P_R^*) over p^* portion of time slots and keep silent for the rest of slots, where*

$$P_S^* = \max \left(P_{ee}, \frac{2h_{RD}}{h_{SR} + h_{RD}} (P_0 - \alpha) \right), \quad (20)$$

$$P_R^* = \frac{h_{SR}}{h_{RD}} P_S^*, \quad (21)$$

and $p^* = \frac{2P_0}{P_S^* + P_R^* + 2\alpha}$. With the optimal power allocation, the average throughput $\mathcal{C}_{DF}(P_0)$ defined in (9) is given as

$$\mathcal{C}_{DF}(P_0) = \begin{cases} \frac{h_{RD} \mathcal{C}(P_{ee} h_{SR})}{(h_{SR} + h_{RD}) P_{ee} + 2h_{RD} \alpha} \cdot P_0 & 0 \leq P_0 \leq \frac{h_{SR} + h_{RD}}{2h_{RD}} P_S + \alpha \\ \frac{1}{2} \mathcal{C} \left(\frac{2h_{SR} h_{RD}}{h_{SR} + h_{RD}} (P_0 - \alpha) \right) & P_0 > \frac{h_{SR} + h_{RD}}{2h_{RD}} P_S + \alpha. \end{cases} \quad (22)$$

Proof. Since objective function (17) is quasiconcave over $P_S > 0$, it is increasing if $0 \leq P_0 \leq \frac{h_{SR} + h_{RD}}{2h_{RD}} P_S + \alpha$ and decreasing if $P_0 > \frac{h_{SR} + h_{RD}}{2h_{RD}} P_S + \alpha$. Thus, with the constraint (18), $P_S^* = \max \left(P_{ee}, \frac{2h_{RD}}{h_{SR} + h_{RD}} (P_0 - \alpha) \right)$ achieves the maximum value of problem (17)–(18).

Then, $P_R^* = \frac{h_{SR}}{h_{RD}} P_S^*$ is obtained from $P_S h_{SR} = P_R h_{RD}$. Substituting P_S^* and P_R^* into objective function (17), the average throughput $\mathcal{C}_{DF}(P_0)$ is obtained as (22).

Thus, Proposition 1 is proved. \square

Remark 1. Based on Proposition 1, it is worth noting that the transmission scheme given in Proposition 1 is to transmit with an on-off structure when the average power budget P_0 is small to maximize the EE of the considered relay channel, and transmit constantly when the average power budget P_0 is large to maximize the spectral efficiency. It is also worth noticing that P_{ee} can be efficiently obtained by a simple bisection search.

4.3 Asymptotic Analysis

In this subsection, the asymptotic performance for the considered relaying scheme is analyzed.

Low SNR Regime: As $P_0 \rightarrow 0$, the average throughput for the considered relaying scheme at the low SNR regime is given in (22):

$$\mathcal{C}_{DF}(P_0) = \frac{\mathcal{C}(P_{ee}h_{SR})h_{RD}}{(h_{SR} + h_{RD})P_{ee} + 2h_{RD}\alpha} \cdot P_0. \quad (23)$$

It is interesting to note that $\mathcal{C}_{DF}(P_0)$ is a linear function of the average power budget P_0 at the low SNR regime. The scaling factors $\frac{\mathcal{C}(P_{ee}h_{SR})h_{RD}}{(h_{SR} + h_{RD})P_{ee} + 2h_{RD}\alpha}$ is the maximum EE of the considered relaying scheme.

High SNR Regime: Based on the results in (22), as $P_0 \rightarrow \infty$, the average throughput of the considered relaying scheme at the high SNR regime is asymptotically given as

$$\mathcal{C}_{DF}(P_0) \approx \frac{1}{2} \log_2 \left(\frac{2h_{SR}h_{RD}}{h_{SR} + h_{RD}} (P_0 - \alpha) \right). \quad (24)$$

It is obvious that the multiplexing gain of the considered relaying scheme is $\frac{1}{2}$, which is due to the half duplex constraint.

5 Numerical Results

In this section, simulations are performed to compare the performances of the proposed optimal power allocation for the two-hop relay transmission (THRT) and various suboptimal schemes.

- DLT: denotes direct link transmission, whose power allocation is to transmit with power value $P_S^* = \max(P_{ee1}, P_0 - \alpha_A)$ over p^* portion of time slots and keep silent for the rest of the slots, where $P_{ee1} \triangleq \arg \max_{P_S > 0} \frac{\mathcal{C}(P_S h_{SD})}{P_S + \alpha_A}$ and $p^* = \frac{P_A}{P_S^* + \alpha_A}$, α_A is the total power consumption for DLT, and h_{SD} is the channel gain of the source-destination link.
- CDLT: denotes constant direct link transmission, which transmits only with the direct link every time slot. The power allocation for CDLT is given as

$$P_S^* = \begin{cases} P_0 - \alpha_A & P_0 > \alpha_A \\ 0 & \text{otherwise.} \end{cases} \quad (25)$$

- CTHRT: denotes constant two-hop relay transmission, where the source transmits to the destination via the relay every time slot. The power allocation for CTHRT is given as

$$(P_S^*, P_R^*) = \begin{cases} \left(P_S^*, \frac{h_{SR}}{h_{RD}} P_S^* \right) & P_0 > \alpha \\ (0, 0) & \text{otherwise,} \end{cases} \quad (26)$$

where $P_S^* = \max\left(P_{ee}, \frac{2h_{RD}}{h_{SR} + h_{RD}}(P_0 - \alpha)\right)$.

5.1 Average Power Budgets vs. Throughput

Figure 2 compares the performances of several transmission schemes at both low and high SNR regimes. The circuit power consumptions are set as $\alpha = 0.18$ W and $\alpha_A = 0.2$ W. The channel gains are set as $h_{SD} = 1$, $h_{SR} = 10$, $h_{RD} = 3$. It is easy to see that in the low SNR regime, the throughput curve of THRT outperforms other throughput curves, which suggests that THRT is more energy efficient than other transmission schemes. As P_0 increases, the curves of THRT and CTHRT coincide, and the curves of DLT and CDLT coincide. when $P_0 = 0.8$ W, throughput performance of THRT/CTHRT is about 0.3 b/s/Hz larger than that of DLT/CDLT.

In high SNR regime, the curves of THRT and CTHRT coincide, and the curves of DLT and CDLT coincide. The curves of DLT and CDLT outperforms the curves of THRT and CTHRT, which is due to the $\frac{1}{2}$ multiplexing gain of THRT and CTHRT. As P_0 increases, the throughput gap between the curves of

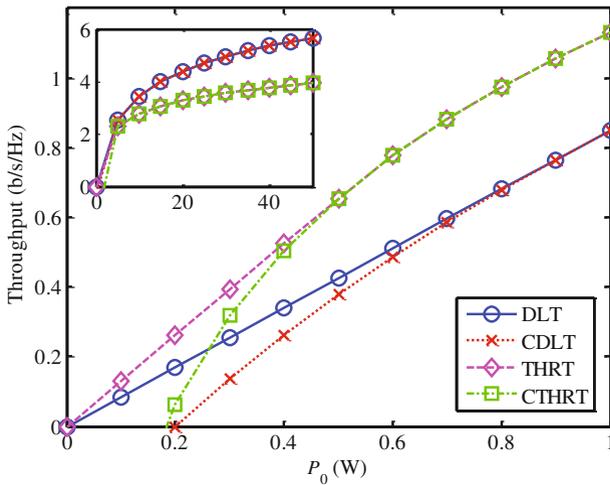


Fig. 2. The relationship between the power budget and the average throughput in low SNR and high SNR regimes.

DLT/CDLT and THRT/CTHRT enlarges, and the throughput performance of DLT/CDLT approaches twice as much as that of THRT/CTHRT.

5.2 Channel Gain h_{SR} vs. Throughput

In this subsection, the average throughput of THRT is compared with different channel gain h_{SR} and circuit power consumption α . The channel gain is set as $h_{RD} = 2$. The average power budget is set as $P_0 = 1$ W. The circuit power consumptions are set as $\alpha = 0.12, 0.16, 0.2, 0.24, 0.28$ W, respectively.

Figure 3 shows that the increase in h_{SR} leads to the increase in the average throughput. Besides, it can be concluded from the figure that as α increases, more power is consumed on the circuits, which leads to the decrease in the average throughput.

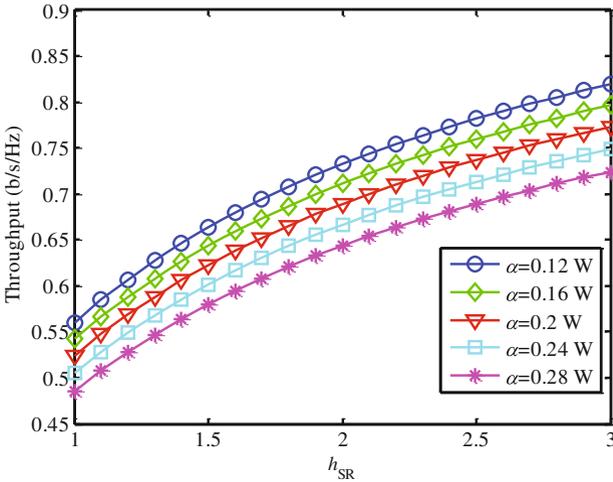


Fig. 3. The relationship between the channel gain h_{SR} and the average throughput with different circuit power consumption α .

6 Conclusion

In this paper, the throughput optimal power allocation for a two-hop relay channel with non-ideal circuit power was studied. By discovering the structure of the optimal power allocation, the non-convex problem was transformed into a quasi-concave problem. The closed-form solution showed that the source and the relay transmit with certain probability, which is determined by the average power budget, circuit power consumption, and channel gains. Then, asymptotic analysis was given, and finally, numerical results showed that the proposed optimal power allocation outperforms other suboptimal schemes.

References

1. Li, G.Y., Xu, Z., Xiong, C., Yang, C., Zhang, S., Chen, Y., Xu, S.: Energy-efficient wireless communications: tutorial, survey, and open issues. *IEEE Wirel. Commun. Mag.* **18**(6), 28–35 (2011)
2. Cui, S., Goldsmith, A.J., Bahai, A.: Energy-constrained modulation optimization. *IEEE Trans. Wirel. Commun.* **4**(5), 2349–2360 (2005)
3. Miao, G., Himayat, N., Li, G.Y.: Energy-efficient link adaptation in frequency-selective channels. *IEEE Trans. Commun.* **58**(2), 545–554 (2010)
4. Xiong, C., Li, G.Y., Zhang, S., Chen, Y., Xu, S.: Energy- and spectral-efficiency tradeoff in downlink OFDMA networks. *IEEE Trans. Wirel. Commun.* **10**(11), 3874–3886 (2011)
5. Xu, J., Zhang, R.: Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power. *IEEE J. Sel. Areas Commun.* **32**(2), 322–332 (2014)
6. Laneman, J.N., Tse, D.N.C., Wornell, G.W.: Cooperative diversity in wireless networks: efficient protocols and outage behavior. *IEEE Trans. Inf. Theory* **50**(12), 3062–3080 (2004)
7. Høst-Madsen, A., Zhang, J.: Capacity bounds and power allocation for wireless relay channels. *IEEE Trans. Inf. Theory* **51**(6), 2020–2040 (2005)
8. Madan, R., Mehta, N.B., Molisch, A.F., Zhang, J.: Energy-efficient cooperative relaying over fading channels with simple relay selection. *IEEE Trans. Wirel. Commun.* **7**(8), 3013–3025 (2008)
9. Zhou, Z., Zhou, S., Cui, J., Cui, S.: Energy-efficient cooperative communication based on power control and selective single-relay in wireless sensor networks. *IEEE Trans. Wirel. Commun.* **7**(8), 3066–3078 (2008)
10. Brante, G., Kakitani, M.T., Souza, R.D.: Energy efficiency analysis of some cooperative and non-cooperative transmission schemes in wireless sensor networks. *IEEE Trans. Commun.* **59**(10), 2671–2677 (2011)
11. Boyd, S., Vandenberghe, L.: *Convex Optimization*. Cambridge University Press, Cambridge (2004)