Multi-diffusion Degree Centrality Measure to Maximize the Influence Spread in the Multilayer Social Networks

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Abstract. In this work, we study the influence maximization in multilayer social networks. This problem is to find a set of k persons, called seeds, that maximizes the information spread in a multilayer social network. In our works, we focus in the determination of the seeds by proposing a centrality measure called *Multi-Diffusion Degree* (denoted by C_{dd}^{MLN}) based on *Independent Cascade* model. We consider the $top-K$ persons as the most influential. This centrality measure uses firstly, the diffusion probability for each person in each layer. Secondly, it uses the contribution of the first neighbors in the diffusion process. To show the performance of our approach, we compare it with the existing heuristics like *multi degree centrality*. With software R and *igraph package*, we show that *Multi-Diffusion Degree* is more performant than the benchmark heuristic.

Keywords: Centrality measure · Diffusion probability · Influence maximization · Mapping matrix · Multilayer social network

1 Introduction

Nowadays the social networks become more and more popular and varied. For example: *facebook*, *viadeo*, *linkedin*, *twitter*, $\cdots$. In these networks, often we find the same persons. A person may have an account in many social networks. With the unified authentication, the e-mail address, the similarity [1], $\cdots$, we can identify the same persons in different social networks. So, we can see these social networks as an aggregation of several networks Fig. 2. We call it *MultiLayer*

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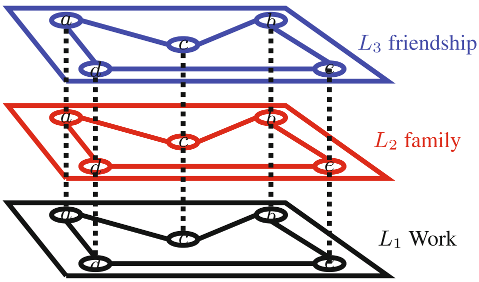

Fig. 1. $MLSN$ dialed of three layers

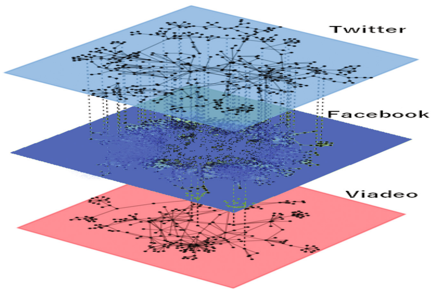

Fig. 2. Multilayer social network (3-layers)

$Social\ Network$ [2,3] and we denote $MLSN$. These networks appear in different contexts: according to their natures (online, offline, hybrid), according to them semantic natures (contact, communication, time, context, etc.). Social network can dial several types of relationships. In the analysis of social networks, it is important to differentiate between these links. So, each nature of link can be seen as a layer and all as a $MLSN$. In the Fig. 1, we have an example of $MLSN$ dialed of three types of relationships: family, work and friendship. We can explore these networks in many fields: in the field of air transport [2], in the theory of online games [4]. A complete example of multiplex (or multilayer) network model can be found in [5]. Several of these works are mainly theoretical. A MSN uses a multidimensional set where each dimension is a relationship between two persons. Recently, the MultiLayer Networks were applied to the study of strength of social ties in multilayer interactions [6]. The social networks analysis (SNA) which attracts many attention thanks to its varied fields of application. For example in marketing, the use of the online social networks gives a big potential. It is more effective than traditional techniques of marketing. For a good visibility of a new product, organizations can use the publicity word of mouth in the social networks [7,8]. This approach is known under the expression of influence maximization problem in the social networks [9]. The problem consists to find a small set of $k - persons$ (i.e. the seeds) in the social network that maximizes the influence spread in a small delay. But The social networks increase in a considerable way. The same persons in several networks can be identified with its e-mail, the unified authentication technology. So, these networks can be an aggregation of one social network with several types of relations. Each types of relation is considered as a layer. The resultant social network is known under the name of MultiLayer Social Network denoted by $MLSN$. The influence maximization problem can be applied in these networks. The goal is to find the most influential persons in the $MLSN$. Mathematically, we can define this problem by the Eq. (1)

$$S_k^* = argmax_{S \subseteq V,\ |S|=k}\ \sigma(S) \tag{1}$$

where:

- V is the set of persons of the MutiLayer Social Network ($MLSN$)
- S is subset of V
- $\sigma(S)$ is an activation function that gives the influenced number of persons by the seeds S
- S_k^* the set of persons that maximises the diffusion in $MLSN$

As application examples, a politician, during the electoral campaigns, wants that his program will be known by many voters. He can search in $MLSN$ (like $twitter \cup facebook \cup viadeo$, A divided network according to the type of links, etc.) the most influential individuals and proposes them his program. These individuals will influence their neighbors. These latter, in turn, influence their neighbors, $\cdots$. In marketing field, if a company wants to sell product, it may find the most influential costumers in a $MLSN$ (like $twitter \cup facebook \cup viadeo$) and gives them the product freely. These costumers will influence their neighbors, so now.

This paper is organized as follows. First, we will develop an introduction, a related work and we will give our contribution. Secondly, we will model the $MLSN$ with the graphs. Thirdly, we will propose an heuristic to give the most influential persons by developing the benchmark spread models. Finally, before to conclude and to give some future works, some simulations will be made to show the performance of our approach.

2 Related Work and Contribution

Several works are effected in influence maximization in the single and multilayer social networks. Some works focus in the spread models [9,10,21] while others in the determination of seeds. [12,16,17,20,22]. In this same point of view, some studies have been done in the goal to treat the network before to determine the seeds [13]. In this latter, the authors purpose to prevent the information feedback toward the seed nodes. Kempe et al. [16] are the first to attack the influence maximization problem. It's very difficult to choose the $k-persons$ that maximise the $\sigma(S)$ function. They show that, if $\sigma(S)$ function is modular and monotone, with the Greedy hill climbing algorithm under the LT and IC model, an approximation of 63% is guaranteed. Some heuristics like *degree*, *closeness*, $\cdots$ centrality [3,14], *eigenvector* centrality [15], consider the $top-k$ persons as the most influential in the network. But most of these works are applicable in the single networks. Yet, the results in single networks can not be used in multilayer networks. It is important to observe that results for single networks do not always generalize to multilayer networks. As an example, in [17], the authors show that the $k-shell$ index [18] proposed to identify the influential persons in single networks loses its effectiveness in interconnected networks, so they introduce a new measure which considers both structural and spreading properties. So far the works in the process of influence maximization in the multilayer social networks do not focus on diffusion probability and contribution of first neighbors. In this paper, to consider these deficient, we propose an heuristic called centrality of *Multi-Diffusion Degree* and we denote it by C_{dd}^{MLN}. This centrality measure is based on the work of [12].

3 Multilayer Social Networks Modeling

In this part, we give a modeling of multilayer social networks. It's very important to model the system before to exploit it. The goal of this modeling is to give an heuristic which gives the most influenced persons that maximizes the influence for a small delay. A system that has several interaction can be modeling by a multilayer network.

Example, Let, an aggregation of the social networks *facebook*, *viadeo* that represent respectively the first and the second layer (see Fig. 3).

In this same problem, the age group is very important to maximize the influence. The social network will be parted according to there age group. Each group is considered as a layer.

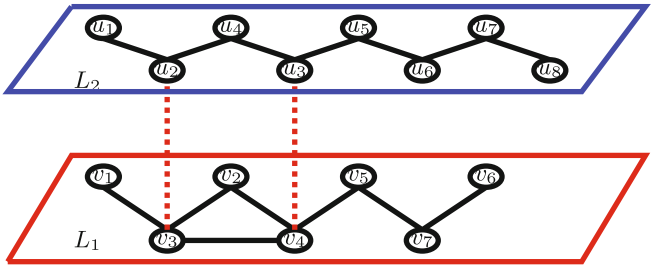

Fig. 3. Two layers social network

A person is modeling by a node and the link between two persons is modeling by an edge. The $k-th$ layer of a multilayer social network is represented by a graph denoted by $L_k(V_k, E_k)$.

- $V_k = (V_k^1, V_k^2, V_k^3, \cdots, V_k^{n_k})$ represents the set of persons of the layer k
- E_k represents the set of links of two persons of the layer k.

A multilayer social network is represented by a MultilLayer Network denoted by MLN. It is defined by MLN=$(L_1, L_2, L_3, \cdots, L_n, MM)$.

MM represents the union of mapping matrices between the layers. To build the mapping matrice between the layers $k-th$ and $k'-th$ denoted by $MM_{k'}^{k}$, we define an equivalence relation as follow:

$V_k^i \Re V_{k'}^j$ if:

1. $V_k^i \in L_k$, $V_{k'}^j \in L_{k'}$
2. (V_k^i - $V_{k'}^j$) a mapping edge (the same persons of the layer L_k and $L_{k'}$)

$\Re$ is an equivalence relation because it is reflexivity, The symmetry and transitivity. We consider the $MLSN$ of the Fig. 3, $L1$ represents $facebook$ and $L2$ $viadeo$. $v3$ and $u2$ represent the same person respectively in $facebook$ and $L2$ $viadeo$. We have also, $v4$ and $u3$ that represent the same person. $v7$ in $facebook$ hasn't a representative in the others layers. So, we have: $v3 \Re v3$ and $v3 \Re u2$. To build the mapping matrix $MM_{k'}^{k}$, we consider the equivalent relations between the persons of layers $k-th$ and $k'-th$. The mapping matrix $MM_{k'}^{k}$ of $k-th$ and $k'-th$ layers is defined below:

$$MM_{k'}^{k} = \begin{array}{c} \\ V_k^1 \\ V_k^2 \\ V_k^3 \\ \vdots \\ V_k^{n_k} \end{array} \begin{array}{c} \begin{array}{ccccc} V_{k'}^1 & V_{k'}^2 & V_{k'}^3 & \cdots & V_{k'}^{n_{k'}} \end{array} \\ \begin{pmatrix} a_{1,1}^{k,k'} & a_{1,2}^{k,k'} & a_{1,3}^{k,k'} & \cdots & a_{1,n_{k'}}^{k,k'} \\ a_{2,1}^{k,k'} & a_{2,2}^{k,k'} & a_{2,3}^{k,k'} & \cdots & a_{2,n_{k'}}^{k,k'} \\ a_{3,1}^{k,k'} & a_{3,2}^{k,k'} & a_{3,3}^{k,k'} & \cdots & a_{3,n_{k'}}^{k,k'} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n_k,1}^{k,k'} & a_{n_k,2}^{k,k'} & a_{n_k,3}^{k,k'} & \cdots & a_{n_k,n_{k'}}^{k,k'} \end{pmatrix} \end{array}$$

where $a_{i,j}^{k,k'} = 1$ if $V_k^i \Re V_{k'}^j$ else 0

After building MM, we define the equivalence class of each node v_k^i denoted by $class(v_k^i)$. For all mapping matrix $M_{k'}^{k}$ with k' a layer different to the layer k, if $MM_{k'}^{k}(v_k^i,\ v_{k'}^j) = 1$ then $v_{k'}^j$ belongs to $class(v_k^i)$. In Table 1, we have the mapping matrix of the $MLSN$ of the Fig. 3. As $\Re$ is an equivalence relation then it is réflexive and symmetric. In $MM = MM_1^1 \cup MM_2^1 \cup MM_1^2 \cup MM_2^2$.

MM_1^1 *and* MM_2^2 represent the unit matrix which do not have big importance on this model. MM_2^1 *et* MM_1^2 are transposed, they have the same information. So the mapping matrix is defined by the Eq. (2).

$$MM = \bigcup_{\substack{k,k' \in \{1 \cdots n\} \\ k \succ k'}} MM_{k'}^{k} \tag{2}$$

where n is the number of layers. The mapping matrices of the Fig. 3 is therefore reduced to the matrix MM_1^2.

Table 1. $MM = MM_1^2$

$$MM_1^2 = \begin{array}{c} \\ V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \end{array} \begin{array}{c} \begin{array}{cccccccc} u1 & u2 & u3 & u4 & u5 & u6 & u7 & u8 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

In MM, we have $class(u1) = \{u1\}$. it has not representative in the others layers. All elements of the column of $u1$ are 0. $class(v3) = \{v3, u2\}$ because $MM_2^1(v3, u2) = 1$.

4 *Multi-diffusion Degree* Centrality Measure

In this part, we propose metric to determine the persons that maximize the influence in the $MLSN$ based on the Independent Cascade Model (ICM) that is a spread model. First, we develop the two benchmark spread models. Then, we propose the heuristic that gives the seeds persons.

4.1 Spread Models

In the influence maximization problem, it's very important to have a spread model that is also a NP *hard* problem. In these works, there are two benchmark heuristics that are the Linear Threshold Model (LTM) [16] and the Independent Cascade Model (ICM) [16,21]. These two spread models are defined in single social networks. In [10], the authors propose an adaptation of the ICM in multilayer networks. In [11], the authors propose also an adaptation of the LTM.

LTM: In this model, a node u, inactive at time t, can be activated by its neighbors v active. Let $p_{v,u}$ the diffusion probability of v on u, let θ_u the activation Threshold of v (Social resistance) chosen randomly between [0,1]. If the sum of the influence factor of all active neighbors of v is bigger than the threshold activation θ_u, so u becomes active and forever. The activated node participates it also in the activation of its inactive neighbors. A recent works [11] in multilayer social networks, a node participates in the activation of all its neighbors in all layers. They define for each node v, an activation threshold denoted $Th(v)$ (Eq. 3)

$$Th(v) = a(\theta^G + \theta_v^L) \tag{3}$$

where a is an activation factor to adjust the threshold, θ^G the global threshold and θ_v^L the local threshold in the layer of v.

Mathematically, they define the LTM by the Eq. (5). The Eq. (4) represents the condition of Th.

$$\sum_{v\ active\ and\ v \in N(u)} p_{(v,u)} \prec 1 \tag{4}$$

$$\sum_{v\ active\ and\ v \in N(u)} p_{(v,u)} \succ Th(u) \tag{5}$$

ICM: In this model, a node u can try to activate one time these inactive neighbors. Let $p_{u,v}$ probability that the node u speeds up the node v. At time t, if v is active, it can activate these inactive neighbors at time $t+1$. In the $MLSN$, we have some works in the $MLSN$. The information spread in all layer via the node that are some representative [10]. A node can activate these neighbors in all layers.

4.2 *Multi-Diffusion Degree*

In this paper, we propose an heuristic for maximizing the influence in the $MLSN$ based on the works of [12] that use the neighborhoods of level ℓ in the single social networks. The proposed heuristic uses the neighborhoods of level 1. We call it $Multi\text{-}Diffusion\ Degree\ centrality$ and denote it C_{dd}^{MLN}. We propose a mathematical model for this heuristic. Let the node v_k^i, the $i-th$ node of the layer k. To determine its centrality measure ($C_{dd}^{MLN}(v_k^i)$), we define $P_{v_k^i}$ that is the diffusion probability of v_k^i in the layer k. In the Eq. (6), we give the contribution of v_k^i in the layer k in the diffusion of information.

$$P_{v_k^i} * C_d^k(v_k^i) \tag{6}$$

where $C_d^k(v_k^i)$ is the degree centrality measure (number of neighbors) of v_k^i in the layer k. But the v_k^i can have some representatives in the others layers. we determine the contribution of each representative. For each node v_k^i, we consider the contribution of all members of $class(v_k^i)$ in the information diffusion. So, the contribution is the sum of contributions of each representative of $class(v_k^i)$ (the same person in all layers). We use the equivalence relation defined above to determine these representatives. The Eq. (6) will be applied to each member of $class(v_k^i)$. The importance of a person may be different from one layer to another. The diffusion probability is defined for each layer. And the number of neighbors in the same layer is used (the degree centrality measure in the single networks). The Eq. (7) gives the contribution of v_k^i in all layers in the diffusion process.

$$\sum_{v_{k'}^{i'} \in class(v_k^i)} P_{v_{k'}^{i'}} * C_d^{k'}(v_{k'}^{i'}) \tag{7}$$

After we determine the contribution of v_k^i in all layers of the multilayer social network, we determine the contribution of the neighbors of each representative of $class(v_k^i)$ in the same layer and in the other layers.

In the same layer, we have the contribution of each neighbor of $v_k^{i'}$ in the Eq. (6). This contribution of all neighbors of $v_k^{i'}$ in the same layer is the sum of the individual contributions. It is defined by the Eq. (8).

$$\sum_{v_k^{i'} \in N^k(v_k^i)} P_{v_k^{i'}} * C_d^k(v_k^{i'}) \tag{8}$$

where $N^k(v_k^i)$ represents the set of neighbors of v_k^i in the layer k.

But, neighbor nodes can be in many layers. So, them contribution doesn't limit in them layer. They participate in the spread process in all layers. For each neighbor, we determine its equivalence class and we determine its contribution in its layer itself. Then, we define a set $N(class(v_k^i))$ that is the set of neighbors of all representatives of v_k^i. We have in the Eq. (9) the contribution of all neighbors of all representatives of v_k^i in them layer.

$$\sum_{v_{k'}^{j} \in N(class(v_k^i))} P_{v_{k'}^{i'}} * C_d^{k'}(v_{k'}^{i'}) \tag{9}$$

$v^j_{k'}$ also can have some representatives in other layers. So, we consider its equivalence class. In Eq. (10), we have the contribution of $v^j_{k''}$, neighbor of a representative in all layers.

$$\sum_{v^l_{k''} \in class(v^j_{k'})} P_{v^l_{k''}} * C_d^{k''}(v^l_{k''}) \tag{10}$$

In Eq. (9), we have the contribution of each neighbors of the representative but in the same layer where is the representative. Yet, a neighbor can have some representatives in the other layers. In Eq. (10), we have the contribution of a neighbor in all layers. The contribution of all the neighbors in all layers is defined in the Eq. (11).

$$\sum_{v^j_{k'} \in N(class(v^i_k))} \left(\sum_{v^l_{k''} \in class(v^j_{k'})} P_{v^l_{k''}} * C_d^{k''}(v^l_{k''}) \right) \tag{11}$$

The Eq. (11) presents some redundancies. Many persons can be some neighbors of several networks. So, they are evaluated many times. For example, in the Fig. 4, we have $class(V3) = \{V3, U2\}$ and $class(V4) = \{V4, U4\}$. $V3$ and $V4$ are neighbors in the layer $L1$. $U2$ and $U4$ are neighbors in the layer $L2$. So, them contribution will be calculated two times. To prevent these redundancies, we build a set that is the union of all class of each neighbor. So, in a set, there isn't repetition, so each neighbor will be evaluated one time. In the Eq. (12), we have the contribution of all neighbors without the redundancies.

$$\sum_{\substack{v^l_{k''} \in \bigcup class(v^j_{k'}) \\ v^j_{k'} \in N(class(V^i_k)}} P_{v^l_{k''}} * C_d^{k''}(v^l_{k''}) \tag{12}$$

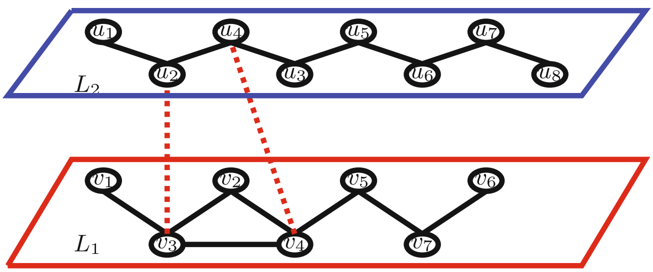

Fig. 4. Redundancy between two nodes

Now, the *Multi-Diffusion Degree centrality* of v^i_k denoted by $C_{dd}^{MLN}(v^i_k)$ is the sum of the contribution of v^i_k in all layers (Eq. 7) and the contribution of all neighbors i all layers (Eq. 4). We define this centrality measure in the Eq. (13). The $top-k$ will be considered the most influential persons in the $MLSN$.

$$C_{dd}^{MLN}(v_k^i) = \sum_{v_{k'}^{i'} \in class(v_k^i)} P_{v_{k'}^{i'}} * C_d^{k'}(v_{k'}^{i'}) + \sum_{\substack{v_{k''}^{l} \in \bigcup class(v_{k'}^{j}) \\ v_{k'}^{j} \in N(class(V_k^i))}} P_{v_{k''}^{l}} * C_d^{k''}(v_{k''}^{l}) \quad (13)$$

5 Experiments and Results

In many works, like [10,11], some proofs show that the multilayer social networks are more effective than if we consider them as a single network. So, to show the performance of our approach, we compare our approach to some heuristics defined in the multilayer networks. In our simulations, we give the influenced number of persons by our approach and the *multi-degree centrality* defined in [20]. We select the $top-k$ given by our approach and the benchmark approach under the IC defined in [10] model. We determine the number of influenced nodes by the two set of seeds.

Table 2. The characteristics of the both multilayer networks

Networks	Aggregation	RT layer	RP layer	MT layer
Can. 2013	N = 348537	N = 340349	N = 85867	N = 233735
	M = 991855	M = 496982	M = 83535	M = 411338
NYC. 2014	N = 102439	N = 94574	N = 7928	N = 50054
	M = 353496	M = 213754	M = 8063	M = 131679

5.1 Data

In our simulation, we use the two multilayer social networks $Cannes2013$[1] and $NYCLIMATEMARCH2014$ (see Footnote 1) [19]. The characteristics of these two networks are detailed in the Table 2. These two networks are extracted in the $Twitter$ network. They include three layers denoted by $\{RT, RP, MT\}$.

A user can ReTweet (RT) another user's tweet. This means that the user is endorsing a piece of information shared by the other user, and is rebroadcasting it to her/his own followers.

A user can RePly (RP) to another user's tweet. This represents an exchange from a user to another as a reaction of the information contained in a user's tweet.

A user can MenTion (MT) another user in a tweet. This represents an explicit share of a piece of information with the mentioned user.

[1] http://deim.urv.cat/manlio.dedomenico/data.php.

Here, for each event, we build a multilayer network composed by L = 3 layers $\{RT, RP, MT\}$, corresponding to the three actions that users can perform in Twitter, and N nodes, being N the number of Twitter users interacting in the context of the given event. A directed edge between user i and user j on the RT layer is assigned if i retweeted j. Similarly, an edge exists on RP layer if user i replied to user j, and on MT layer if i mentioned j.

5.2 Parameters and Benchmarks

We use the *multi-degree* centrality heuristic adapted by M. Magnani et al. [20] as benchmark. In this heuristic, they search all neighbors of v and its representatives in all layers. It is given by the Eq. (14).

$$\delta(v) = |P_{eqIMi}(\bigcup_{i \in [1..n], (u,v) \in E_i} u)| \tag{14}$$

In this equation, we determine the equivalence class of the node v. For each representative, they use the degree centrality defined by Kempe et al. [16] for each representative in of each layer where is this equivalent node. To show the performance of our approach, we determine the seeds given by our heuristic by selecting the $top-k$ and the seeds given by the benchmark heuristic by selecting also the $top - k$. We measure the number of influenced nodes by these two heuristics by using the IC, defined in [10], as spread model. A spread probability is generated for each node randomly between 0 and 1. The iteration numbers is fixed at 3 for *NYClimateMarch*2014 network and 4 for *Cannes*2013 network.

5.3 Results

To show the performance of our model, we determine the $top-k$ (seed set) given by our heuristic and that defined in [20] (benchmark model). We determine the influenced number of nodes by each seed set under the IC model. In the Figs. 5 and 6, we use the multilayer social network *Cannes*2013. In the Figs. 7 and 8, we use the multilayer social network *NYClimateMarch*2014.

In the Figs. 5 and 7, we have determined the influenced number of nodes according to the number of seeds given by our heuristic and the benchmark heuristic. Here, we take various seeds S_k given by these two approaches. The values of k varies between 5 and 30. After the number of iterations fixed for each network, we determine the number of influenced nodes by these two seed sets. For the two experiment networks, in each set S_k, our heuristic spreads more information than the base model. In the theoretical part, we use of neighbors of level 2. The seed nodes are the nodes that are the most neighbors of levels one and two. This theoretical result is justified by the simulations of these two figures.

In the Figs. 6 and 8, we determine the number of influenced nodes according to the iteration numbers by using the S_{30} sets (given by our heuristic and the benchmark heuristic). The results show that for each iteration, our heuristic gives

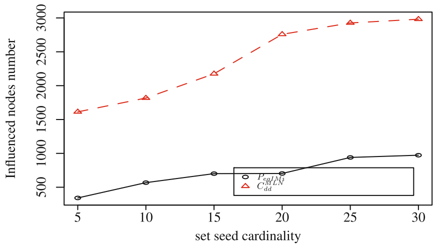

Fig. 5. Number of influenced nodes according to the seeds given by C_{dd}^{MLN} and P_{eqIMi} heuristics

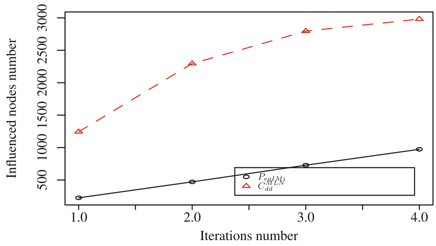

Fig. 6. Number of influenced nodes according to the iteration numbers with 30 seeds given by C_{dd}^{MLN} and P_{eqIMi} heuristics

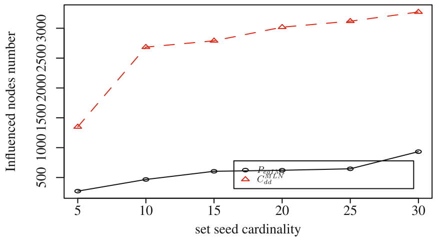

Fig. 7. Number of influenced nodes according to seeds given by C_{dd}^{MLN} and P_{eqIMi} heuristics

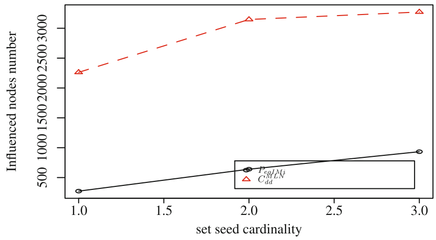

Fig. 8. Number of influenced nodes according to the iteration numbers with 30 seeds given by C_{dd}^{MLN} and P_{eqIMi} heuristics

better results than the benchmark heuristic. In the theoretical part, a probability that a node spreads the information was taken into account. We don't only look at the node that has more neighbors but the one that does more pressure on its neighbors. From the first iteration, our heuristic spreads more information than benchmark heuristic.

6 Conclusion

In this paper, we treat the influence maximization problem in multilayer social networks. First, we model this network by using the equivalence class and generated the mapping matrices between all layers. Then, we defined a new heuristic that uses neighbors of level 2 and a spread probability of each person in each layer. This heuristic is based on the propagation model IC. The software R and *igraph* package are used to show the performance of our approach. In the future work, it is interested to adapt this centrality measure under LT model.

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